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Optimum Structural Design Considering Costs Caused by Failure of Structures

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This paper deals with an optimum structural design based on reliability analysis. An expected total cost is defined as a sum of the structural cost and the expected loss caused by failure of a structure. The optimum design problem is set up to minimize the expected total cost. A feature of this problem lies in that the optimum value of reliability is determined together with the optimum structure. An algorithmic procedure is presented to solve the problem by applying stochastic programming and a uni-dimensional search technique. Design examples are provided of a three, and a thirteen, member trusses.

1. Introduction

Loads acting on structures and strength of the structural elements are sometimes subject to random variations. In such a case, structural reliability, or alternatively, the probability of failure has been used as a criterion for structural safety. Applying reliability analysis, optimum design problems have been studied[1~9] to determine the structure minimizing the structural cost or weight.

The authors treated in the previous papers[10),11)] a problem to determine the optimum structure minimizing the structural cost or weight under the specified failure probability of the structure and proposed an efficient algorithmic procedure to solve the problem by applying stochastic programming. However, there are some cases where the allowable failure probability can not be specified. In such cases, alternative formulations of the optimum design problems are necessary.

In this paper, a problem is considered to determine simultaneously the optimum value of failure probability and structure when the costs caused by failure of the structure are specified. For this purpose, defined is the expected total cost which is taken as a sum of the structural cost and the expected costs due to failure of the structure. An algorithmic procedure is developed and numerical examples are presented.

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2. Statement of Problem

Consider a structural system in which safety margins of failure modes are described by a linear combination of the resistances of elements and loads acting on the structure. That is, the safety margins of failure modes are given by

$$Z_i = \sum_{j=1}^{n} a_{ij} R_j - \sum_{j=1}^{l} b_{ij} L_j \qquad (i = 1, 2, \cdots, m), \tag{1}$$

where R_j = structural resistance of the j-th element,
L_j = load acting on the structure,
a_{ij} = resistance coefficient determined by the position and condition of the j-th element related to the i-th failure mode,
b_{ij} = load coefficient determined by the position and magnitude of the j-th load related to the i-th failure mode,
n = number of structural elements,
l = number of loads,
m = number of failure modes.

Failure of the structure occurs if any value of Z_i $(i = 1, 2, \cdots, m)$ is negative, *i.e.*, any one of failure modes happens. When structural resistances, R_j's, and loads, L_j's, exhibit statistical variations and thus they are treated as random variables, safety margins, Z_i's, become also random variables. Hence safety of the structure must be evaluated in statistical terms. Let F_i be the event of failure of mode i and $\bar{F}_i$ survival of mode i. The failure probability of the structure can be written as

$$\begin{aligned} P_f &= \mathrm{Prob}\,(F_1) + \mathrm{Prob}\,(\bar{F}_1 \cap F_2) + \mathrm{Prob}\,(\bar{F}_1 \cap \bar{F}_2 \cap F_3) + \cdots \\ &\qquad + \mathrm{Prob}\,(\bar{F}_1 \cap \bar{F}_2 \cap \cdots \cap \bar{F}_{m-1} \cap F_m) \\ &= 1 - \mathrm{Prob}\,(\bar{F}_1 \cap \bar{F}_2 \cap \cdots \cap \bar{F}_m). \end{aligned} \tag{2}$$

Structural resistance, R_j, is a function of dimension of the element, A_j, such as cross-sectional area and strength of the material, C_{yj}, (*e.g.*, yield stress) to be used, both of which are in general random variables. However, only C_{yj}'s are treated as random variables in this paper, while A_i's are given as deterministic variables. As the design variables, the resistances of structural elements are adopted, and the dimensions of structural elements are assumed to be determined by the mean values of structural resistances and strengths of the materials, $\bar{R}_j$ and $\bar{C}_{yj}$, *i.e.*,

$$A_j = A_j(\bar{R}_j, \bar{C}_{yj}). \tag{3}$$

The structural cost is a function of the dimensions of structural elements when the materials to be used are specified, and thus from Eq. (3) it can be written as

$$H_C = H_C(\bar{R}_1, \bar{R}_2, \cdots, \bar{R}_n). \tag{4}$$

Now consider the case where failure probability P_f is determined by specifying only the mean values of structural resistances, $\bar{R}_j$'s, if the probabilistic natures of the loads, $\bar{L}_j$'s, are given. Such a case is experienced when R_j's are Gaussian random variables with known coefficients of variation.

