A New Method for Evaluating Lower and Upper Bounds of Failure Probability in Redundant Truss Structures

A New Method for Evaluating Lower and Upper Bounds of Failure Probability in Redundant Truss Structures

Yoshisada MUROTSU*, Hiroo OKADA*, Kazukuni NIWA**
and Shigeru MIWA***

(Received June 15, 1979)

Failure criteria of redundant truss structures are generated by using Matrix Method. Failure probability of the structure is estimated with its lower and upper bounds. An efficient method is proposed which evaluates the bounds by selecting only the dominant modes of failure, repeating branching and bounding operations in search of the modes. Numerical examples are provided to demonstrate the validity of the proposed method.

1. Introduction

Many studies have been made of reliability analysis of structural systems[1)~8)]. However, they are limited to simple types of structures. This may be caused by the following two reasons, viewed from methodologies required for reliability analysis. There are many modes of failure in structural systems and generally they are neither statistically independent nor exclusive events. Consequently, in order to exactly evaluate the failure probability, some methods are needed for calculating joint probabilities with their correlation considered. Another reason is due to a fact that there are no systematic procedures developed for generating the failure criteria of the structures – particularly of those with redundancy. For the former problem, approximation methods are proposed by the present authors and their validity has been demonstrated.[9),10),11)] An Approach to the latter problem is also proposed which generates the failure criteria by using Matrix Method. Based on the criteria, the failure probability is estimated by evaluating its lower and upper bounds.[12)] However, there is some room left to be improved for the evaluation of the bounds.

In this paper, an efficient method is proposed which selects only the dominant modes of failure by repeating branching and bounding operations in search of the modes and finally evaluates the lower and upper bounds by calculating their failure probabilities. To illustrate the applicability of the proposed method, numerical examples are provided of a statically indeterminate 16-member truss structure with three degrees of redundancy.

* Department of Naval Architecture, College of Engineering.
** Graduate Student, Department of Aeronautical Engineering.
*** Graduate Student, Department of Naval Architecture.

2. Generation of Failure Criteria of Truss Structures

There are many modes of failure in structural systems, depending on the configuration of the systems, the loading conditions, *etc.* It is not easy to find all possible modes of failure for the structures composed of many members. A systematic method to generate the failure criteria is presented in the following of the structural members and the structural systems.

Consider a truss structure which consists of n members. The configuration and materials to be used are also assumed to be specified. Failure of a member occurs when the internal force exceeds the strength of the member. The safety margin defined as difference between the strength and the internal force is expressed in the form:

$$Z_i = R_i(C_{yi}, A_i) - S_i(A_1, A_2, \cdots, A_n; L_1, L_2, \cdots, L_l) \tag{1}$$

where
- Z_i : safety margin of the i-th member
- R_i : strength of the i-th member
- S_i : internal force of the i-th member
- C_{yi} : allowable stress of the i-th member determined by the material to be used
- A_i : cross sectional area of the i-th member
- L_j : external load applied to the structure $(j = 1, 2, \cdots, l)$
- n : number of members
- l : number of loads

The strength R_i in equation (1) is easily determined by specifying the material and dimension of the member. On the other hand, the internal force S_i is complex to evaluate, and thus it is derived by applying Matrix Method[13].

Let $\{Q_i\}$ and $\{\delta_i\}$ denote the nodal force and displacement vectors of the i-th member in the local coordinate system shown in Fig. 1. The member stiffness equation is written as

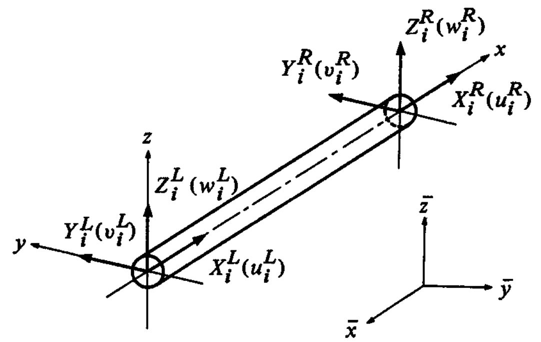

Fig. 1. Local and global coordinate systems.

$$\{Q_i\} = [k_i]\{\delta_i\} \tag{2}$$

where $$\{Q_i\}^T = (X_i^L, Y_i^L, Z_i^L, X_i^R, Y_i^R, Z_i^R) \tag{3}$$

$$\{\delta_i\}^T = (u_i^L, v_i^L, w_i^L, u_i^R, v_i^R, w_i^R) \tag{4}$$

and $[k_i]$ is the member stiffness matrix given by

$$[k_i] = \frac{E_i A_i}{l_i}\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

