Effects of Carrier Offset on Interference in FDM-FM Transmission Systems

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Using the direct method of analysis, the second and third order intermodulation noises in a FDM-FM signal passed through a transmission system in which a AM-PM converter follows a linear network is expressed in terms of the parameters of the system and modulating signal. When the transfer function of the network is represented by a polynomial series up to the fifth order in angle frequencies, the resulting intermodulation noise for both cases with and without pre-emphasis are presented. Further, the effects of carrier frequency offset on signal-to-interference ratio in the case of a single-pole filter are investigated by the analysis carried out in this paper and Monte Carlo method.

1. Introduction

The distortion in a FM signal passed through a transmission system is one of the important subjects on the design of the system. Therefore, this subject has been studied by many authors[1)~7)], and the many efforts have been made to express intermodulation noise as a function of the given transmission deviation. Also, computer-simulation methods for the distortion have been developed because of the difficulty of its analysis[8)9)].

When the transfer function of a network in the system is represented by a polynomial series in angle frequencies, the authors have pointed out that the series must approximate sufficiently the transfer function within a frequency band of the spectral distribution of FM signal which influences significantly the distortion[6)].

Further, the authors have shown that the output power spectrum consists of the components of four kinds by applying the so-called direct method to the spectral analysis for the case of Gaussian noise modulation[6)]. Their components are the linear signal, the cross-power signal, intermodulation noise components, and the spectral component due to the term falling on the linear term from the third-order distortion. Bedrosian and Rice have expressed the output power spectrum by the components of three kinds: the linear signal, the cross-power signal, and intermodulation noise and presented their numerical results for the case of a symmetrical single-pole filter[4)].

In this paper, when the transfer function is represented by a polynomial series up to the fifth order, the second and third order intermodulation noises in the system with AM-PM converter are presented for the cases of a bandlimited, zero-mean,

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white Gaussian noise and the one colored by pre-emphasis network. For the case of a single-pole filter of interest in FM communication systems, the effects of carrier frequency offset on signal-to-interference ratio (SIR) are investigated, and examined by Monte Carlo computations. Further, it is illustrated that the interference quantity improved by inserting CCIR pre-emphasis network depends on the carrier offset. These results will be useful in evaluating FM systems in which carrier offsets must be taken into consideration.

2. The Second and Third Order Intermodulation Noises

Let $e_i(t)$ denote FM signal at the input of a transmission system in a complex form as follows:

$$e_i(t)=\exp[j\{\omega_c t+\Delta\omega\int^t \phi(t)\,dt+\theta_0\}], \tag{1}$$

where ω_c is a carrier angle frequency, $\Delta\omega$ an angle deviation coefficient, $\phi(t)$ a modulating signal, and θ_0 a constant phase angle. Let the normalized transfer function $Z_N(j\omega)$ of a linear network in the system be represented by a polynomial series up to the Nth-order in radian frequencies, within a frequency band, as follows:

$$Z_N(j\omega)=1+\sum_{n=1}^{N}(\alpha_n+j\beta_n)(\omega-\omega_c)^n, \tag{2}$$

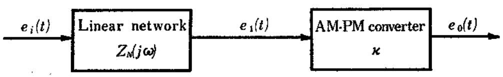
where N is an arbitrary integer and the frequency band is that of spectral distribution of FM signal which influences significantly the distortion noise of the output power spectrum. When the distortion up to the third order is considered, one half of the frequency band is given by

$$f_e=3\cdot\max(B, D_v), \tag{3}$$

where B is the top frequency of a baseband signal and D_v the rms frequency deviation. Hence, the coefficients α_n and β_n in the polynomial series are determined by Taylor expansion or least-square method or other methods so that the series approximates sufficiently the linear network within the frequency band f_e.

Let $a(t)$ and $\theta(t)$ be the amplitude modulation and phase components of FM signal $e_1(t)$ at the output of the network, respectively, as shown in Fig. 1.