Let $H_f(P_f)$ denote the expected costs caused by failure of the structure, such as cost of reconstruction, cost of compensation, cost due to loss of social prestige, etc., when failure probability is P_f.

The expected total cost H_T is defined by

$$H_T = H_C + H_f(P_f). \tag{5}$$

The problem to be considered is as follows:

PROBLEM "Given the configuration of the structure and the materials to be used, determine the structural resistances, $\bar{R}_j$'s, to minimize the expected total cost."

It should be noted here that by solving the problem the optimum value of failure probability, or alternatively reliability, of the structure is determined together with the optimum structure.

3. Solution of Problem

It takes much time to calculate multi-dimensional probability distribution functions for evaluating failure probability of the structure, P_f, in Eq. (2). Further probability thus evaluated is an approximate one, using any method so far developed for calculating multi-dimensional probability distribution functions. Thus, it is desirable to employ a search method to attain the optimum solution without using the derivative of P_f, which requires much processing time and may result in accumulation of errors. For the purpose, consider a subproblem:

SUBPROBLEM "Specified the allowable probability level, P_{fa}, determine the optimum values of the resistances, $\bar{R}_j$'s, to minimize the structural cost, H_C, under the constraint:

$$P_f \leq P_{fa} \qquad (6)."$$

This subproblem is equivalent to the problem treated in the previous paper,[10,11] and can be solved efficiently by the algorithmic procedure proposed previously. An important property of the solution to the subproblem, which will be proved in the following section (see *LEMMA* 1), is that the solution to the subproblem is attained on the boundary, *i.e.*, $P_f = P_{fa}$. Consequently, the solution to the original problem is obtained by sequentially solving the subproblem. The algorithmic procedure is given as follows:

Step 1: Specify the initial value of P_{fa}.

Step 2: For the given value of P_{fa}, solve the subproblem and calculate the expected total cost corresponding to the optimum solution thus obtained. If optimality condition for the original problem is satisfied, stop the calculation. Otherwise, go to *Step* 3.

Step 3: Applying a uni-dimensional search technique,[13] the value of P_{fa} is adjusted so as to minimize the expected total cost, H_T. Go to *Step* 2.

The flow chart for the above procedure is given in Fig. 1, and the mathematical background is given in the next section.

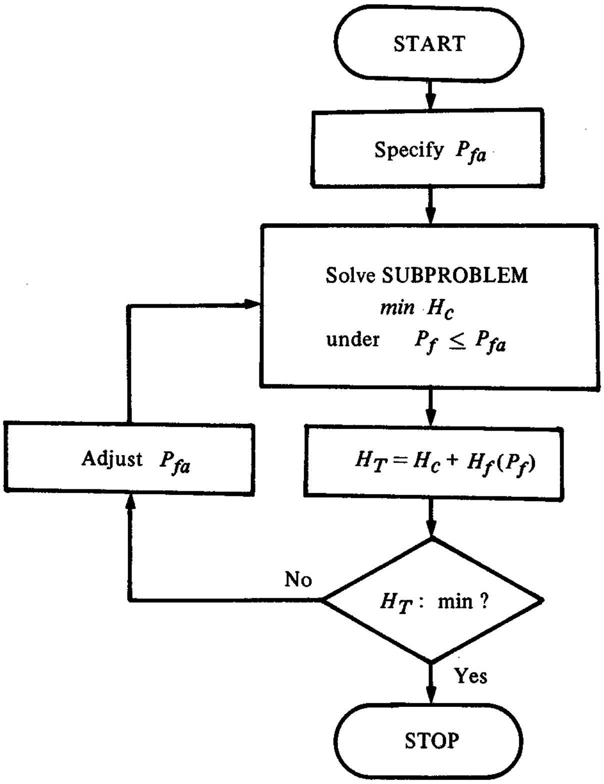

Fig. 1. Algorithmic procedure for solving problem.

