Some Examples on Computer Graphics (Part 1, In the Case of Linear Change Phenomena)

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This paper describes examples of the use of Computer Aided Graphic System (CAGS) for the applications in order to visualize some of mechanical deformations and other pattern transformations. The pattern to be processed by computer is coded to the generated mesh elements to expand Fourier series as digital input data, and the output data are displayed as a figure by CAGS.

The developed program shows the validity to provide easy usage and effective fruits for the students who have a little knowledge on FORTRAN.

1. Introduction

Today, students of engineering department in college or university are said to be required to have enough knowledge of using computer graphic system as well as programming some algorithms, as the need arises, before they enter into industries or institutes. The lecture on "Computer Graphic Exercise" of which one of the authors is in charge for the students of Engineering Department, is given to the sophomores who completed the course of Monge Descriptive Geometry in the freshman for a period of one year semester. It can be said, in general, that effective teaching of computer programming requires the ability to let the student understand well the actions of picture analysis related with Descriptive Geometry. It is needless to speak about the necessity of visualization in the different technical scientific fields such as in anatomy, surgery, X-ray dianostics, biology, biochemistry, astronomy, aeronautics, architecture, ship building engineering, modern chemistry in general, especially in crystallography and so on.

For example, Vant Hoff (1852–1911) in Holland made it clear that the lactic acid $\mathbf{CH_3CH(OH)\cdot COOH}$ (*D*-lactic acid) has two kinds of chemical compounds of sarcolactic acid (*L*-lactic acid) and fermentation lactic acid (*DL*-lactic acid), even though three kinds of chemical compounds are considered as shown in Fig. 1 (a), (b) and (c). The discovery is said to be due to the result that Vant Hoff displayed the ligands (atoms of bor) in three-dimensional way, and he found that two of them were situated at the same relation by exchanging atoms for **OH** and **COOH** as shown in Fig. 2 (a) and (b). Such problem

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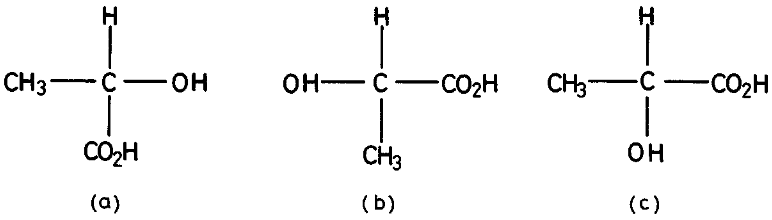

Fig. 1. Chemical compounds of lactic acid.

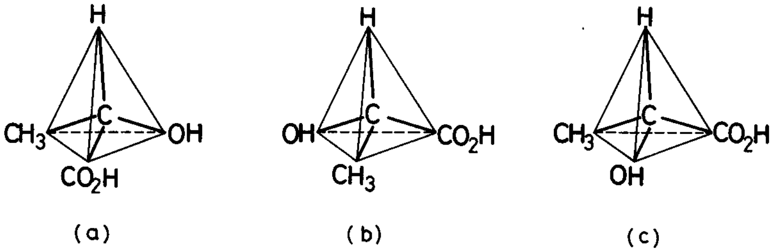

Fig. 2. Three-dimensional display of **$CH_3CH(OH)\cdot COOH$.**

in this field can be treated rather simply if a good knowledge of synthetic geometry is applied, and at present, students may easily understand the difference between the L-, and DL-lactic acid of (a) and (c) in Fig. 2. Problems on crystalline structure, other constructive figures in geodesy and so on, can sometimes be thoroughly understood only by a little knowledge of a well developed capacity of such visualization.

This paper shows some examples of the computer graphic exercise to investigate the actions of intermediate repeating figures between an original drawing A and a final drawing B, which can give some significant fields as mentioned above. The usage of the program itself is not so difficult to the students because all arguments of the subroutine programs are only concerned with the positions of the input and output figure data of the given drawings A and B.

