Cost-Effectiveness Analysis of Material Testing in Structural Design

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(Received November 15, 1978)

This paper deals with two problems which one encounters in designing of structural systems on a basis of reliability analysis. First is how to select the underlying distribution of data of material strengths to which the structural reliability is sensitive. Second is the determination of the optimum sample size in material testing from economical considerations. The present work describes the illustrative design problems which show the effect of selection of distributions and determination of optimum sample size in view of the resultant costs of structure and material test.

1. Introduction

Quality control in the field of structural systems is in a dawn stage of development compared with one in other fields. It is generally recognized, however, that safety of structure must be statistically evaluated which depends on the behavior of material strength and load. Hence reliability or failure probability has been proposed as a reasonable criterion for structural safety [1–5]. Thus most traditional procedure for analysis of data consists of choice of the underlying distribution, estimation of the population parameters, and test of goodness of fit between theoretical and observed distribution.

Lack of rational criterion for selection of the underlying distribution in the above procedure has permitted the flexible choice. This flexibility yields often the disconcerting aspect that none of several hypothetical distributions can be rejected in the statistical tests on a given level of significance. The designed reliability of the structure, however, is intrinsically very sensitive to the underlying distribution. Another problem stems from the fact that there is no good criterion for determining the optimum sample size in material testing.

Concretely speaking, the following problems remain unsolved in the traditional procedure: i) optimum selection of the underlying distribution for structural design, and ii) determination of the optimum sample size in test of material. A related work is only suggested by Shinozuka *et al.* [6] concerning the proof-load test.

The present paper provides a new methodology which is within a general framework of reliability-based and economical analysis. Effectiveness of selecting the distribution is

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evaluated on the resultant structural cost and the weight in a reliability design, and determination of the optimum sample size is solved by relating it to the trade-off between costs of structural system and material testing. Numerical example in tensile test of fiber reinforced plastics (FRP) is also presented to illustrate the proposed procedure.

2. Cost-Effectiveness Analysis

2.1 Mathematical formulation

Let R be the strength of materials which is a random variable with the cumulative distribution function (cdf) $F_R(x;\theta)$. When population parameters θ are estimated from sample of size n, the nominal strength R_γ corresponding to 100γ percent point is given by

$$R_\gamma = F_R^{-1}(\gamma;\hat{\theta}), \tag{1}$$

where $\hat{\theta}$ stand for the estimates of θ.

The estimators are essentially random variables, and the true values of population parameters remain unknown in so far as the sample size is finite. Consequently, the upper or lower confidence limit $\theta^*_{n,\epsilon}$ in safety side must be adopted instead of $\hat{\theta}$ in Eq. (1). The equation then becomes

$$R^*_{n,\gamma} = F_R^{-1}(\gamma;\theta^*_{n,\epsilon}). \tag{2}$$

Introduction of Eq. (2) which supersedes Eq. (1) lies in its potentiality to make it possible to express the effect of sample size in $\theta^*_{n,\epsilon}$ or $R^*_{n,\gamma}$. Which of the upper and lower confidence limits should be chosen as $\theta^*_{n,\epsilon}$ with respect to the structural safety depends on the shape of distribution function. Since, on the other hand, the load acting on the structure L is random vairable with cdf $F_L(x;\zeta)$, the nominal load $L^*_{n_1,\gamma_1}$ can be represented by

$$L^*_{n_1,\gamma_1} = F_L^{-1}(\gamma;\zeta^*_{n_1,\epsilon_1}), \tag{3}$$

where $\zeta^*_{n_1,\epsilon_1}$ are the confidence limits of parameters ζ.

Now suppose that the applied load L^* to a structure and the allowable stress R^* are specified in structural design. The design value of cross-sectional area A can be calculated from

$$A = \frac{L^* \cdot S_F}{R^*}, \tag{4}$$

where S_F is a factor of safety. As regards the structural weight w, Eq. (4) yields the relation:

$$\begin{aligned} w &= d\,l\,A \\ &= d\,l\,S_F \cdot \frac{L^*}{R^*}, \end{aligned}$$

where l and d denote length and specific weight of structural element, respectively. Substitutaion of $R^*_{n,\gamma}$ and $L^*_{n_1,\gamma_1}$ given by Eqs. (2) and (3) into the above R^* and L^* enables us to obtain the structural weight w_n as follows:

$$w_n = d\, l\, S_F \cdot \frac{L^*_{n_1,\gamma_1}}{R^*_{n,\gamma}}\,. \tag{5}$$

The structural cost H_C which is proportional to the structural weight w_n results in

$$H_C = c_1\, d\, l\, S_F \cdot \frac{L^*_{n_1,\gamma_1}}{R^*_{n,\gamma}}\,, \tag{6}$$

where c_1 is a constant.