In equation (5), $E_i{}'$ and l_i indicate a modulus of elasticity and length of the i-th member. The displacement and nodal force vectors are related to those referred to the global coordinate system by the transformation:

$$\begin{aligned} \{\delta_i\} &= [T_i]\{d_i\} \\ \{Q_i\} &= [T_i]\{\bar{Q}_i\} \end{aligned} \tag{6}$$

where $\{d_i\}$ and $\{\bar{Q}_i\}$ are the displacement and nodal force vectors of the i-th member referred to the global coordinate system and $[T_i]$ the transformation matrix. Consequently, the member stiffness equation in the global coordinate system is given by

$$\{\bar{Q}_i\} = [\bar{k}_i]\{d_i\} \tag{7}$$

where

$$\begin{aligned} [\bar{k}_i] &= [T_i]^{-1}[k_i]\,[T_i] \\ &= [T_i]^T[k_i]\,[T_i] \end{aligned}$$

The stiffness equations of all the members are formed in the similar manner and they are transformed from the local coordinate system to the global. Next, the global nodal displacement vector $\{d\}$ is formed by rearranging the displacement vectors $\{d_i\}$ of the individual members. The global nodal force vector $\{L\}$ corresponding to $\{d\}$ is also defined, which includes the applied external loads. Further, the total structure stiffness matrix $[K]$ is generated by superimposing the individual member stiffness matrices. Then, the total structure stiffness equation is written as

$$[K]\{d\} = \{L\} \tag{8}$$

where $[K]$ is given by

$$[K] = \sum_{i=1}^{n}{}'\,[\bar{k}_i] = \sum_{i=1}^{n}{}'\,[T_i]^T[k_i]\,[T_i] \tag{9}$$

In equation (9), the summation $\sum_{i=1}^{n}{}'$ denotes superimposing the member stiffness matrices of all the members. The displacement vector of the i-th member $\{\delta_i\}$ in the local coordinate system is related to that in the global coordinate system $\{d_i\}$ by the transformation (6), while $\{d_i\}$ is determined by solving equation (8). Consequently, the nodal force $\{Q_i\}$ is given in the form:

$$\{Q_i\} = [C_i]\{L\} \tag{10}$$

where $[C_i] = [k_i][T_i][K_i]^{-1}$, and $[K_i]^{-1}$ is the matrix formed by extracting the elements concerned with the i-th member from the matrix $[K]^{-1}$.

In case of the truss structure, the internal force to be considered is the nodal force X_i^R and it is written in the form:

$$S_i = X_i^R = \sum_{i=1}^{l} b_{ij}L_j \tag{11}$$

where b_{ij} is the element of the matrix $[C_i]$ referred to X_i^R and L_j. It should be noted that the coefficients b_{ij} of a statically determinate truss are constant while those of a statically indeterminate truss become functions of the cross sectional areas A_i of the members.

Substituting equation (11) into equation (1) enables the failure criterion of the i-th member to be determined as follows:

$$Z_i = \text{sign}\left(\sum_{j=1}^{l} b_{ij}L_j\right) \cdot \left[C_{yi}A_i - \sum_{j=1}^{l} b_{ij}L_j\right] \tag{12}$$

where sign$(\cdot)$ denotes the sign of $(\cdot)$. The yield stress $\pm\sigma_Y$ is taken as the allowable stress C_{yi} in equation (12) when the member fails in tension or compression failure while the buckling stress $-\sigma_C$ is taken when instability in the compression member is considered.

Next, consider a failure criterion of the structural system. In case of a statically determinate truss, the structural failure arises when any one member is subject to failure. Consequently, the failure criterion is given by

$$Z_i \leqq 0 \qquad \text{for} \quad \forall i \in \{1, 2, \cdots, n\} \tag{13}$$

In case of a statically indeterminate truss, failure in any one member does not necessarily result in complete failure of the structural system. Structural failure is assumed to occur in the following manner. When any one member fails, redistribution of the internal forces arises among the members in survival and a member next to fail is determined. After repeating the similar processes, complete failure of the structural system results when the members up to some specified number p_k are lost. Complete failure of the structure is determined by investigating singularity of the total structure stiffness matrix $[K^{(p_k)}]$ formed with the members in survival. That is, the criterion for complete failure is given by

$$|K^{(p_k)}| = 0 \tag{14}$$

where $|(\cdot)|$ is the determinant of a matrix $(\cdot)$.