4. Mathematical Background of Algorithmic Procedure

Let the design variables be expressed by n-dimensional vector $R = (R_1, R_2, \cdots, R_n)^T$ and its design space be a subspace of n-dimensional Euclidean space E^n, *i.e.*, $\Gamma \subset E^n$, where superscript T means to take transpose of vector. The structural cost, H_C, and failure probability, P_f, is a function of the design vector R, and thus they are rewritten as

$$H_C = H_C(R), \qquad P_f = P_f(R) \quad \text{for} \quad R \in \Gamma .$$

In structural systems, the structural costs increase in general as the design variables are taken to be large, while failure probabilities decrease for the cases considered. Hence the following conditions are satisfied in general:

($C1$) $H_C(R)$ is componentwise increasing, *i.e.*, for some $j \in [1, 2, \cdots, n]$ and for any R^1 and $R^2 \in \Gamma$ such that $R^2 - R^1 = (R_j^2 - R_j^1)e_j > 0$, $H_C(R)$ is increasing along $[R^1, R^2]$. e_j is a n-dimensional unit vector with the j-th element of unit and all others of zero.

($C2$) $P_f(R)$ is componentwise decreasing, *i.e.*, $P_f(R)$ is decreasing along $[R^1, R^2]$ as defined in ($C1$).

The following lemma holds for the solution to *SUBPROBLEM*:

LEMMA 1: The solution to the subproblem is attained on the boundary of the probability constraint, *i.e.*, $P_f(R^*) = P_{fa}$.

PROOF. For any vector R^1 contained in an open set:

$$G \triangleq [R \mid P_f(R) < P_{fa}],$$

i.e., $R^1 \in G$, there exists a number $\epsilon > 0$ which defines the ϵ-neighbourhood of R^1:

$$O_\epsilon(R^1) \triangleq [R \mid \| R^1 - R \| < \epsilon] \subset G.$$

Consider a vector R^0 whose elements R_j^0 are identical with those of R^1 except the i-th element, *i.e.*,

$$R_i^0 = R_i^1 - \epsilon/2, \quad R_j^0 = R_j^1 \quad (j = 1, 2, \cdots, n, \; j \neq i),$$

and which satisfies

$$R^0 \in O_\epsilon(R^1) \subset G.$$

From the condition ($C1$), the following inequality holds between the structural costs corresponding to R^0 and R^1

$$H_C(R^1) > H_C(R^0).$$

Consequently, R^1 can not be an optimum solution to *SUBPROBLEM* (*q. e. d.*).

For the expected total cost, the following lemma holds:

LEMMA 2: If the failure probability, P_f, is specified to be P_{fa}, the expected total cost, H_T, is minimum for the solution to the subproblem.

PROOF. For the specified value of P_f, the second term of H_T is constant, *i.e.*, $H_f(P_f) = H_f(P_{fa}) =$ constant. Then

$$\begin{aligned} H_T^o(P_{fa}) &\triangleq \min_{R \in \Gamma, P_f = P_{fa}} [H_C + H_f(P_f)] \\ &= \min_{R \in \Gamma, P_f = P_{fa}} H_C + H_f(P_{fa}) \\ &= \min_{R \in \Gamma, P_f \leq P_{fa}} H_C + H_f(P_{fa}). \end{aligned}$$

The last relation follows from *LEMMA* 1 (*q. e. d.*).

From *LEMMA*s 1 and 2, the following lemma concerning the solution of the original problem holds:

LEMMA 3: The solution to the original problem is obtained by sequentially solving the subproblem.