As presented previously in [7,8], the factor of safety S_F is closely associated with failure probability P_F or reliability P_S of the structure, and is very sensitive to the shapes of distributions of R and L. Eq. (6) gives a criterion for selection of the underlying distribution based on the resultant structural cost H_C. Hence in consideration of the relation between S_F and P_F, effect of selecting the underlying distribution of R and L can be discussed on

i) the resultant structural cost H_C under a constant value of P_F, and
ii) the resultant failure probability P_F under a constant value of S_F.

The larger the sample size becomes, the narrower confidence interval of the estimate gets, and the closer $\theta^*_{n,\epsilon}$ approaches to $\hat{\theta}_n$ under a generally satisfied condition. For a given value of γ, therefore, the nominal strength $R^*_{n,\gamma}$ obtained by Eq. (2) has larger value while the nominal load $L^*_{n_1,\gamma_1}$ by Eq. (3) has smaller one as the sample size increases without loss of generality. It should be noted in connection with Eqs. (5) and (6) that the structural weight is a decreasing function of n under a given value of S_F or P_F and the structural cost H_C decreases as n increases. On the other hand, the cost of testing material H_S is represented as a function of n. Hence

$$H_S = H_S(n)\,. \tag{7}$$

Let C_T denote the total cost of structural system defined by

$$C_T = H_C(S_F,\, L^*_{n_1,\gamma_1},\, R^*_{n,\gamma}) + H_S(n)\,, \tag{8}$$

where H_C and H_S can be obtained from Eqs. (6) and (7). There are two approaches available for evaluation of C_T with respect to sample size n, *i.e.*, to evaluate C_T under given values of P_F and S_F. The determination of the optimum sample size n_{opt} can be formulated as either of the following two mathematical programming problems:

P1: Determine n_{opt} to minimize C_T for a given value of P_F, or
P2: Minimize C_T with respect to n for a given value of S_F,

where cdf's of the load and strength $F_L(x;\zeta)$ and $F_R(x;\theta)$ are assumed to be specified. The relation among C_T, H_C, H_S, and n under a given value of P_F is schematically illustrated in Fig. 1 with the value of n_{opt} which minimizes C_T. In the figure, the cost of testing material H_S is drawn as a linear function of n. A uni-dimensional search technique is applied to the determination of the optimum sample size.

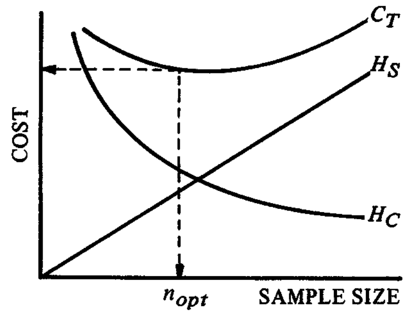


Fig. 1. Schematic representation of C_T, H_C, H_S, and sample size.

2.2 Determination of nominal strength

The determination of $R^*_{n,\gamma}$ in Eq. (2) necessitates the procedure of constructing $\theta^*_{n,\epsilon}$ from data of material testing. Among many estimators of the population parameters, MLE (maximum likelihood estimator) is adopted in this paper because of its analytical reasonability and BAN (best asymptotic normal) property. In what follows, the procedures to determine $\theta^*_{n,\epsilon}$ and $R^*_{n,\gamma}$ by MLE are derived for three types of distribution, *i.e.*, normal, doubly exponential, and Weibull distributions.