Then, the phase rate of FM signal $e_0(t)$ passed through the transmission system is given by $\overset{(1)}{\theta(t)}+\kappa\overset{(1)}{a(t)}$, where κ is a constant AM-PM conversion coefficient in



$$e_i(t)=\cos\{\omega_0 t+\Delta\omega\int^t \phi(\tau)d\tau+\theta_0\},$$
$$e_1(t)=\exp[a(t)]\cos\{\omega_c t+\theta(t)\},$$
$$e_0(t)=\exp[\overline{a}(t)]\cos\{\omega_c t+\theta(t)+\kappa a(t)\}.$$

Fig. 1. Model of FM transmission system

radians and the superscript (1) denotes the first derivative with respect to time t[2)3)]. These output phase rate terms $\overset{(1)}{\theta(t)}$ and $\overset{(1)}{a(t)}$ can be represented by the power series of $\Delta\omega$ as follows:

$$\overset{(1)}{\theta(t)}=\sum_{n=1}^{\infty} D_n \Delta\omega^n, \quad \overset{(1)}{a(t)}=\sum_{n=1}^{\infty} A_n \Delta\omega^n, \tag{4}$$

where the nth order distortion terms D_n and A_n are represented by the nth terms of $\overset{(1)}{\phi(t)}, \overset{(2)}{\phi(t)}, \dots : \overset{(l_1)}{\phi(t)} \cdot \overset{(l_2)}{\phi(t)} \cdots \overset{(l_n)}{\phi(t)}$ $(l_1, l_2, \dots, l_n=0, 1, 2, \dots)$ and α_i, β_j $(i, j=1,2,\dots, N)$ and their higher order terms: $\alpha_{i_1}\cdot\alpha_{i_2}\cdots\beta_{j_1}\cdot\beta_{j_2}\cdots (i_1, \dots, j_1, \dots =1, 2, \dots)$, up to the terms determined from the order N of the polynomial of the transfer function.

Now, let us consider the distortion up to the third order, then the output phase rate is represented by the terms $D_n \Delta\omega^n$ and $A_n \Delta\omega^n$ up to $n=3$.

Let a modulating signal be a stationary, differentiable, bandlimited, zero-mean Gaussian noise having a variance σ^2 and a spectral density $W(f)$. Further, let $w(f)$ be normalized spectral density such as $w(f)=W(f)/\sigma^2$ and $\phi(t)$ be the normalized noise represented by a Fourier series in a complex form such as $\phi(t)=2^{-1}\sum_{m_1} C_{m_1}\cdot\exp[j\{2\pi f_{m_1}t-\varphi_{m_1}\}]$ under conditions $C_{m_1}=C_{-m_1}$ and $\varphi_{m_1}=-\varphi_{-m_1}$, where $C_{m_1}=\sqrt{2w(f_{m_1})\Delta f}$, $f_{m_1}=m_1\Delta f$, $\Delta f=1/T$, and T is a period of $\phi(t)$.

Then, upon replacing $\Delta\omega$ by σ, Eq. (4) can be applied to the spectral analysis for the case of noise modulation.

For a transfer function represented by the polynomials up to the fourth order in the amplitude and phase characteristics versus the angle frequency, Cross has tabulated the amplitude modulation and phase terms which contribute to the distortion up to the third order and derived intermodulation noise by using the correlation method in the analysis. When α_n and β_n up to $n=4$ are composed of the coefficients in the series of these characteristics by Taylor expansion, the distortion terms obtained from Eq. (4) include his results in the table.

In this paper, we assume that the transfer function is sufficiently approximated by the polynomial series up to the fifth order; that is, α_n and β_n up to $n=5$, within a given frequency band f_e. Applying the direct method to the spectral analysis as described in Ref. (6), the power spectrum $W_0(f)$ of the output phase rate can be expressed by

$$W_0(f)=W^L(f)+W^{L'}(f)+W^C(f)+W^I(f)\,, \tag{5}$$

where $W^L(f)$ is the linear signal component, $W^{L'}(f)$ the component arising from the third order distortion term, $W^C(f)$ the cross-power component between the linear and third order distortion terms, $W^I(f)$ intermodulation noise. Further, $W^I(f)$ consists of the second order intermodulation noise $W^{I''}(f)$ and the third order one $W^{I'''}(f)$; that is, $W^I(f)=W^{I''}(f)+W^{I'''}(f)$.