2. Theory and Method

To describe the program for analysing input data, a mesh generation of a pattern must be executed. The meshing method is applied and the mesh is divided into a two-dimensional rectangular array of quadrilateral elements, having m-rows and n-columns as shown in Fig. 3.

If we take the coordinates of the nodes parallel to the (x, y) axis, each node of an element is uniquely identified by an integer pair (m, n) representing its row-column location in the grid.

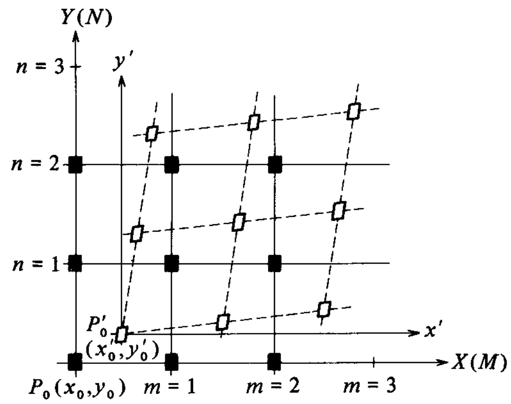

Fig. 3. Rectangular array of quadrilateral elements of (m, n) pair.

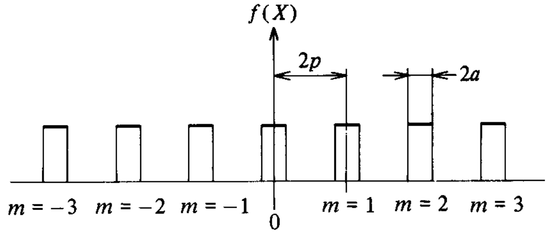

Fig. 4. Amplitude and phase function of a mesh.

Then the amplitude and phase of the node as shown in Fig. 4 can be functioned as;

$$f(x) = \begin{cases} 1; & for \quad (2mp - a) \leqq x \leqq (2mp + a) \\ 0; & for \quad (2mp + a) < x < 2(m+1)p - a \end{cases} \tag{1}$$

where $2p$ is a pitch constant and $2a$ is a width of the node in the grid.

Only one dimensional case of x-direction is shown for the simplitity, but it can be expanded to the two-dimensional case[1)] without loss any precision. Then the whole nodes' distribution is given as the following Fourier series;

$$f(x, y) \sim \frac{a}{p} \left[1 + 2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{m,n} \cdot \cos \omega(mx + ny)\right], \tag{2}$$

where ω is an angular frequency of $\omega = \pi/p$. Then Fourier coefficient is given as;

$$c_{m,n} = \frac{1}{p} \left[\frac{\sin ma\omega}{ma\omega}\right]. \tag{3}$$

This coefficient can be regarded as constant, because $a\omega \to 0$ when $p \gg a$ and knowing $(\sin Z)/Z = 1$ at $Z = 0$. Then the boundary data set of a given figure can be stored to the computer memories at inputing them through punched cards (see the *Hardware* in Appendix).

When the pattern is deformed due to some reasons, each node on the grid displaces to the another position (refer to Fig. 3) by the vector $G(x, y)$. Then the deformations as the scalar components to x- and y-direction become;

$$\left.\begin{aligned} G_u(x, y) &= x' - x \\ G_v(x, y) &= y' - y . \end{aligned}\right\} \tag{4}$$

Then the displacements of the g_x and g_y will be

$$\left.\begin{aligned} g_x &= \int_{x_1}^{x_2} [\partial G_u(x, y)/\partial x] \cdot dx \\ g_y &= \int_{y_1}^{y_2} [\partial G_v(x, y)/\partial y] \cdot dy . \end{aligned}\right\} \tag{5}$$

Finally, the Eq. (1) becomes,

$$f_1(x, y) = C_o \sum_{m=1}^{M} \sum_{n=1}^{N} \cos \omega \, [m(x - g_x) + n(y - g_y)] \, . \tag{2'}$$

In the equation C_o is a constant, and M and N are integers having large numbers being related to the divided end nodes of (M, N) for the given figures. If the functions of g_x and g_y are known, the deformed pattern can be displayed by X–Y plotter after calculating the Eq. (2′) and finding the distances between the coordinates of the nodes (see the *Software* in Appendix).