2.2.1 Normal distribution

For a normal variate R with mean μ and variance σ^2, ML-estimates of μ and σ^2 are

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} R_i ,$$

and

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{1}(R_i - \hat{\mu})^2 , \qquad (9)$$

respectively. From Eq. (9), confidence limits for $\hat{\mu}$ and $\hat{\sigma}$ are determined by

$$\mu^*_{n,\epsilon} = \hat{\mu} - \frac{t_{\epsilon/2}(n-1)}{\sqrt{n-1}} \cdot \hat{\sigma} ,$$

and

$$\sigma^*_{n,\epsilon} = \frac{\sqrt{n}}{\chi_{1-\epsilon}(n-1)} \cdot \hat{\sigma} , \qquad (10)$$

which depend on sample size n and confidence level ϵ. $t_{\epsilon/2}(n-1)$ and $\chi_{1-\epsilon/2}(n-1)$ in Eq. (10) designate the $100(\epsilon/2)$ percent point of the Student's t-distribution with $(n-1)$ degrees of freedom and the $100(1-\epsilon/2)$ percent point of the χ-distribution with $(n-1)$ degrees of freedom, respectively. Use of $\mu^*_{n,\epsilon}$ and $\sigma^*_{n,\epsilon}$ in Eq. (10) aids one in evaluating the nominal strength $R^*_{n,\gamma}$ by means of

$$R^*_{n,\gamma} = \mu^*_{n,\epsilon} + u_\gamma \sigma^*_{n,\epsilon} , \tag{11}$$

where u_γ is 100γ percent point of the standard normal distribution. Concretely substituting Eq. (10) into (11), we have

$$R^*_{n,\gamma} = \hat{\mu} \left[1 - \left(\frac{t_{\epsilon/2}(n-1)}{\sqrt{n-1}} - \frac{\sqrt{n}}{\chi_{1-\epsilon/2}(n-1)} \cdot u_\gamma \right) \hat{CV} \right] , \tag{12}$$

where $\hat{CV}$ is an estimate of coefficient of variation. The value calculated by Eq. (12) is recommendable for an allowable stress in the structural design.

2.2.2 Doubly exponential distribution

The probability density function (pdf) of asymptotic smallest value distribution which is typical of the doubly exponential distribution is

$$f(x) = \frac{1}{a} \exp \left[\left(\frac{x-b}{a}\right) - \exp\left(\frac{x-b}{a}\right) \right] , \tag{13}$$

where a and b stand for scale and location parameters. The likelihood function of Eq. (13) and its logarithmic form are written as

$$L = \frac{1}{a^n} \exp \left[\sum_{i=1}^{n} \frac{x_i - b}{a} - \sum_{i=1}^{n} \exp\left(\frac{x_i-b}{a}\right) \right] ,$$

and

$$\ln L = -n \cdot \ln a + \sum_{i=1}^{n} \frac{x_i - b}{a} - \sum_{i=1}^{n} \exp\left(\frac{x_i-b}{a}\right) . \tag{14}$$

ML-estimates of a and b are given [9] as the solutions of the simultaneous equations:

$$a = \frac{\sum_{i=1}^{n} x_i \exp\left(\frac{x_i}{a}\right)}{\sum_{i=1}^{n} \exp\left(\frac{x_i}{a}\right)} - \bar{x} ,$$

and (15)

$$b = a \left[\ln \sum_{i=1}^{n} \exp\left(\frac{x_i}{a}\right) - \ln n \right] ,$$

where

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i .$$

The asymptotic information matrix of MLE becomes

$$I \cong - \begin{pmatrix} \dfrac{\partial^2 \ln L}{\partial a^2} & \dfrac{\partial^2 \ln L}{\partial a \partial b} \\ \dfrac{\partial^2 \ln L}{\partial b \partial a} & \dfrac{\partial^2 \ln L}{\partial b^2} \end{pmatrix}_{\substack{a=\hat{a} \\ b=\hat{b}}}$$

whose inversion gives the asymptotic variance-covariance matrix. Thus, the asymptotic forms of Var $(\hat{a})$ and Var $(\hat{b})$ result in

$$\mathrm{Var}(\hat{a}) = \frac{1}{\frac{n}{\hat{a}^2}\left[1 + \frac{1}{\hat{a}^2} \cdot \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{b})^2 \exp\left(\frac{x_i - \hat{b}}{\hat{a}}\right) - \left(1 + \frac{\bar{x}}{\hat{a}} - \frac{\hat{b}}{\hat{a}}\right)^2\right]} , \tag{16}$$

and

$$\mathrm{Var}(\hat{b}) = \frac{1 + \frac{1}{\hat{a}^2} \cdot \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{b})^2 \exp\left(\frac{x_i - \hat{b}}{\hat{a}}\right)}{\frac{n}{\hat{a}^2}\left[1 + \frac{1}{\hat{a}^2} \cdot \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{b})^2 \exp\left(\frac{x_i - \hat{b}}{\hat{a}}\right) - \left(1 + \frac{\bar{x}}{\hat{a}} - \frac{\hat{b}}{\hat{a}}\right)^2\right]} .$$