Each spectrum can also be expressed by the components due to the distortion term $\overset{(1)}{\theta(t)}$ in the network, due to the AM-PM conversion $\kappa\overset{(1)}{a(t)}$, and due to their interaction. The subscripts D, A, and DA are used to classify those components in the following expressions.

In noise loading test on the transmission system, the spectra $W^L(f)$, $W^{L'}(f)$, and $W^C(f)$ are made to vanish by slotting the input. Therefore, let us investigate only intermodulation noise here.

Let $D_n^r(f_1, \ldots, f_n)$ and $A_n^r(f_1, \ldots, f_n)$ denote respectively the cosine-sine components of the nth order distortion terms D_n and A_n, independent of the spectral density of the modulating signal, where r is equal to c or s and the superscripts c and s express respectively the cosine- and sine-components. Further, for simplifying the expressions, we use the following notation:

$$F_n^r(f, \ldots, f_n) = \left\{ \begin{matrix} D_n^r(f_1, \ldots, f_n) \\ A_n^r(f_1, \ldots, f_n) \end{matrix} \right\} \tag{6}$$

Then, the second and third order intermodulation noises can be expressed as follows:

$$W^I(f) = W^{I''}(f) + W^{I'''}(f), \tag{7}$$

$$W^{I''}(f) = W_D^{I''}(f) + W_A^{I''}(f) + W_{DA}^{I''}(f) \tag{8}$$

$$\left.\begin{aligned} &\left. \begin{matrix} W_D^{I''}(f)/\sigma^4 \\ W_A^{I''}(f)/(\kappa^2\sigma^4) \end{matrix} \right\} = 2^{-1}\int_{-\infty}^{\infty} [F_2^c(x, f)\{F_2^c(x, f) + F_2^c(f-x, f)\} \\ &\qquad + F_2^s(x, f)\{F_2^s(x, f) + F_2^s(f-x, f)\}]\, w(x) \cdot w(f-x)\, dx, \\ &W_{DA}^{I''}(f) = \kappa\sigma^4 \int_{-\infty}^{\infty} [D_2^c(x, f)\{A_2^c(x, f) + A_2^c(f-x, f)\} \\ &\qquad + D_2^s(x, f)\{A_2^s(x, f) + A_2^s(f-x, f)\}]\, w(x)\, w(f-x)\, dx, \end{aligned}\right\} \tag{9}$$

$$W^{I'''}(f) = W_D^{I'''}(f) + W_A^{I'''}(f) + W_{DA}^{I'''}(f), \tag{10}$$

$$\left.\begin{aligned} &\left. \begin{matrix} W_D^{I'''}(f)/\sigma^6 \\ W_A^{I'''}(f)/(\kappa^2\sigma^6) \end{matrix} \right\} = 2^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [F_3^c(x, y, f)\{F_3^c(x, y, f) \\ &\qquad + F_3^c(x, f+x-y, f) + F_3^c(y-x, f-x, f) + F_3^c(y-x, y, f) \\ &\qquad + F_3^c(f-y, f-x, f) + F_3^c(f-y, f+x-y, f)\} + F_3^s(x, y, f) \\ &\qquad \cdot \{F_3^s(x, y, f) + F_3^s(x, f+x-y, f) + F_3^s(y-x, f-x, f) \\ &\qquad + F_3^s(y-x, y, f) + F_3^s(f-y, f-x, f) + F_3^s(f-y, f+x-y, f)\}] \\ &\qquad \cdot w(x)\, w(y-x)\, w(f-x)\, dx dy, \\ &W_{DA}^{I'''}(f) = 2^{-1}\kappa\sigma^6 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [D_3^c(x, y, f)\{A_3^c(x, y, f) \\ &\qquad + A_3^c(x, f+x-y, f) + A_3^c(y-x, f-x, f) + A_3^c(y-x, y, f) \\ &\qquad + A_3^c(f-y, f-x, f) + A_3^c(f-y, f+x-y, f)\} + D_3^s(x, y, f) \\ &\qquad \cdot \{A_3^s(x, y, f) + A_3^s(x, f+x-y, f) + A_3^s(y-x, f-x, f) \\ &\qquad + A_3^s(y-x, y, f) + A_3^s(f-y, f-x, f) + A_3^s(f-y, f+x-y, f)\}] \\ &\qquad \cdot w(x)\, w(y-x)\, w(f-y)\, dx dy. \end{aligned}\right\} \tag{11}$$

When a network in the system is symmetrical with respect to carrier frequency, the spectra $W_D^{\Gamma''}(f)$, $W_{DA}^{\Gamma''}(f)$, $W_A^{\Gamma'''}(f)$, and $W_{DA}^{\Gamma''}(f)$become zero; that is, the second order intermodulation noise is caused by the amplitude modulation after AM-PM converter, and the third order intermodulation noise is equal to that of the output phase rate of the network.