3. Deformation of Grid

Suposing the deformation of the grid is within the elastic limit, then the components of x- and y-direction of the vector $G(x, y)$ will be linearly propositional to $u(x, y)$ and $v(x, y)$. So that we get[2)]

$$\epsilon_x = \partial u/\partial x \, , \qquad \epsilon_y = \partial v/\partial y \tag{6}$$

and

$$\gamma_{x,y} = \partial v/\partial x + \partial u/\partial y \, , \tag{7}$$

where ϵ_x and ϵ_y are the strain to the directions of x-, and y-axis, and $\gamma_{x,y}$ is the shearing strain.

But when the deformation is beyond the elastic limit and large strain or deformation is introduced to the grid, then the phenomenon is not linear, but non-linear. So it is important to know the function of $G_u(x, y)$ and $G_v(x, y)$, and to take ϵ'_x, ϵ'_y and $\gamma'_{x,y}$ which are the effective strains as,

$$\left.\begin{aligned} \epsilon_x' &= \partial g_x/\partial x \ , \qquad \epsilon_y' = \partial g_y/\partial y \\ \gamma_{x,y}' &= \partial g_y/\partial x + \partial g_x/\partial y \ . \end{aligned}\right\} \tag{8}$$

We get,

$$f_2(x', y') = \sum_{m=1}^{M} \sum_{n=1}^{N} \cos \omega \left\{ m\,[x'(1-\epsilon_x')] + n\,[y'(1-\epsilon_y')] \right\} . \tag{9}$$

The Eq. (9) can be applied to, not only for elastic and plastic deformations, but also some other hysteresis changes in every scientific fields, some of which are as shown in the following examples.

4. Test Run of Programming

Knowing that a figure is composed of many digital data of the coordinates (x, y), the input of the two blocks of data belonging to an original drawing A and the final drawing B is fed to the computer, according to the Eqs. (2′) and (9). Then the output of the block of data belonging to the intermediate drawings C_k can be calculated by the computer according to the following equation,

$$\left.\begin{aligned} (g_x)_k &= (m_k g_{xA} + n_k g_{xB})/(m_k + n_k) \\ (g_y)_k &= (m_k g_{yA} + n_k g_{yB})/(m_k + n_k) \ , \end{aligned}\right\} \tag{10}$$

where (g_{xA}, g_{yA}) and (g_{xB}, g_{yB}) are the displacements in the drawings A and B, and (m_k, n_k) is ratio to divide two points between $P_A(x, y)$ and $P_B(x, y)$ into $m_k : n_k$, $k = 1, 2, \cdots, K$.

As a test run of the programming, a rubber plate with a hole under an uniform elastic deformation is considered as an example. The surface of the plate has a two-dimensional quadrilateral mesh and each point of the cross node is uniquely identified as m-row and n-column before and after loading on it. Suposing the deformation is within the elastic limit, the intermediate drawings of C_k (where $k = 1, 2, \cdots, 5$) are shown in Fig. 5. The program which was developed for this study shows the validity of proper working by means of CAGS.

In engineering, some phenomena which change linearly are most common, that is, the displacement occurs mostly within the plastic limit in general, where this method can be widely applicable to the fields of this kind.