For the doubly exponential variate x, or strength of a structural element, with parameters a and b, we have

$$\mathrm{E}(X) = b - Ca , \qquad (C = 0.577\ldots : \text{Euler's constant})$$

and

$$\mathrm{V}(X) = \frac{\pi a}{\sqrt{6}} .$$

As is well known, it is usually assumed in the standard design procedures that the structural design must be based on a strength which is smaller than E(X), and a variance of strength which is larger than V(X) [1]. In other words, the safety design asserts that $a^*_{n,\epsilon}$ must be evaluated to be larger than a, while $b^*_{n,\epsilon}$ smaller than b. It follows from the BAN property that the confidence limits of ML-estimates are

$$a^*_{n,\epsilon} = \hat{a} + u_{1-\epsilon/2} \cdot \sqrt{\mathrm{Var}(\hat{a})} ,$$

and

$$b^*_{n,\epsilon} = \hat{b} + u_{\epsilon/2} \cdot \sqrt{\mathrm{Var}(\hat{b})} . \tag{17}$$

Using $a^*_{n,\epsilon}$ and $b^*_{n,\epsilon}$, we have the nominal strength $R^*_{n,\gamma}$ which corresponds to 100γ percentile of doubly exponential distribution and also to Eq. (2) such that

$$R^*_{n,\gamma} = b^*_{n,\epsilon} + a^*_{n,\epsilon} \cdot \ln\left[-\ln(1-\gamma)\right] . \tag{18}$$

2.2.3 Weibull distribution

The pdf of Weibull distribution is

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right],$$

where α and β are shape and scale parameters. The likelihood function and the resulting logarithmic form are

$$L = \left(\frac{\alpha}{\beta}\right)^n \prod_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha-1} \exp\left[-\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha}\right],$$

and

$$\ln L = n(\ln\alpha - \ln\beta) + (\alpha - 1)\sum_{i=1}^{n}(\ln x_i - \ln\beta) - \sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha}. \tag{19}$$

ML-estimates of α and β are obtained as the solutions of the simultaneous equations:

$$\frac{n}{\alpha} + \sum_{i=1}^{n}(\ln x_i - \ln\beta) - \sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha}\ln\left(\frac{x_i}{\beta}\right) = 0,$$

and (20)

$$-\frac{n\alpha}{\beta} + \frac{\alpha}{\beta}\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha} = 0.$$

Any numerical method must be applied because Eq. (20) can not be solved explicitly. In this situation $\mathrm{Var}(\hat{\alpha})$ and $\mathrm{Var}(\hat{\beta})$ turn out to be

$$\mathrm{Var}(\hat{\alpha}) = \frac{1}{n\left[\frac{1}{\hat{\alpha}^2} + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\hat{\beta}}\right)^{\hat{\alpha}}\left[\ln\left(\frac{x_i}{\hat{\beta}}\right)\right]^2 - \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\hat{\beta}}\right)^{\hat{\alpha}}\ln\left(\frac{x_i}{\hat{\beta}}\right)\right]^2\right]},$$

and (21)

$$\mathrm{Var}(\hat{\beta}) = \frac{\frac{1}{\hat{\alpha}^2} + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\hat{\beta}}\right)^{\hat{\alpha}}\left[\ln\left(\frac{x_i}{\hat{\beta}}\right)\right]^2}{\frac{n\hat{\alpha}^2}{\hat{\beta}^2}\left[\frac{1}{\hat{\alpha}^2} + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\hat{\beta}}\right)^{\hat{\alpha}}\left[\ln\left(\frac{x_i}{\hat{\beta}}\right)\right]^2 - \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\hat{\beta}}\right)^{\hat{\alpha}}\ln\left(\frac{x_i}{\hat{\beta}}\right)\right]^2\right]}.$$