3. Intermodulation Noise for the Case of Gaussian Noise Modulation

(A) Case without pre-emphasis

As mentioned in Section 2, when the spectral density of a modulating signal and the parameters of a transmission system are given, intermodulation noise can be derived from Eqs. (7)~(11). A modulating signal of particular interest is band-limited, zero-mean Gaussian noise having a uniform spectral density

$$W(f)=\begin{cases} \dfrac{\sigma_0^2}{B-A}, & A\leq f\leq B, \\ 0, & f \text{ elsewhere}, \end{cases} \tag{12}$$

corresponding to a multichannel communication signal, where A and B denote the lowest and top baseband frequencies respectively, and σ_0^2 the variance of the noise.

By putting $\sigma=\sigma_0$ and $w(f)=1/(B-A)$, the first terms of the second order intermodulation noise components, which are applicable to the interference calculation in the case where the distortion is small in comparison, are written from Eq. (9) as follows:

$$\left.\begin{aligned} W_D^{\Gamma''}(f)&=B^{-2}\sigma^4(2\pi f)^2(2B-f)\Gamma_D, && A\leq f\leq 2B, \\ W_A^{\Gamma''}(f)&=\kappa^2B^{-2}\sigma^4(\pi f)^2(2B-f)\Gamma_A, && A\leq f\leq 2B, \\ W_{DA}^{\Gamma''}(f)&=\kappa B^{-2}\sigma^4(2\pi f)^2(2B-f)\Gamma_{DA}, && A\leq f\leq 2B, \end{aligned}\right\} \tag{13}$$

where Γ_D, Γ_A and Γ_{DA} are given by α_n and β_n as described in Eq. (A-1) in Appendix. Further, the third order intermodulation noise components can be written from (11) as follows:

$$\left.\begin{aligned} W_D^{\Gamma'''}(f)&=\begin{cases}2B^{-3}\sigma^6(\pi f)^2G_1(f)\Delta_D, & A\leq f\leq B,\\ B^{-3}\sigma^6(\pi f)^2G_2(f)\Delta_D, & B\leq f\leq 3B,\end{cases} \\ W_A^{\Gamma'''}(f)&=\begin{cases}2\cdot 3^{-1}\kappa^2B^{-3}\sigma^6(\pi f)^2G_1(f)\Delta_A, & A\leq f\leq B,\\ 3^{-1}\kappa^2B^{-3}\sigma^6(\pi f)^2G_2(f)\Delta_A, & B\leq f\leq 3B,\end{cases} \\ W_{DA}^{\Gamma'''}(f)&=\begin{cases}\kappa B^{-3}\sigma^6(2\pi f)^2G_1(f)\Delta_{DA}, & A\leq f\leq B,\\ 2\kappa B^{-3}\sigma^6(\pi f)^2G_2(f)\Delta_{DA}, & B\leq f\leq 3B,\end{cases} \end{aligned}\right\} \tag{14}$$

where Δ_D, Δ_A, Δ_{DA}, $G_1(f)$, and $G_2(f)$ are given by Eqs. (A-2) and (A-4).

Thus, the second and third order intermodulation noises can be obtained by substituting Eqs. (13) and (14) in Eqs. (8) and (10), respectively.