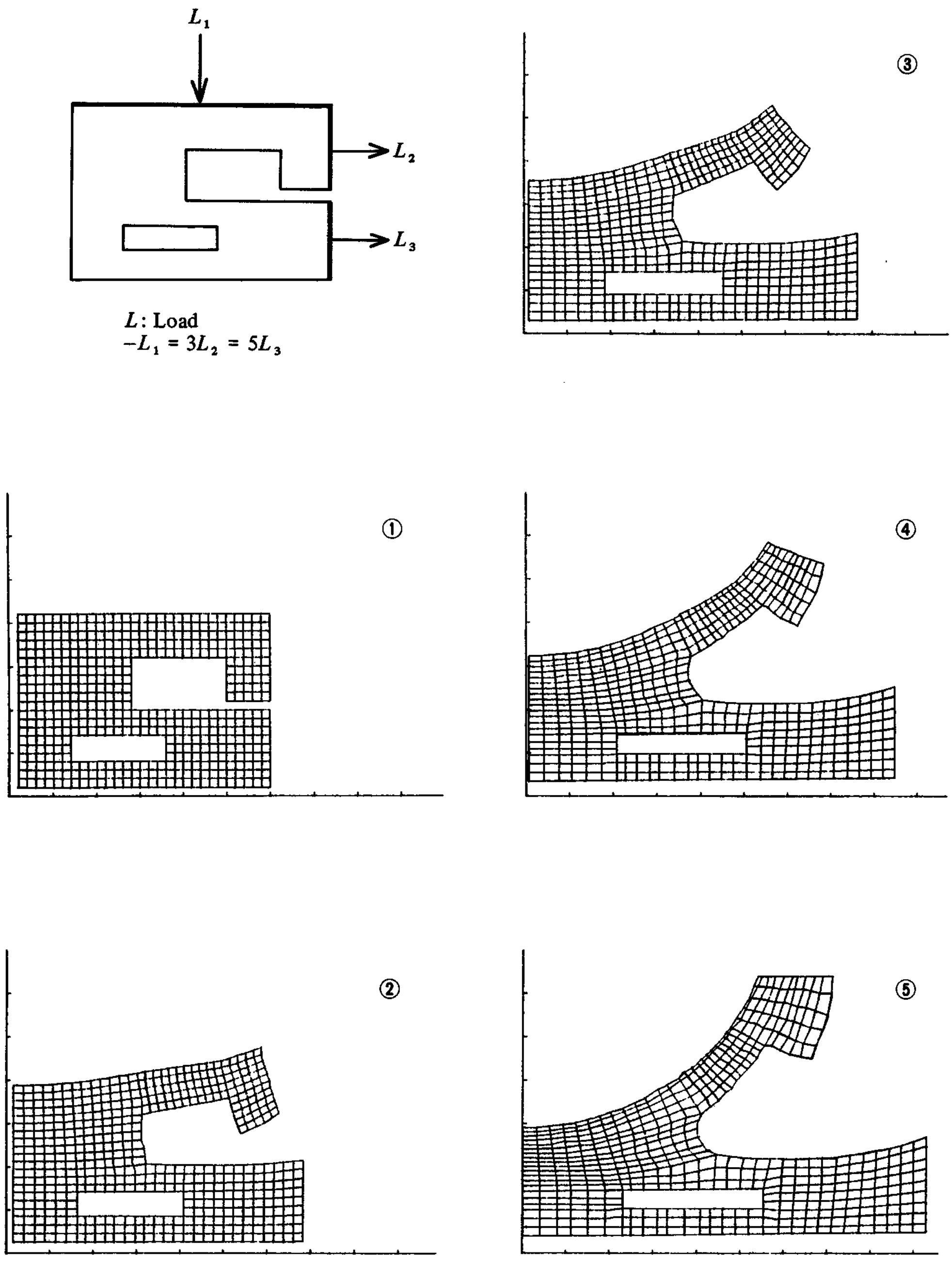

Fig. 5. Elastic deformation of a rubber plate with a hole.

5. Applications on Intermitted Patterns

Applications were performed for the purpose of exhibiting details of the changing characteristics of a meshed structural component to evaluate large displacement.

A body of a car was crushed imaginatively and, the node data of the mesh were input for the calculation to the computer, and the repairing processes were shown automatically by the CAGS. In fact, such a crushed body has, so called, *time delay* in its damaged steps, but the result shown in Fig. 6 seems that the steps are not so far from those of the real case.