Since logarithmic transformation of a Weibull variate follows the doubly exponential probability law, the following relations hold between Weibull and doubly exponential parameters [10]:

$$\alpha = \frac{1}{a},$$

and

$$\beta = \ln b.$$

In line with the discussion about $a^*_{n,\epsilon}$ and $b^*_{n,\epsilon}$ in Eq. (17), it is seen intuitively that both $\alpha^*_{n,\epsilon}$ and $\beta^*_{n,\epsilon}$ should be smaller than $\hat{\alpha}$ and $\hat{\beta}$ for the safety design. Hence

$$\alpha^*_{n,\epsilon} = \hat{\alpha} + u_{\epsilon/2} \cdot \sqrt{\mathrm{Var}(\hat{\alpha})} \ ,$$

and (22)

$$\beta^*_{n,\epsilon} = \hat{\beta} + u_{\epsilon/2} \cdot \sqrt{\mathrm{Var}(\hat{\beta})} \ .$$

With $\alpha^*_{n,\epsilon}$ and $\beta^*_{n,\epsilon}$ thus determined, the nominal strength $R^*_{n,\gamma}$ in the case of Weibull distribution can be evaluated by

$$R^*_{n,\gamma} = \beta^*_{n,\epsilon} \left[-\ln(1-\gamma) \right]^{1/\alpha^*_{n,\epsilon}} \ . \tag{23}$$

3. Illustrative Example

Tensile data of FRP (fiber reinforced plastics) whose frequency histogram is shown in Fig. 2 are analyzed as an illustrative example. 30 specimens are prepared from a chopped-strand-mat FRP sheet of constant thickness (8 mm). The details of test materials and experimental procedure are described in reference [11]. The Traditional statistical analysis of test data is performed in accordance with three steps:

1. Postulate normal, doubly exponential, and Weibull types as the underlying distributions of FRP tensile strength.
2. Calculate the ML-estimates by Eq. (9), (15), and (20).
3. Test goodness of fit between the postulated distributions and the actual data.

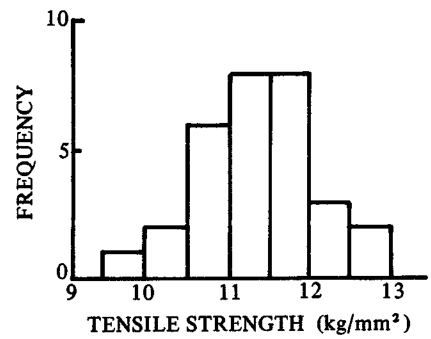


Fig. 2. Frequency histogram of tensile strengths of 30 dumbell specimens from a Mat FRP. [Percentage of volume content of glass fiber: 20.1%]

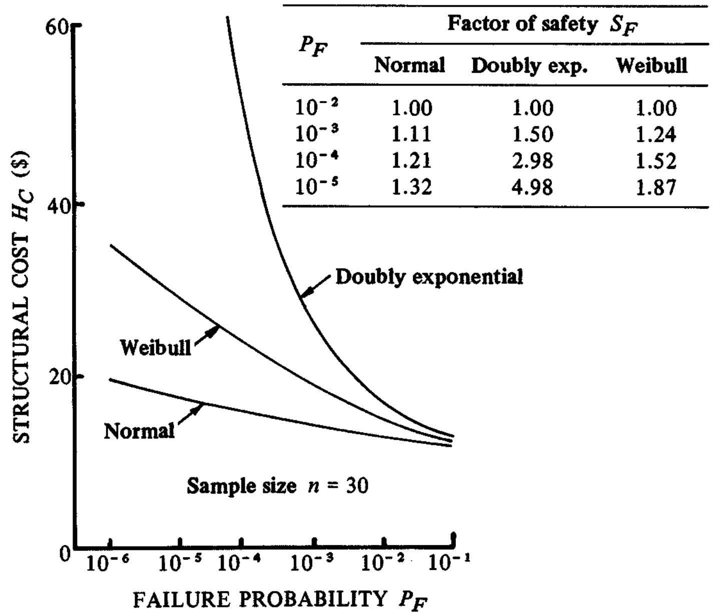
The outcome of the statistical analysis based on the above steps is tabulated in Table 1, which tells that none of the postulated distributions can be rejected by test of the null hypotheses. In consequence cost-effectiveness analysis discussed in Section 2 is carried out to investigate the effect of underlying distribution on the resultant structural cost for the actual data of FRP. The confidence limits and nominal strengths calculated are represented in Table 1. Fig. 3 in which the nominal load is assumed to be deterministic

Table 1. Calculated estimates and significance test results for the tensile strength of FRP.