Consider now a combination of the members to determine complete failure of the truss, which is denoted as $\{r_{k1}, r_{k2}, \cdots, r_{kp_k}\}$ according to the sequential order of failure $(r_{kp_k} \in \{1, 2, \cdots, n\},\ k \in \{1, 2, \cdots, m\})$. m is the total number of the combinations of the members to cause complete failure of the structure, which depends on the configuration of the structure, the number of the members n and the degrees of redundancy s. The upper bound of m is given by $m = {}_nC_{(s+1)} = n(n-1)\cdots(n-s) / \{(s+1)s \cdots 2 \cdot 1\}$. In the state where p members, *i.e.*, $r_{k1}, r_{k2}, \cdots, r_{kp}$ $(p < p_k)$, fail, the residual strength of those members are added to the nodes as external forces, corresponding to the types of failure. Then, stress analysis of the structure composed of the members in survival is carried out once again by applying Matrix Method, and the safety margins of the members in survival are determined as follows:

$$Z_i^{(p)} = \mathrm{sign}\,(\sum_{j=1}^{l} b_{ij}L_j^{(p)}) \cdot [C_{yi}A_i - \sum_{j=1}^{l} b_{ij}L_j^{(p)}] \tag{15}$$

$$\text{for } i \in \{1, 2, \cdots, n\},\ i \bar{\in} \{r_{k1}, r_{k2}, \cdots, r_{kp}\}$$

where $L_j^{(p)}$ represents the resultant external forces with the artificial forces added corresponding to the residual strengths of the members in failure. For example, when a member of a brittle material fails in tension, the residual strength is put to zero while in case of a ductile material the strength of the member $R_{(\cdot)}$ is taken as the residual strength.

As mentioned above, complete failure of the redundant truss occurs if all of p_k members are subject to failure. Hence, a criterion for complete failure of the structure is expressed by using the safety margins of the members to fail as

$$Z_{r_{kp}}^{(p-1)} \leqq 0 \qquad (p = 1, 2, \cdots, p_k) \tag{16}$$

The criterion (16) depends on the sequential order of failure of the members. Therefore, it is seen that there exist $(p_k!)$ failure modes for a combination of the members $\{r_{k1}, r_{k2}, \cdots, r_{kp_k}\}$ which make complete failure of the statically indeterminate truss. Consequently, the total number of the failure modes becomes $\sum_{k=1}^{m} p_k!$.

3. Method for Evaluating Lower and Upper Bounds

Let $W_{k1}, W_{k2}, \cdots, W_{kp_k!}$ be $(p_k!)$ failure events corresponding to a combination of the members $\{r_{k1}, r_{k2}, \cdots, r_{kp_k}\}$ which make complete failure of the redundant truss. When the sequential order of failure of the members is $r_{k1}, r_{k2}, \cdots, r_{kp_k}$, the failure event W_{kq} is expressed as

$$W_{kq} = F_{r_{k1}}^{(0)} \cap F_{r_{k2}}^{(1)} \cap \cdots \cap F_{r_{kp_k}}^{(p_k-1)} \qquad \forall q \in \{1, 2, \cdots, p_k!\} \tag{17}$$

where $F_{r_{kp+1}}^{(p)}$ is the event that the safety margin $Z_{r_{kp+1}}^{(p)}$ of the member r_{kp+1} given by equation (15) becomes negative. In the statically indeterminate truss, complete failure of the structure arises when any one event W_{kq} $(k = 1, 2, \cdots, m, q = 1, 2, \cdots, p_k!)$ among $\sum_{k=1}^{m} p_k!$ events occurs. Consequently, the failure probability is given by

$$P_f = \text{Prob}\ [\bigcup_{k=1}^{m} \bigcup_{q=1}^{p_k!} W_{kq}] \tag{18}$$

In order to exactly calculate equation (18), all the failure events must be specified by investigating all the possible failure paths to complete failure of the truss. However, it is too complicate to carry out in practice. Therefore, instead of directly calculating equation (18), the failure probability is estimated by evaluating its lower and upper bounds.

A lower bound P_{fL} is estimated as

$$P_{fL} = \max_{k,q} \text{Prob}\ [W_{kq}] \tag{19}$$

An upper bound P_{fU} is evaluated by the probability that any one member of the statically indeterminate truss fails:[12)]

$$P_{fU} = \sum_{i=1}^{n} \text{Prob}\ [F_i] \tag{20}$$

Equation (20) is easily evaluated by calculating one-dimensional probability distribution function.

When all the modes of failure are counted, another upper bound is evaluated by

$$P_{fU} = \sum_{k=1}^{m} \sum_{q=1}^{p_k!} \text{Prob}\ [W_{kq}] \tag{21}$$

An algorithmic procedure is given below which systematically evaluates a lower and upper bounds by selecting only the dominant modes of failure. An underlying idea of the algorithm is based on the concept of Branch and Bound method[14)] in combinatorial programming problems.