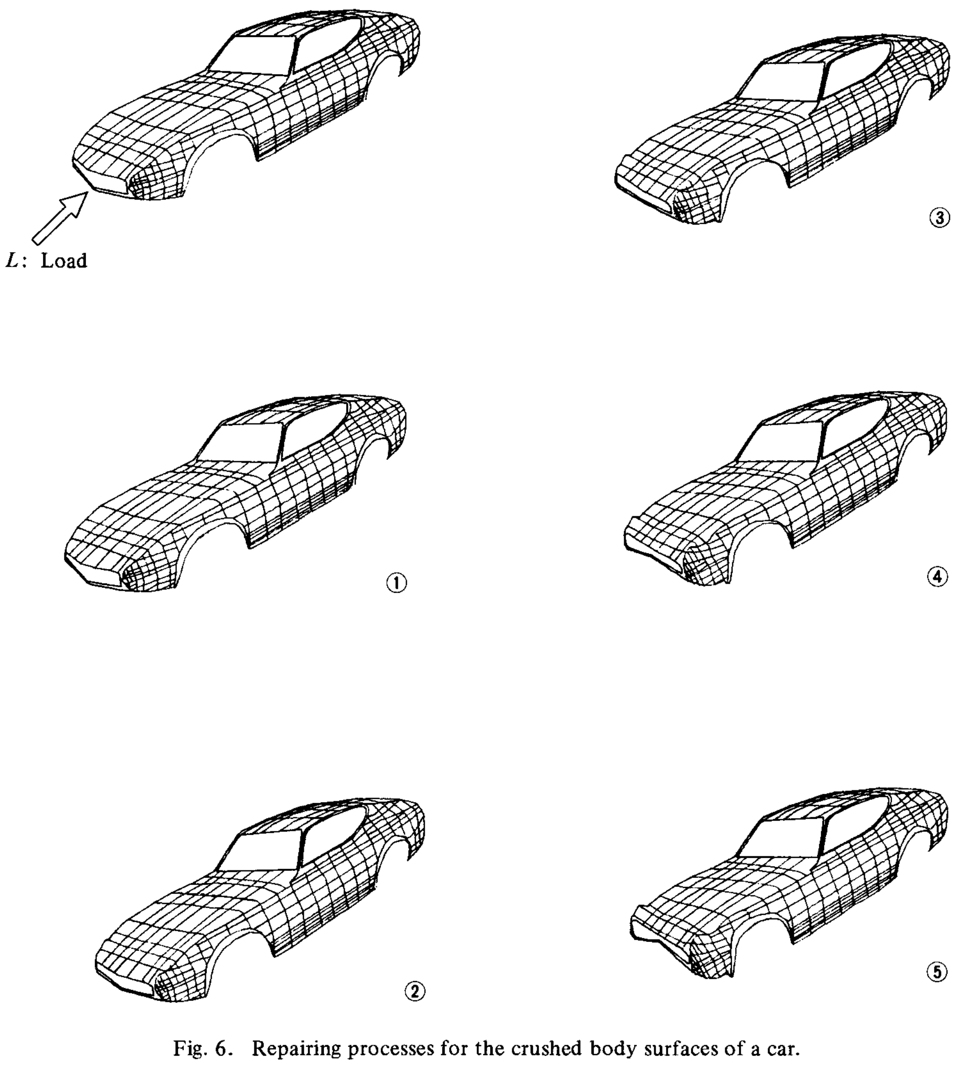

Fig. 6. Repairing processes for the crushed body surfaces of a car.

This method can be applicable not only to the engineering, but also to a part of arts and studies of Chinese characters (*Kanji*), by removing the mesh lines and connecting the points of nodes.