PROOF. From *LEMMA*s 1 and 2, the following relation results:

$$\begin{aligned} H_{T_{min}} \triangleq \min_{R\in\Gamma} H_T &\triangleq \min_{P_{fa}} H_T^o(P_{fa}) \\ &= \min_{P_{fa}} [\min_{R\in\Gamma, P_f=P_{fa}} (H_C + H_f(P_f))] \\ &= \min_{P_{fa}} [\min_{R\in\Gamma, P_f=P_{fa}} H_C + H_f(P_{fa})] \\ &= \min_{P_{fa}} [\min_{R\in\Gamma, P_f\le P_{fa}} H_C + H_f(P_{fa})] \qquad (q.\ e.\ d.). \end{aligned}$$

Denote the structural cost corresponding to the optimum solution of the subproblem for a specified value of P_{fa} as $H_C^o(P_{fa})$, *i.e.*,

$$H_C^o(P_{fa}) = \min_{R\in\Gamma, P_f(R)\le P_{fa}} H_C(R)$$

The following lemma holds:

LEMMA 4: $H_C^o(P_{fa})$ is a decreasing function of P_{fa}.

PROOF. For the specified values of the allowable failure probability such that $P_{fa}^1 < P_{fa}^2$, consider the corresponding feasible regions:

$$F^1 \triangleq [R \mid P_f(R) \le P_{fa}^1], \quad F^2 \triangleq [R \mid P_f(R) \le P_{fa}^2].$$

The condition $(C2)$ yields

$$F^1 \subset F^2 .$$

Hence $$H_C^o(P_{fa}^1) > H_C^o(P_{fa}^2)$$

from *LEMMA* 1 (*q. e. d.*).

Finally it is clear from *LEMMA* 3 that the following proposition holds concerning the algorithmic procedure for solving the original problem:

PROPOSITION: The solution to the original problem is obtained by the procedure given in Section 3, performing uni-dimensional search with respect to the allowable failure probability.

It should be remarked here that the algorithm does not always work well if the expected total cost, H_T, is not unimodal with respect to the allowable failure probability, P_{fa}. In that case, optimization should be started from a number of initial values of P_{fa}, and search for the global minimum is to be carried out since the solution from any one initial value may be a local minimum.

5. Numerical Examples

5.1 Design of three member statically indeterminate truss

Consider a plastic design of an statically indeterminate three member truss structure shown in Fig. 2. The failure of the structure occurs when any two members among three

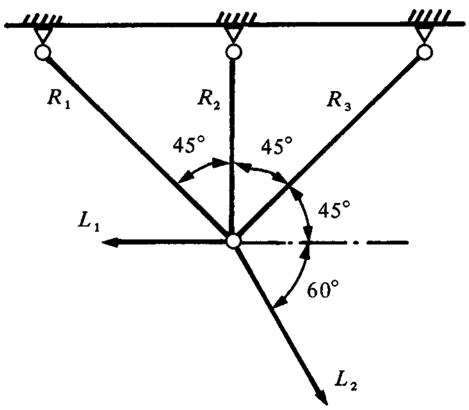

Fig. 2. Three member statically indeterminate truss.

collapse. Thus the following three failure modes are considered as primary modes of failure and their safety margins, Z_i's, are given by

i) Members 1 and 2 collapse both in tension:

$$Z_1 = R_1 + \frac{\sqrt{2}}{2}R_2 + \frac{\sqrt{2}}{2}L_1 - \frac{\sqrt{2}(\sqrt{3}+1)}{2}L_2, \tag{7}$$

ii) Members 2 and 3 collapse both in tension:

$$Z_2 = \frac{\sqrt{2}}{2}R_2 + R_3 - \frac{\sqrt{2}}{2}L_1 - \frac{\sqrt{2}(\sqrt{3}-1)}{4}L_2, \tag{8}$$

and *iii*) Members 1 and 3 collapse in compression and tension, respectively:

$$Z_3 = \frac{\sqrt{2}}{2}R_1 + \frac{\sqrt{2}}{2}R_3 - L_1 + \frac{1}{2}L_2. \tag{9}$$