<u>Step 0</u> (Setting initial values)

Specify the number of the members n and the degrees of redundancy s. Let k be a parameter to denote the stage of the structural failure. The number of the members in survival in the k-th stage is designated as $\bar{t}_k$. Set $k = 1$, $\bar{t}_k = n$, $P_{fL} = 0$ and $P'_{fU} = 0$.

<u>Step 1</u> (Selecting a member to fail)

i) If $\bar{t}_k > 0$, generate the safety margins $Z_i^{(k-1)}$. Select a member next to fail by the criterion:

$$\text{Prob}\ [F_{r_1}^{(0)}] = \max_i \text{Prob}\ [F_i^{(0)}] \qquad \text{for}\ \ k = 1 \tag{22}$$

$$\text{Prob}\ [F_{r_1}^{(0)} \cap F_{r_k}^{(k-1)}] = \max_i \text{Prob}\ [F_{r_1}^{(0)} \cap F_i^{(k-1)}] \quad \text{for}\ \ k \geq 2 \tag{23}$$

Put $\bar{t}_k = \bar{t}_k - 1$ and go to *Step* 2.

ii) If $\bar{t}_k = 0$, put $k = k - 1$.

a) If $k > 0$, go to *Step* 4.

b) If $k = 0$, go to *Step* 5.

Step 2 (Checking structural failure)

i) If $s = 0$, go to *Step* 5.

ii) If $s > 0$, check the singularity of the total stiffness matrix $[K^{(k)}]$ formed with the members in survival, *i.e.*, those except the members $r_1, r_2, \cdots, r_k$ in failure. When $|[K^{(k)}]| = 0$, structural failure occurs and then go to *Step* 3. Otherwise, go to *Step* 1 with $k = k + 1$ and $\bar{t}_k = n - k + 1$.

Step 3 (Calculating failure probability of a failure mode)

Calculate the failure probability of a failure mode by

$$P'_{fL} = \text{Prob}\ [F_{r1}^{(0)} \cap F_{r2}^{(1)} \cap \cdots \cap F_{r_k}^{(k-1)}] \tag{24}$$

Put $P'_{fU} = P'_{fU} + P'_{fL}$. If $P'_{fL} > P_{fL}$, set $P_{fL} = P'_{fL}$ and go to *Step* 4. Otherwise, go to *Step* 1.

Step 4 (Determing members to eliminate)

Eliminate the members from consideration which satisfy

$$\text{Prob}\ [F_i^{(0)}]\ /P_{fL} < 10^{-\gamma} \qquad \text{for } k = 1 \tag{25}$$

$$\text{Prob}\ [F_{r1}^{(0)} \cap F_i^{(k-1)}]\ /P_{fL} < 10^{-\gamma} \qquad \text{for } k \geqq 2 \tag{26}$$

where γ is a given constant (see Appendix). The maximum contribution of the excluded failure modes to the structural failure probability is given by Prob $[F_i^{(0)}]$ for $k = 1$ and Prob $[F_{r_1}^{(0)} \cap F_i^{(k-1)}]$ for $k \geqq 2$. Consequently, put

$$P'_{fU} = P'_{fU} + \text{Prob}\ [F_i^{(0)}] \quad \text{for } k = 1 \tag{27}$$

$$= P'_{fU} + \text{Prob}\ [F_{r_1}^{(0)} \cap F_i^{(k-1)}] \quad \text{for } k \geqq 2 \tag{28}$$

Go to *Step* 1 with $\bar{t}_k = \bar{t}_k - t_k$

Step 5 (Determing an upper bound)

i) If $s = 0$, put $P_{fU} = P_f^{(1)} = \sum_{i=1}^{n} \text{Prob}\ [F_i^{(0)}]$,

$$P_{fL} = \max_i \text{Prob}\ [F_i^{(0)}]\ .$$

ii) If $s > 0$, compare P'_{fU} to $P_f^{(1)} = \sum_{i=1}^{n} \text{Prob}\,[F_i^{(0)}]$.
If $P'_{fU} < P_f^{(1)}$, $P_{fU} = P'_{fU}$. Otherwise, $P_{fU} = P_f^{(1)}$.

P_{fL} and P_{fU} thus obtained yield a lower and upper bounds of the failure probability of the structure. A flow chart illustrating the algorithmic procedure mentioned above is given in Fig. 2.