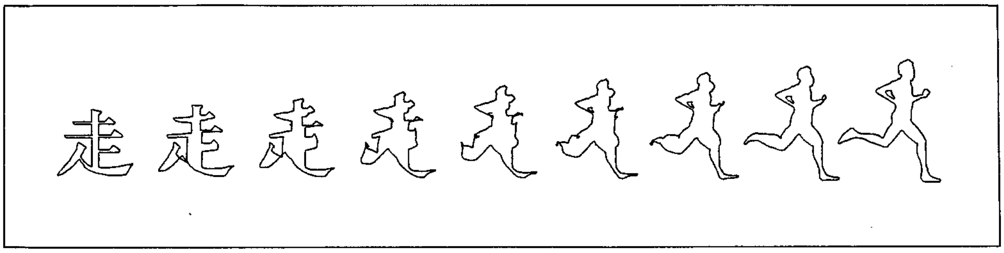

Fig. 7. A verification of the hieroglyph showing a *Kanji* "Run" and a running man.

The left hand side in Fig. 7 is a Chinese character of "Run", and the right hand side is a pattern of a running man. The former is gradually changing its shape and finally becomes quite near to the shape of the latter. This is one example of a hieroglyph, and the same can be applied to the others with the CAGS.

It is well known that some of the Chinese characters were gradually changed to the Japanese letters (*Kana Moji*) in ancient times. Some examples for the Japanese letters are compared with the changes of, so called "*Hentai Kana*" as shown at each upper rows in Fig. 8. The comparison with the changes of upper and low rows shows the validity of the coincidence between the two. The same method can be applied to the other letters by using the CAGS.

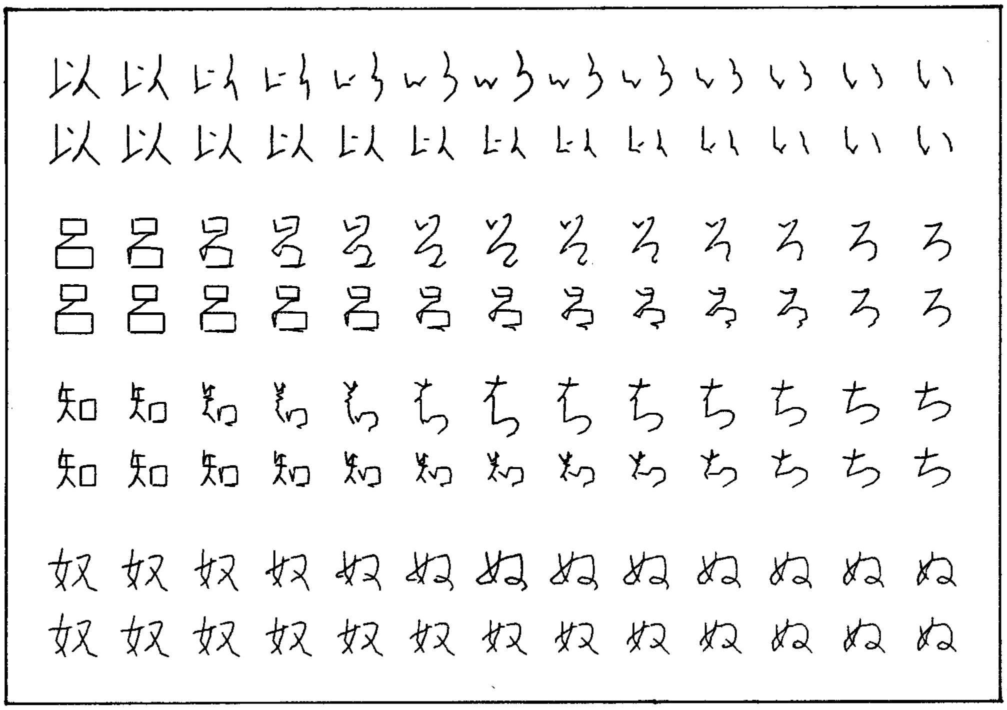

Fig. 8. A comparison with computer aided Japanese letters and shapes of *Hentai Kana* in ancient times.