	Normal	Doubly exponential	Weibull
ML-estimates:	$\hat{\mu} = 11.32$ $\hat{\sigma} = 0.71$	$\hat{a} = 0.63$ $\hat{b} = 11.66$	$\hat{\alpha} = 11.39$ $\hat{\beta} = 11.65$
χ^2:	2.76	1.19	1.05
χ^2_4 (0.05):	9.49	9.49	9.49
Decision:	cannot reject	cannot reject	cannot reject
Confidence limit: ($\epsilon = 0.01$)	$\mu^*_{n,\epsilon} = 10.96$ $\sigma^*_{n,\epsilon} = 1.07$	$a^*_{n,\epsilon} = 0.86$ $b^*_{n,\epsilon} = 11.35$	$\alpha^*_{n,\epsilon} = 11.02$ $\beta^*_{n,\epsilon} = 11.32$
Nominal strength: ($\gamma = 0.01$)	8.47 (8.30)	7.39 (7.24)	7.46 (7.31)

Nominal strength: kg/cm^2 (x 10^2 MPa)

illustrates the effect of underlying distribution on the resultant structural cost corresponding to various values of failure probability. The discrepancies in the resultant structural cost become eminent as the failure probability decreases. This fact calls designer's attention to the selection of underlying distribution because the failure probability between $P_F = 10^{-4}$ and 10^{-6} is generally recommendable in the actual design.



P_F	Factor of safety S_F		
	Normal	Doubly exp.	Weibull
10^{-2}	1.00	1.00	1.00
10^{-3}	1.11	1.50	1.24
10^{-4}	1.21	2.98	1.52
10^{-5}	1.32	4.98	1.87

Fig. 3. Effect of underlying distributions on the resultant structural cost H_C in actual data of FRP. [The value of nominal load = 11.30 kgf (111.53 N), $c_1 ld$ = 10.0 \$/mm^2]

Further consider the relationship between the resultant structural cost and sample size of testing materials. Since the true distribution of the material strength is unknown in practice and the experiment with large sample size is generally expensive, Monte Carlo simulation is desirable to the study. Each group of 1000 random numbers is separately generated from normal, doubly exponential, and Weibull populations by setting parameters equal to ML-estimates given in Table 1. Table 2 shows the confidence limits and nominal strengths calculated for several samples ranging between 20 and 1000. It is immediately perceivable that confidence limits approach closely to the values of ML-estimates shown in Table 1 and the nominal strength increases gradually as sample size becomes large. The relationship between the resultant structural cost and sample size for the cases of $P_F = 10^{-2}$ and 10^{-6} is shown in Fig. 4 which illustrates the fact that the structural cost decreases as sample size increases. The tendency is remarkably recognized when $P_F = 10^{-6}$, especially in the case of doubly exponential distribution.

Table 2. Confidence limits ($\epsilon = 0.01$), and nominal strengths $R^*_{n,\gamma}$ ($\gamma = 0.01$).

Sample size	Normal			Doubly exponential			Weibull		
n	$\mu^*_{n,\epsilon}$	$\sigma^*_{n,\epsilon}$	$R^*_{n,\gamma}$	$a^*_{n,\epsilon}$	$b^*_{n,\epsilon}$	$R^*_{n,\gamma}$	$\alpha^*_{n,\epsilon}$	$\beta^*_{n,\epsilon}$	$R^*_{n,\gamma}$
20	10.85	1.21	8.03	0.94	11.30	6.99	9.37	11.26	6.89
40	11.01	1.00	8.68	0.86	11.41	7.46	11.57	11.38	7.64
60	11.07	0.93	8.90	0.82	11.45	7.70	12.71	11.42	7.95
80	11.11	0.89	9.04	0.79	11.48	7.84	13.30	11.45	8.11
100	11.13	0.87	9.10	0.77	11.50	7.94	13.85	11.47	8.23
200	11.19	0.82	9.28	0.73	11.55	8.20	14.96	11.52	8.47
300	11.21	0.99	9.37	0.72	11.57	8.28	15.36	11.55	8.56
500	11.24	0.77	9.45	0.70	11.59	8.38	15.82	11.57	8.65
1000	11.27	0.75	9.52	0.68	11.62	8.48	16.28	11.59	8.74