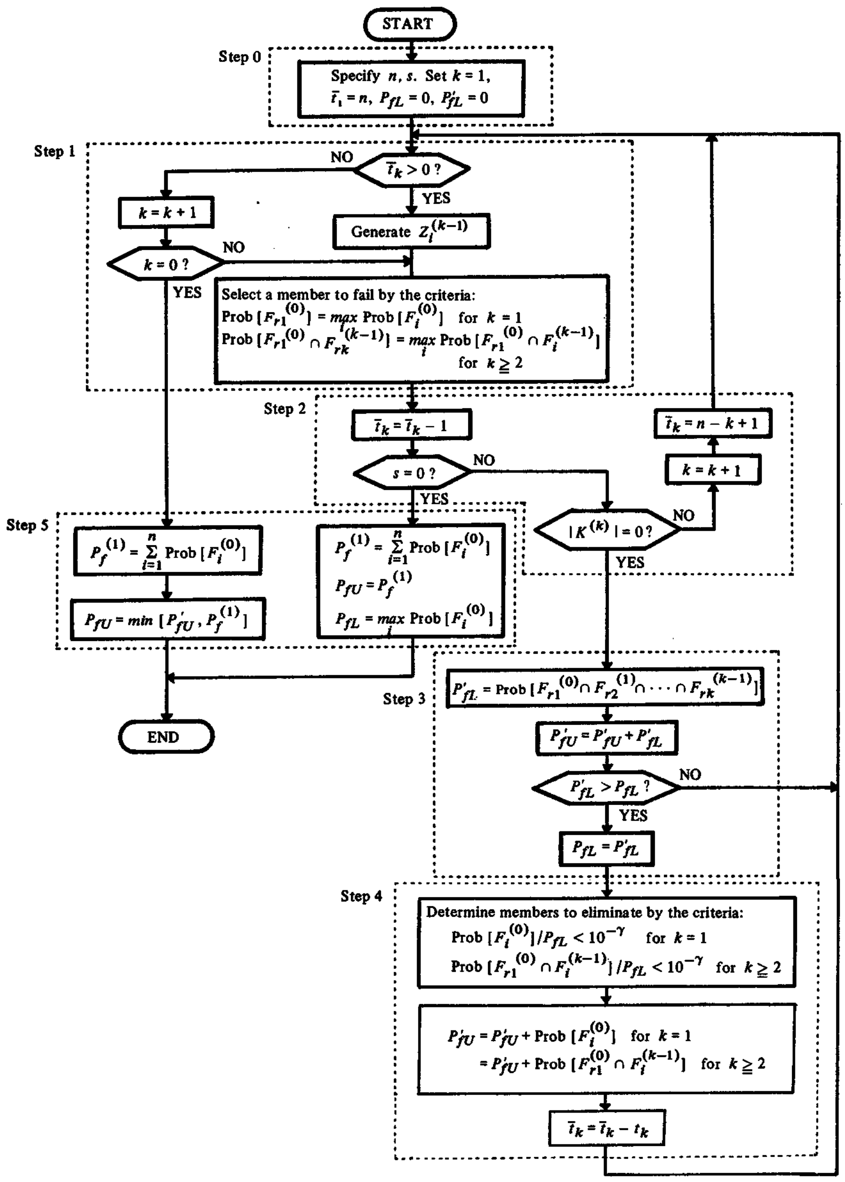

Fig. 2. Flow chart illustrating algorithmic procedure to evaluate lower and upper bounds.

4. Numerical Examples

Numerical examples are presented to demonstrate the applicability of the methods proposed for generating the failure criteria and for performing reliability analysis. It is assumed that allowable stresses of the members and the applied external loads are statistically independent Gaussian random variables, while the dimensions of the members such as cross sectional area, length, *etc.* are deterministic. Then, the safety margins given by equations (12) and (15) become Gaussian random variables and the failure probability is evaluated by calculating multi-dimensional Gaussian distribution functions.[9),10),11)]

Consider a 16-member truss structure with three degrees of redundancy shown in Fig. 3. The data concerned are given in Table 1. Buckling failure modes are tentatively

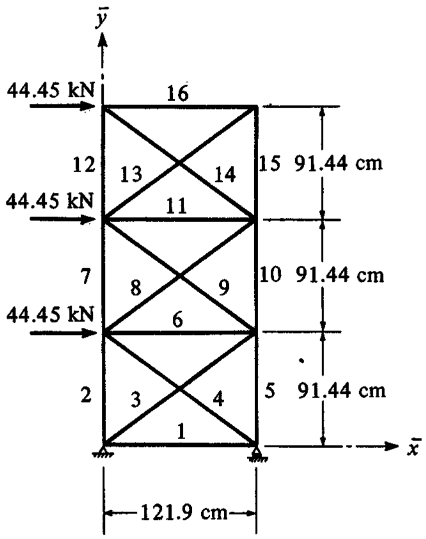

Fig. 3. Statically indeterminate 16-member truss with three degrees of redundancy.