(B) Case with pre-emphasis

The power spectral density of the Gaussian noise colored by the pre-emphasis network having the characteristic $K_0 \exp[\alpha f/B]$ is expressed as follows:

$$W(f)=\begin{cases} \dfrac{K_0\sigma_0^2}{B-A}e^{\alpha f/B}, & A\leq f\leq B, \\ 0, & f \text{ elsewhere}, \end{cases} \tag{15}$$

where σ_0^2 is the variance of the input Gaussian noise; $K_0=0.3$ and $\alpha=2.1$ for the CCIR network[3]. Then, the variance σ^2 and the normalized power spectrum $w(f)$ can be written as

$$\sigma^2=K_0\sigma^2/K, \tag{16}$$

$$w(f)=\begin{cases} \dfrac{K}{B-A}e^{\alpha f/B}, & A\leq f\leq B, \\ 0, & f \text{ elsewhere}, \end{cases} \tag{17}$$

where $K=\alpha/(e^{\alpha}-1)$.

The half frequency band f_e for the case with pre-emphasis is given by Eq. (3), where rms frequency deviation D_v is equal to $\sigma/(2\pi)$ from Eq. (16).

From Eqs. (9), (16) and (17), the first terms of the second order intermodulation noise components can be written as follows:

$$\left.\begin{aligned} W_D^{I''}(f) &= \begin{cases} \lambda^{-1}B^{-2}K^2\sigma^4(2\pi f)^2\, E_1(f)\Gamma_D, & A\leq f\leq B, \\ B^{-2}K^2\sigma^4(2\pi f)\, E_2(f)\Gamma_D, & B\leq f\leq 2B, \end{cases} \\ W_A^{I''}(f) &= \begin{cases} \kappa^2\lambda^{-1}B^{-2}K^2\sigma^4(\pi f)^2\, E_1(f)\Gamma_A, & A\leq f\leq B, \\ \kappa^2B^{-2}K^2\sigma^4(\pi f)^2\, E_2(f)\Gamma_A, & B\leq f\leq 2B, \end{cases} \\ W_{DA}^{I''}(f) &= \begin{cases} \kappa\lambda^{-1}B^{-2}K^2\sigma^4(2\pi f)^2\, E_1(f)\Gamma_{DA}, & A\leq f\leq B, \\ \kappa B^{-2}K^2\sigma^4(2\pi f)^2\, E_2(f)\Gamma_{DA}, & B\leq f\leq 2B, \end{cases} \end{aligned}\right\} \tag{18}$$

where $\lambda=2\alpha$: $E_1(f)$ and $E_2(f)$ are given by Eq. (A-3).

Further, upon using $G_3(f)$, $G_4(f)$, $G_5(f)$ in Eq. (A-4), each component of the third order intermodulation noise can be written as follows:

$$\left.\begin{aligned} W_D^{I'''}(f) &= \begin{cases} \lambda^{-2}B^{-3}K^3\sigma^6(\pi f)^2\, G_3(f)\Delta_D, & A\leq f\leq B, \\ \lambda^{-2}B^{-3}K^3\sigma^6(\pi f)^2\, G_4(f)\Delta_D, & B\leq f\leq 2B, \\ B^{-3}K^3\sigma^6(\pi f)^2\, G_5(f)\Delta_D, & 2B\leq f\leq 3B, \end{cases} \\ W_A^{I'''}(f) &= \begin{cases} 3^{-1}\kappa^2\lambda^{-2}B^{-3}K^3\sigma^6(\pi f)^2\, G_3(f)\Delta_A, & A\leq f\leq B, \\ 3^{-1}\kappa^2\lambda^{-2}B^{-3}K^3\sigma^6(\pi f)^2\, G_4(f)\Delta_A, & B\leq f\leq 2B, \\ 3^{-1}\kappa^2B^{-3}K^3\sigma^6(\pi f)^2\, G_5(f)\Delta_A, & 2B\leq f\leq 3B, \end{cases} \\ W_{DA}^{I'''}(f) &= \begin{cases} 2\kappa\lambda^{-2}B^{-3}K^3\sigma^6(\pi f)^2\, G_3(f)\Delta_{DA}, & A\leq f\leq B, \\ 2\kappa\lambda^{-2}B^{-3}K^3\sigma^6(\pi f)^2\, G_4(f)\Delta_{DA}, & B\leq f\leq 2B, \\ 2\kappa B^{-3}K^3\sigma^6(\pi f)^2\, G_5(f)\Delta_{DA}, & 2B\leq f\leq 3B. \end{cases} \end{aligned}\right\} \tag{19}$$

Thus, the second and third order intermodulation noises can be obtained by substituting Eqs. (18) and (19) in Eqs. (8) and (10), respectively.