The failure probability of the structure is calculated as

$$\begin{aligned} P_f &= \mathrm{Prob}(Z_1 < 0) + \mathrm{Prob}(Z_1 \geq 0 \cap Z_2 < 0) + \mathrm{Prob}(Z_1 \geq 0 \cap Z_2 \geq 0 \cap Z_3 < 0) \\ &= 1 - \mathrm{Prob}(Z_1 \geq 0 \cap Z_2 \geq 0 \cap Z_3 \geq 0) \end{aligned} \tag{10}$$

In this example, the resistance of the member, R_j, is related to their respective cross sectional area, A_j, and yield stress, C_{yj}, as

$$R_j = C_{yj} A_j . \tag{11}$$

The structural cost is given by

$$H_C = \sum_{j=1}^{3} C_{mj} d_j l_j A_j , \tag{12}$$

where C_{mj} = material cost of the j-th member per unit weight,
d_j = specific weight of the j-th member,
and l_j = length of the j-th member.

The expected loss due to structural failure is given by

$$H_f(P_f) = C_f P_f . \tag{13}$$

Consider the case where the resistances of the members, R_j's, and the loads, L_j's are independent Gaussian random variables and the coefficients of variations, CV_{Rj} and CV_{Lj}, and the means of the loads, $\bar{L}_j$, are given. Then the failure probability of the structure is determined by specifying the mean value of the strengths, $\bar{R}_j$.

For example, when the cross sectional area, A_j, is deterministic variable and the yield stress, C_{yj}, is Gaussian random variable with known mean, $\bar{C}_{yj}$, and coefficient of variation, CV_{yj}, the resistance of the j-th member, R_j, becomes Gaussian random variable with the coefficient of variation equal to CV_{yj} as seen from Eq. (11). Consequently, A_j's are determined as $\bar{R}_j/\bar{C}_{yj}$. Further the safety margins given by Eqs. (7)–(9) become Gaussian random variables, and thus to evaluate failure probability, P_f, three dimensional Gaussian distribution functions need to be calculated. For the purpose, the method developed in the previous paper[12] is used. Data concerned are listed in Table 1.

Table 1. Data concerned for three member truss.

j	l_j in	CV_{Rj}	$\bar{C}_{yj}$ksi	$C_{mj}d_j$\$/in^3	$\bar{L}_j$kips	CV_{Lj}
1	$60\sqrt{2}$	0.05	40	0.03	100	0.2
2	60	0.05	40	0.03	150	0.2
3	$60\sqrt{2}$	0.05	40	0.03		

Fig. 3 illustrates a search procedure in Steps 2 and 3 given in Section 3, using the quadratic approximation[13] for the case of $C_f = 10^3$ \$.

The optimum solutions are listed in Table 2 for various values of C_f. As the value of C_f becomes large, *i.e.*, the cost due to failure of the structure becomes large, the

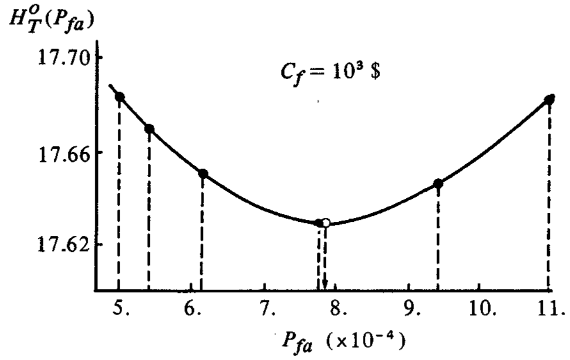

Fig. 3. Sequences searching for optimum failure probability using quadratic approximation method.

Table 2. Optimum solutions for various values of C_f (three member truss).