6. Discussion and Conclusion

With the CAGS, the intermediate patterns can be effectively performed only by processing data of the original and final drawings. Some phenomena which change linearly are most common, and the equation (10) will be widely applicable to many fields. However, some phenomena such as plastic deformation is not a linear but a non-linear change, so that, future work will be done in the area of non-linear applications. When the changing follows, for example, by stress-strain curve (non-linear), then the patterns as Fig. 6 will have more realistic looks than those in the present case.

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References

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2) S. Timoshenko, eta al, *Theory of Elasticity*, p. 6, McGraw-Hill, New York (1951).

Appendix

Hardware and Software of Computer Aided Graphic System

The CAGS hardware, which is shown schematically in Fig. 9 and pictorially in Photo. 1 and 2, consists of a TOSBAC SYSTEM 600 (stands for A-600) computer and

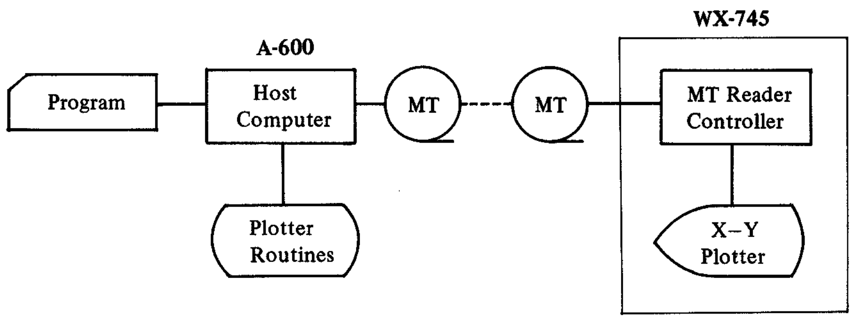

Fig. 9. Schematic drawing of the CAGS.

WATANABE PLOTTER 745 (stands for WX-745) X–Y plotter. The A-600 is used to generate all the pen vector commands using JIS FORTRAN IV level and the WX-745 is capable to plot the vector routines. Three colored ball-pens or ink pens can be chosen as shown in Photo. 3. The CAGS software provides 10 basic and 26 functional sub-routines for the pattern processing, which permit the user to command the CAGS.

The authors handle the CAGS and developed the above mentioned sub-routines for the intermediate pattern processing. The programs are omitted for the save of page waste, and only the flow chart of the outline is shown in Fig. 10.

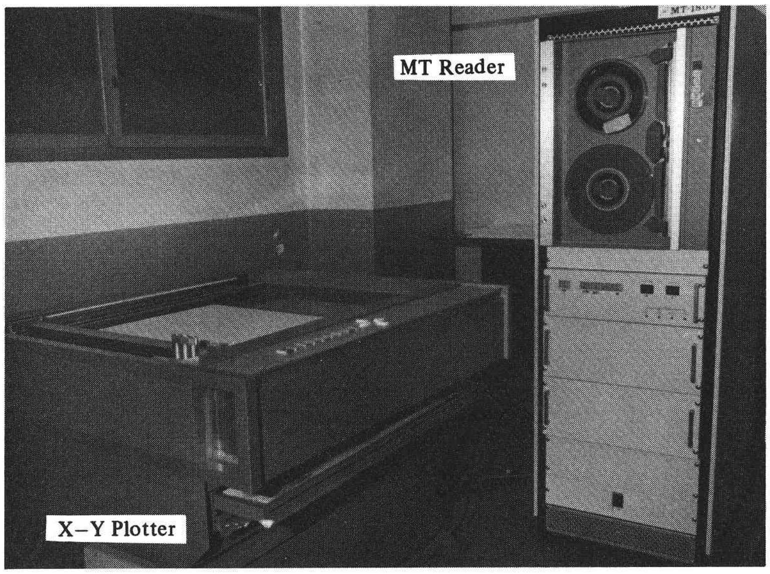

Photo. 1. Hardware aspects of the CAGS.