4. Analytical results

Let SIR(DA) be the ratio of the interference power to the output power in the elementary band $[f, f+\Delta f]$, signal-to-interference ratio (SIR), in the systems.

When the distortion is comparatively small, the output power in the elementary band may be approximated sufficiently by $W(f)\Delta f$. Therefore, SIR(DA) is represented in decibels as follows:

$$\mathrm{SIR(DA)}=10\cdot\log\,[W(f)/W^{I}(f)]. \tag{20}$$

Let SIR(D) and SIR(A) be SIR's for the output phase rate of the linear network and for AM-PM conversion respectively, then they are given by

$$\mathrm{SIR(D)}=10\cdot\log\,[W(f)/\{W_D^{I''}(f)+W_D^{I'''}(f)\}] \tag{21}$$

$$\mathrm{SIR(A)}=10\cdot\log\,[W(f)/\{W_A^{I''}(f)+W_A^{I'''}(f)\}] \tag{22}$$

These ratios are defined in the frequency band $A \leq f \leq B$.

For a single-pole filter of interest in FM systems, let us consider the effects of carrier frequency offset from the midband frequency of the filter on SIR.

Let f_0 be the midband frequency of the filter, f_b be the half 3 dB filter bandwidth, and x_0 be the ratio of the offset to the half 3 dB bandwidth, i.e., $x_0=(f_c-f_0)/f_b$, then each of the real and imaginary parts of the transfer function in the frequency band $[-f_e/f_b, f_e/f_b]$ from Eq. (3) can be expanded in a polynomial series with respect to the ratio of the frequency deviation from the carrier to f_b.

Let a_n and b_n be the coefficients of the series for the real and imaginary parts respectively, then $\alpha_n=a_n/\omega_b^n$ and $\beta_n=b_n/\omega_b^n$. In this section, both a_n and b_n up to $n=5$ are determined by least-square error method in the frequency band [−0.6, 0.6]. The coefficients α_n and β_n up to $n=5$ which are obtained from a_n and b_n are used to calculate SIR.

Fig. 2 shows the analytical results of SIR(D), SIR(A), SIR(DA), and the corresponding simulation results versus the offset. The behaviors of SIR(D) and SIR(A) are symmetric on the offset, while the behavior of SIR(DA) is asymmetric due to the spectra $W_{DA}^{I''}(f)$ and $W_{DA}^{I'''}(f)$. The calculated SIR's are in good agreement with the simulated ones. Fig. 3 shows the interference quantities improved by inserting CCIR pre-emphasis network versus the relative frequency f/f_b for the offsets x_0's.

5. Conclusions

The second and third order intermodulation noises for both cases with and without pre-emphasis in FDM-FM systems with AM-PM converter has been expressed as a function of the coefficients α_n's, β_n's, and the power spectrum of a

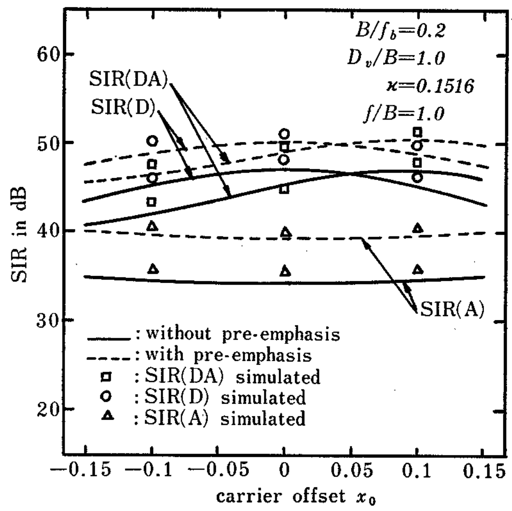


Fig. 2. SIR in a single-pole filter versus carrier offsets; $D_v/B=1$; $B/f_b=0.2$; $f/B=1$; $\kappa=0.1516$.