C_f\$	$\bar{R}_1$ kips	$\bar{R}_2$ kips	$\bar{R}_3$ kips	P_f	H_C^o\$	H_T^o\$
10^2	70.0	132.0	62.4	9.64×10^{-3}	14.71	15.67
10^3	89.2	140.0	70.4	7.79×10^{-4}	16.85	17.63
10^4	105.2	146.8	76.8	6.79×10^{-5}	18.61	19.29
10^5	119.3	152.8	82.4	6.16×10^{-6}	20.15	20.77
10^6	132.0	158.0	87.2	5.71×10^{-7}	21.55	22.13

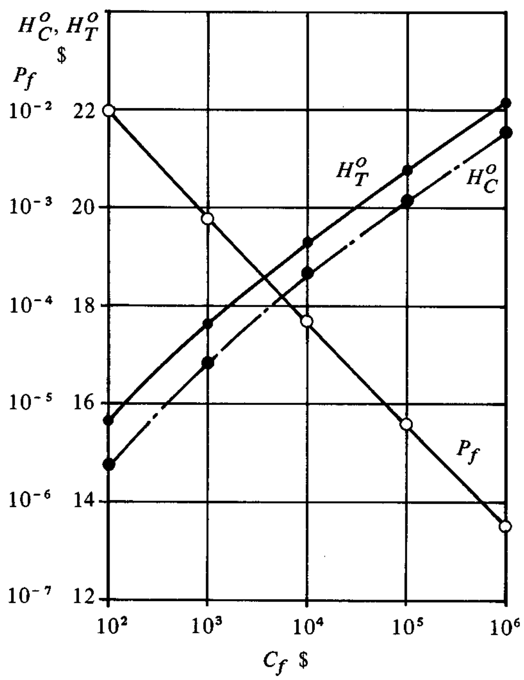

Fig. 4. Effect of cost due to failure of structure on optimum solution (three member truss).

optimum failure probability becomes small, while the structural cost becomes large. This fact is also schematically shown in Fig. 4.

5.2 Design of thirteen member statically determinate truss

Consider a plastic design of a thirteen member statically determinate truss shown in Fig. 5. The safety margins of failure modes are given by

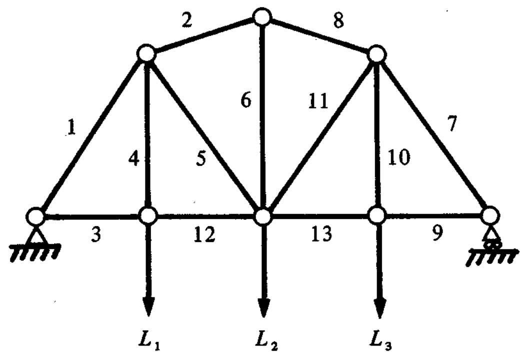

Fig. 5. Thirteen member statically determinate truss ($l_1 = \sqrt{3}\, l_0$, $l_2 = \sqrt{6 - 2\sqrt{6}}\, l_0$, l_0 = 100 in).

$$
\begin{aligned}
z_1 &= R_1 - 0.9186 L_1 - 0.6124 L_2 - 0.3062 L_3 \\
z_2 &= R_2 - 0.3029 L_1 - 0.6058 L_2 - 0.3029 L_3 \\
z_3 &= R_3 - 0.5303 L_1 - 0.3535 L_2 - 0.1768 L_3 \\
z_4 &= R_4 - 1.0000 L_1 \\
z_5 &= R_5 + 0.4186 L_1 - 0.3876 L_2 - 0.1938 L_3 \\
z_6 &= R_6 - 0.1835 L_1 - 0.3670 L_2 - 0.1835 L_3 \\
z_7 &= R_7 - 0.3062 L_1 - 0.6124 L_2 - 0.9186 L_3 \\
z_8 &= R_8 - 0.3029 L_1 - 0.6058 L_2 - 0.3029 L_3 \\
z_9 &= R_9 - 0.1768 L_1 - 0.3535 L_2 - 0.5303 L_3 \\
z_{10} &= R_{10} - 1.0000 L_3 \\
z_{11} &= R_{11} - 0.1938 L_1 - 0.3876 L_2 + 0.4186 L_3 \\
z_{12} &= R_{12} - 0.5303 L_1 - 0.3536 L_2 - 0.1768 L_3 \\
z_{13} &= R_{13} - 0.1768 L_1 - 0.3536 L_2 - 0.5303 L_3 \,.
\end{aligned}
\tag{14}
$$

Failure probability may be evaluated by Eqs. (2) and (14). However, dimension is too high to exactly calculate it, and thus the approximate formula developed in the previous paper[11] will be applied in the following calculations.