excluded. Table 2 compares the calculated lower and upper bounds to those of reference 12. The upper bound is estimated in reference 12 by using $P_f^{(1)}$. It is seen that the lower and upper bounds are improved in case of low failure probabilities. The intervals between the lower and upper bounds are narrow, and thus they give a good estimate of the failure probability of the structure. Fig. 4 illustrates an example of search trees in the evaluation of the bounds by repeating branching and bounding operations in search of the dominant modes of failure. For a truss structure composed of 16 members with three degrees of redundancy, the maximum number of possible failure modes is calculated as $16 \times 15 \times 14 \times 13 = 43680$. The above example shows that only 29 modes of failure are taken into account for evaluating the lower and upper bounds, which saves greatly the computation time.

Table 1. Data of statically indeterminate 16-member truss

(1) Data of materials

Mean value of yield stress;

$\bar{\sigma}_{Y_i} = 2.76 \times 10^8$ *Pa* $(i = 1, 2, \cdots, 16)$

Modulus of elasticity;

$E_i = 2.06 \times 10^{11}$ *Pa* $(i = 1, 2, \cdots, 16)$

(2) Data of dimensions

Member number	*Area* A_i cm^2	*Radius* r_i cm^2	*Thickness* t_i mm
1	3.35	2.43	2.3
2, 5	8.64	4.45	3.2
3, 4, 14	5.76	3.03	3.2
6	2.29	1.70	2.3
7, 8, 10	4.03	2.43	2.8
9	7.35	3.82	3.2
11, 12, 15	1.58	1.36	2.0
13, 16	2.29	2.14	2.3

(3) Data of initial deflection

$\bar{w}_0/S = 0.1,$ $CV_{w_0} = 0.1$

Table 2. Lower and upper bounds of failure probabilities for statically indeterminate 16-member truss when buckling failure modes are excluded ($\gamma = 5$).

$CV_{\sigma_{Y_i}}$	CV_{L_j}	*Lower bounds*		*Upper bounds*	
		Proposed method	*Ref.* 12	P'_{fU}	$P_f^{(1)}$
0.02	0.1	1.95×10^{-10}	4.50×10^{-30}	2.08×10^{-10}	1.51×10^{-9}
0.05	0.1	1.48×10^{-9}	1.68×10^{-10}	3.41×10^{-7}	3.87×10^{-6}
0.1	0.1	7.63×10^{-5}	2.73×10^{-6}	1.80×10^{-3}	3.84×10^{-3}
0.05	0.2	7.63×10^{-5}	1.56×10^{-5}	3.16×10^{-3}	2.68×10^{-3}

Mean value of processing time (Burroughs B-6700) : 1041 *sec/case*

For the calculations presented so far, no consideration has been given to buckling failure of the compression members. Effect of buckling failure is discussed on the failure probability of the statically indeterminate 16-member truss. The members are tubular with dimensions given in Table 1. Buckling stress of the member with initial deflection under compression is assumed to be given as follows[15]:

$$\sigma_C = \frac{1}{2}(\sigma_Y + \sigma_E + \frac{w_0}{S}\sigma_E)[1 - \sqrt{1 - (4\sigma_E \sigma_Y)/(\sigma_Y + \sigma_E + \frac{w_0}{S}\sigma_E)^2}] \quad (29)$$