$R^*_{n,\gamma}$: kg/mm²

Finally consider the problem to determine the optimum sample size when the cost of testing material H_S is proportional to the sample size n, *i.e.*,

$$H_S = c_2 \cdot n$$

where c_2 is a constant. The optimum values which are obtained by a uni-dimensional search technique are shown in Table 3 (a) and (b) for the cases of (a) $c_1 ld = 10.0$ \$/mm^2, $c_2 = 0.5$ \$/sample, and (b) $c_1 ld = 20.0$ \$/mm^2, $c_2 = 0.5$ \$/sample. The values of failure probability are specified in the range from 10^{-2} to 10^{-6}. The values of optimum sample size minimizing total cost for Weibull distribution are smallest of the three distributions, while those for doubly exponential distribution are larger than the others. It should be noted that the smaller the specified failure probability becomes, the larger the value of optimum sample size grows especially for doubly exponential distribution.

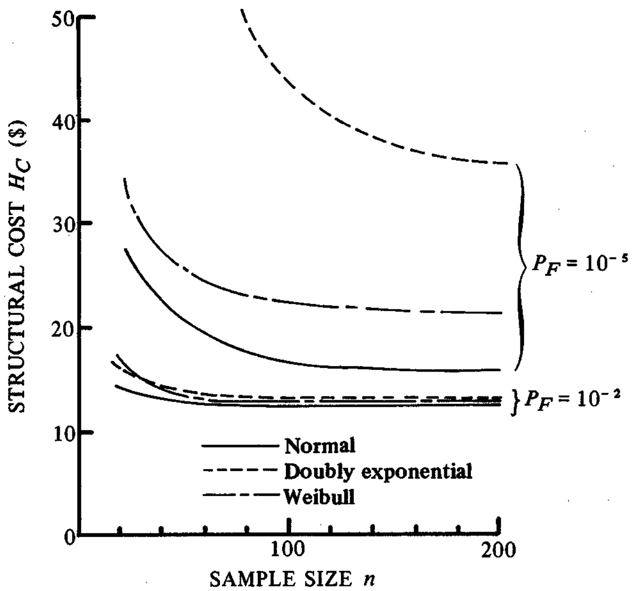


Fig. 4. Relation between structural cost H_C and sample size n. [The value of nominal load = 11.30 kgf (111.53 N), $c_1 ld$ = 10.0 \$/mm²]

Table 3. Optimum sample size n_{opt}.

(a) $c_1 ld$ = 10.0 \$/mm², c_2 = 0.05 \$/sample

P_F	Normal		Doubly exp.		Weibull	
	n_{opt}	C_T	n_{opt}	C_T	n_{opt}	C_T
10^{-2}	35	17.82	28	16.99	36	16.79
10^{-3}	35	21.27	57	22.42	51	20.01
10^{-4}	93	22.86	98	30.70	63	23.45
10^{-5}	104	25.06	198	45.63	78	27.42
10^{-6}	115	27.15	384	81.36	94	31.59

(b) $c_1 ld$ = 20.0 \$/mm², c_2 = 0.05 \$/sample

P_F	Normal		Doubly exp.		Weibull	
	n_{opt}	C_T	n_{opt}	C_T	n_{opt}	C_T
10^{-2}	98	33.02	53	32.28	51	31.36
10^{-3}	99	36.72	88	41.52	62	37.08
10^{-4}	116	40.43	137	56.13	92	43.16
10^{-5}	128	44.35	269	82.07	104	50.14
10^{-6}	167	48.95	402	142.66	122	58.21

c_1: \$/kg, d: kg/mm², l: mm, c_2: \$/sample, C_T: \$

4. Summary

The effect of selection of distribution has been discussed from the viewpoint of reliability and cost-effectiveness analysis, and successfully proposed is the new methodology of determining the optimum sample size. It was also shown that the resultant costs associated with structural design remarkably depend on the postulated strength distribution and sample size in the test of materials.

Acknowledgments

The authors would like to express their sincere thanks to Professor T. Fujii and Mr. Z. Maekawa, Osaka City University, for the permission for us to use their tensile data of FRP. Acknowledgment is also due to the Ministry of Education, Science and Culture of Japan for the financial support given to this study.

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