The structural cost and the expected loss are assumed to be given in the similar manner as in the previous example.

The resistances of structural elements and the loads acting on the structure are assumed to be independent Gaussian random variables. The data concerned are listed in Table 3.

The optimum solution for some values of C_f is given in Table 4. It is seen that the optimum structure is of symmetric form. The minimum expected total cost for the given value of the allowable failure probability, $H_T^o(P_{fa})$, is plotted against the allowable failure probability, P_{fa}, in Fig. 6. It is interesting to note that the minimum expected total cost is not so sensitive to failure probability in this case, which is also true in the previous example as shown in Fig. 3.

Table 3. Data concerned for thirteen member truss.

CV_{Rj}	CV_{yj}	$C_{mj}d_j$	$\bar{L}_j$kips	CV_{Lj}
0.15	40	0.01	20	0.15

Table 4. Optimum solutions for thirteen member truss.

C_f	$\bar{R}_1$	$\bar{R}_2$	$\bar{R}_3$	$\bar{R}_4$	$\bar{R}_5$	$\bar{R}_6$	P_{fa}	H_C^o	H_T^o
10^4	95.63	67.10	59.23	58.08	17.00	40.71	2.15×10^{-4}	25.11	27.26
10^5	116.01	81.35	72.37	70.17	20.52	49.45	2.70×10^{-5}	30.43	33.13

$\bar{R}_1 = \bar{R}_7,\ \bar{R}_2 = \bar{R}_8,\ \bar{R}_3 = \bar{R}_9 = \bar{R}_{12} = \bar{R}_{13},\ \bar{R}_4 = \bar{R}_{10},\ \bar{R}_5 = \bar{R}_{11}$ kips

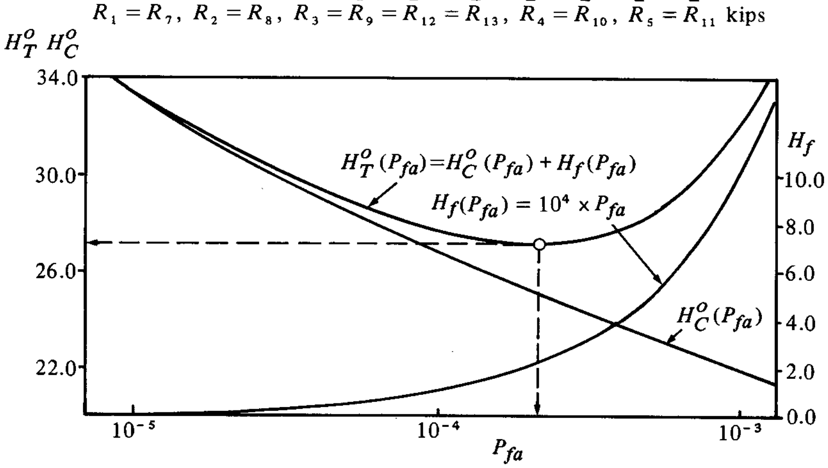

Fig. 6. Minimum expected total cost and minimum structural cost plotted against allowable failure probability (thirteen member truss).

6. Conclusion

An optimum design problem is considered to minimize the expected total cost defined as a sum of the structural cost and the expected loss caused by failure of the structure. A feature of this problem lies in that the optimum value of failure probability is determined together with the optimum values of the structural elements. A method is proposed to solve the problem by applying stochastic programming and a uni-dimensional search technique. Numerical examples are presented to illustrate the design procedures.

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