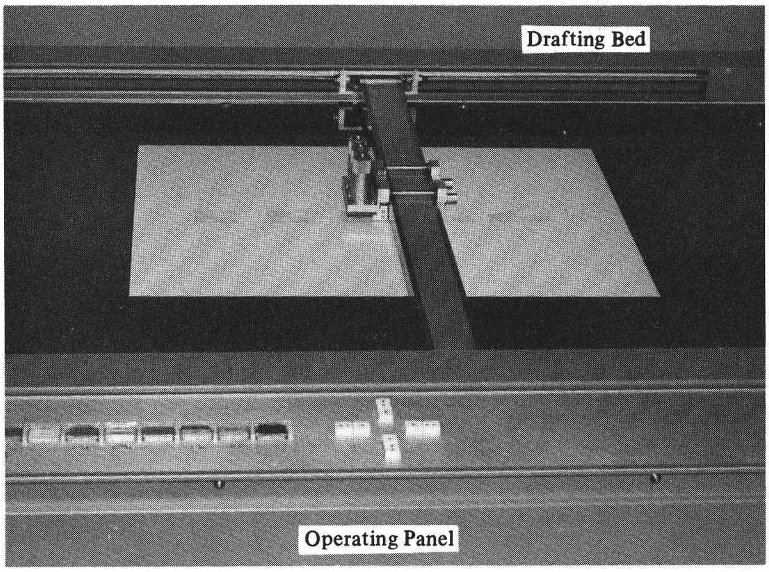

Photo. 2. Outside view of the plotter and its drafting bed.

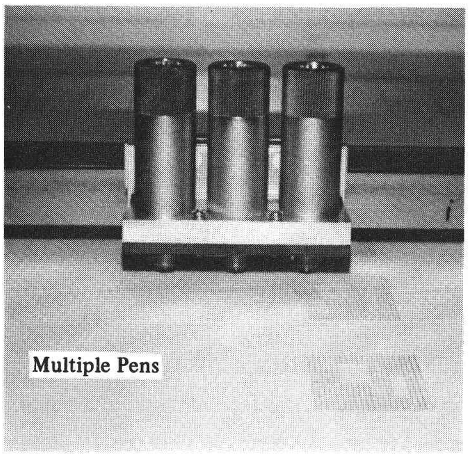

Photo. 3. Close-up of three pens in line.

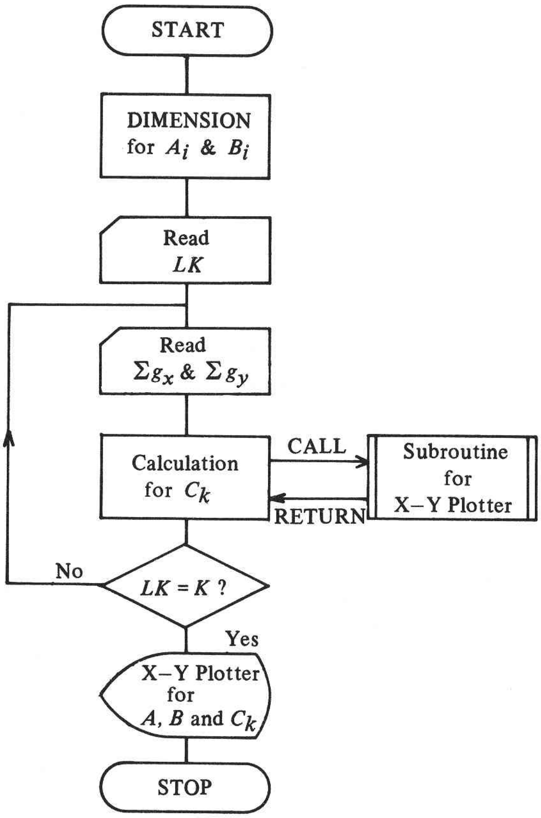

Fig. 10. Flow chart of the newly developed CAGS program.