where σ_Y : yield stress

$\sigma_E = \pi^2 E/(l/S)^2$: Euler's buckling stress

$S = \sqrt{I/A}$: radius of gyration

w_0 : initial deflection

I : geometrical moment of inertia

A : cross sectional area

l : length

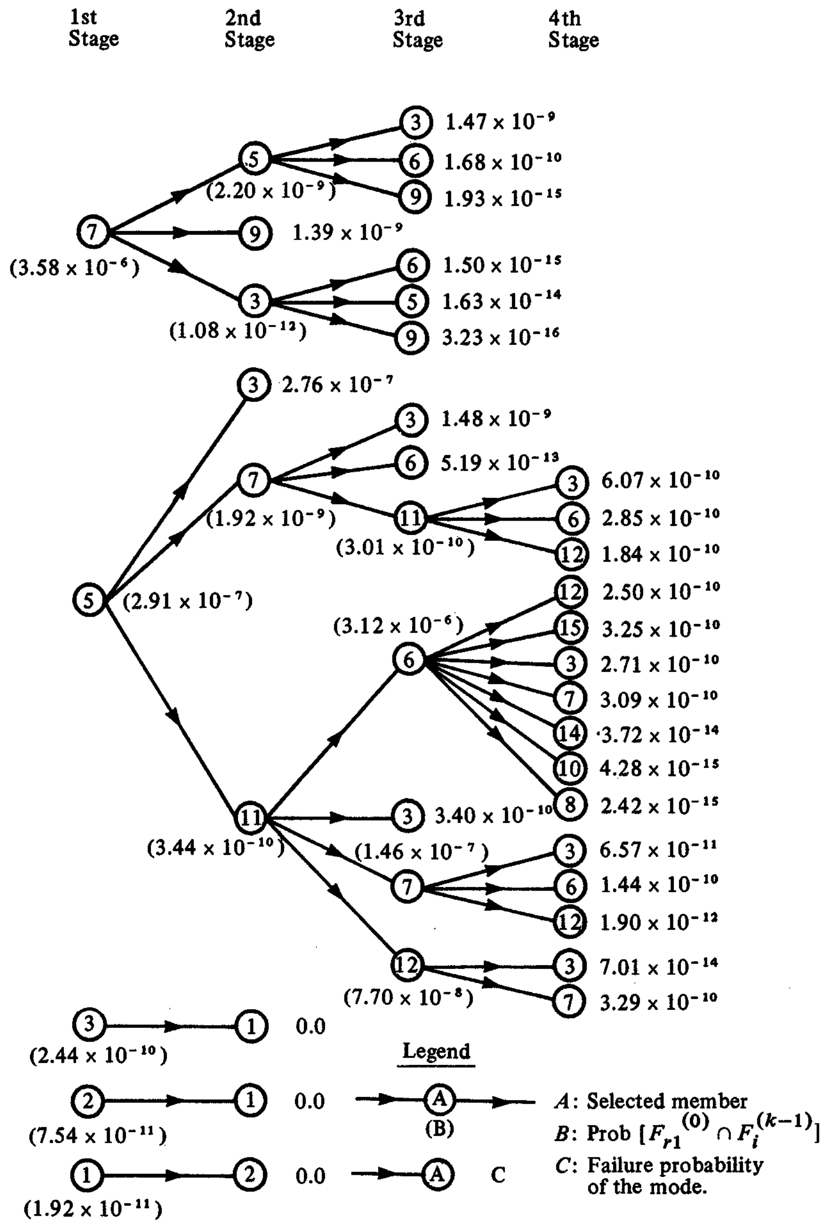

Fig. 4. Search trees in evaluating lower and upper bounds of failure probability.

Yield stress and initial deflection are assumed to be statistically independent Gaussian random variables. By application of the first order approximation, the mean and coefficient of variation of buckling stress are calculated as

$$\left.\begin{aligned} \bar{\sigma}_C &= \frac{1}{2}\left(\bar{\sigma}_Y + \sigma_E + \frac{\bar{w}_0}{S}\sigma_E\right)\left[1 - \sqrt{1 - (4\sigma_E\bar{\sigma}_Y)/(\bar{\sigma}_Y + \sigma_E + \frac{\bar{w}_0}{S}\sigma_E)^2}\,\right] \\ CV_{\sigma_C} &= \frac{\sqrt{(CV_{\sigma_Y}\bar{\sigma}_Y)^2 + (\sigma_E/S)^2(CV_{w_0}\bar{w}_0)^2}}{(\bar{\sigma}_Y + \sigma_E + \frac{\bar{w}_0}{S}\sigma_E)\sqrt{1 - (4\sigma_E\bar{\sigma}_Y)/(\bar{\sigma}_Y + \sigma_E + \frac{\bar{w}_0}{S}\sigma_E)^2}} \end{aligned}\right\} \quad (30)$$

Buckling stress is approximated as a Gaussian random variable with the mean and coefficient of variation given by equation (30). Data concerned are given in Table 1. The estimated lower and upper bounds are given in Table 3 for the various values of the coefficients of variation. The intervals between the lower and upper bounds are comparatively narrow and thus they yield a good estimate of the failure probability. The calculated lower bounds are also compared in the table to those of reference 12. It is seen that the proposed method gives an improved estimate. Comparing the values with those of Table 2 indicates that the failure probability is greatly influenced by inclusion of buckling failure mode of the compression members. Consequently, instability criterion and its variability need to be consolidated for reliability analysis of the truss structures.

Table 3. Lower and upper bounds of failure probabilities for statically indeterminate 16-member truss when buckling failure modes are included ($\gamma = 5$).

$CV_{\sigma Y_i}$	CV_{L_j}	*Lower bounds*		*Upper bounds*	
		Proposed method	*Ref. 12*	P'_{fU}	$P_f^{(1)}$
0.02	0.1	1.43×10^{-9}	1.43×10^{-9}	9.08×10^{-9}	6.51×10^{-8}
0.05	0.1	8.45×10^{-9}	8.45×10^{-9}	3.92×10^{-7}	3.87×10^{-6}
0.1	0.1	7.63×10^{-5}	2.80×10^{-5}	3.55×10^{-3}	5.08×10^{-3}
0.05	0.2	4.15×10^{-3}	1.51×10^{-3}	6.64×10^{-3}	6.34×10^{-3}

Mean value of processing time (Burroughs B-6700) 2203 *sec/case*

5. Conclusion

Reliability analysis of redundant truss structures is carried out by evaluating its lower and upper bounds. Failure criteria of the structures are generated by using Matrix Method. An efficient method is proposed which calculates the lower and upper bounds of the structural failure probability by repeating branching and bounding operations in search of the dominant modes of failure. Through numerical examples, it is shown that the proposed method yields a narrow interval estimate of the structural failure probability and also saves greatly the computation time.

Acknowledgments

The authors would like to express their appreciation to Professors K. Taguchi, Y. Fukumoto and T. Tsumura for their encouragement through the course of this study. This work is partly supported by the Science Research Fund of the Ministry of Education, Science, and Culture of Japan under Grant No. 385007.

References

1) H. Switzky, J. Aircraft, 2–3, 228 (1965).
2) A. M. Freudenthal, J. M. Garrelts and M. Shinozuka, J. Str. Div., Proc. ASCE, **90**, ST-1, 267 (1966).
3) M. Shinozuka and H. Itagaki, Annals of Reliability and Maintainability, **5**, 605 (1966).
4) J. T. P. Yao and H-Y. Yeh, J. Str. Div., Proc. ASCE, **93**, ST-12, 2611 (1969).
5) J. D. Stevenson and F. Moses, J. Str. Div., Proc. ASCE, **96**, ST-11, 2409 (1972).
6) F. Moses and D. E. Kinser, AIAA J., 5–6, 1152 (1967).
7) E. H. Vanmarcke, Computer and Structure, **3**, 757 (1973).
8) A. E. Mansour, J. Ship Research, 18–3, 203 (1974).
9) Y. Murotsu, M. Yonezawa, F. Oba and K. Niwa, Bull. Univ. Osaka Pref., A24–2, 193 (1975).
10) Y. Murotsu, M. Yonezawa, F. Oba and K. Niwa, Proc. 12th International Symposium on Space Technology and Science, Tokyo, 1047 (1977).
11) Y. Murotsu, M. Yonezawa, F. Oba and K. Niwa, Advances in Reliability and Stress Analysis, ASME, (1979).
12) Y. Murotsu, H. Okada, K. Niwa and S. Miwa, 1979 ASME Failure Prevention and Reliability Conference, St. Louis, Mo., U.S.A., (Sept. 1979); ASME paper 79-DET-113.
13) For example, J. S. Przemieniecki, Theory of Matrix Structural Analysis, McGraw-Hill, New York (1968); O.C. Zienkiewicz, The Finite Element Method in Engineering Science, McGraw-Hill, New York (1971).
14) For example, G. H. Brooks and C. R. White, J. Ind. Eng., **16**, 34 (1965).
15) S. T. Timoshenko and J. M. Gere, Theory of Elastic Stability, p. 45, McGraw-Hill, New York (1961).

Appendix – Determination of a bounding criterion.

For a structure composed of n members with s-degrees of redundancy, the maximum number of the failure modes is given by $_nP_{(s+1)} = n(n-1)\cdots(n-s)$, considering the case where structural failure in all the modes occurs if and only if $(s+1)$ members are subject to failure. Consequently, the maximum number of the members to be eliminated in the bounding operations becomes $_nP_{(s+1)}-1$. When the contribution of the eliminated members to the upper bound is to be limited to $P_{fL} \times 10^{-\delta}$, the following condition must be satisfied:

$$[\,_nP_{s+1} - 1\,] \times 10^{-\gamma} < \,_nP_{(s+1)} \times 10^{-\gamma} \leqq 10^{-\delta}$$

$$\therefore \;\gamma \geqq \delta + \sum_{i=0}^{s} \log(n-i) \qquad \text{(A.1)}$$

In case of $n = 16$, $s = 3$ and $\delta = 1$, $\gamma \geqq 5.64$. It should be noted that the condition (A.1) corresponds to the severest case.