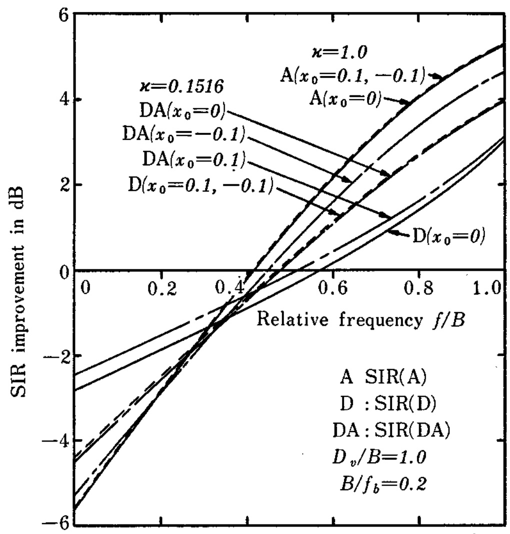


Fig. 3. The improvement quantity of SIR by pre-emphasis in a single-pole filter; $D_v/B=1$; $B/f_b=0.2$; $\kappa=0.1516$ or 1.0.

baseband signal.

It is confirmed that Eqs. (13), (14), (18) and (19) coincide with the first terms of intermodulation noise in the case where the transfer function is represented by the polynomial series of the higher order than the fourth.

The variation of SIR(D) for carrier offset is small in comparison with that of a three-pole filter as shown in Ref. (9), while the variation of SIR(DA) for the same offset amounts to several dB as shown in Fig. 2. Further, the improvement effect of the interference by the pre-emphasis fluctuates with the offset as shown in Fig. 3.

The authors have clarified the effects of carrier offset on the intermodulation noise in a FM system. The above results shows that these effects must be taken into account when evaluating the interference in the system.

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Appendix

In order to simplify the expressions of the second and third order intermodulation noises derived from Eqs. (7)~(11) for both cases with and without pre-emphasis, we use the constants Γ_D, Γ_A, Γ_{DA}, Δ_D, Δ_A, and Δ_{DA} determined by $\alpha_{n's}$ and $\beta_{n's}$ and the functions $E(f)$ and $G(f)$ of frequency f.

They are expressed as follows:

$$\Gamma_D=\beta_2^2,\quad \Gamma_A=4\alpha_2^2-4\alpha_1^2\alpha_2+\alpha_1^4,\quad \Gamma_{DA}=2\alpha_2\beta_2-\alpha_1^2\beta_2. \tag{A-1}$$

$$\left.\begin{aligned}&\Delta_D=3\beta_3^2-2\alpha_1\beta_2\beta_3+3\alpha_1^2\beta_2^2,\\&\Delta_A=9\alpha_3^2+25\alpha_1^2\alpha_2^2+\alpha_1^6-30\alpha_1\alpha_2\alpha_3+6\alpha_1^3\alpha_3-10\alpha_1^4\alpha_2,\\&\Delta_{DA}=3\alpha_3\beta_3-5\alpha_1\alpha_2\beta_3+\alpha_1^3\beta_3-3\alpha_1\alpha_3\beta_2+5\alpha_1^2\alpha_2\beta_2-\alpha_1^4\beta_2.\end{aligned}\right\} \tag{A-2}$$

$$\left.\begin{aligned}&E_1(f)=2B\S_1(f)-(2B-\lambda f)\S_2(f),\\&E_2(f)=(2B-f)\S_2(f).\end{aligned}\right\} \tag{A-3}$$

$$\left.\begin{aligned}&G_1(f)=3B^2-f^2,\quad G_2(f)=9B^2-6Bf+f^2,\\&G_3(f)=6B(B+\lambda B-\lambda f)\S_3(f)+\{12B^2-6\lambda Bf+(\lambda f)^2\}\S_2(f)\\&\qquad -6B(3B-\lambda B+\lambda f)\S_1(f),\\&G_4(f)=6B^2\S_4(f)-\{6B^2+6\lambda B(2B-f)+\lambda^2(3B^2-6Bf+2f^2)\}\S_2(f),\\&G_5(f)=(9B^2-6Bf+f^2)\S_2(f),\end{aligned}\right\} \tag{A-4}$$

where $\S_1(f)=\exp[\lambda(2B-f)/(2B)]$, $\S_2(f)=\exp[\lambda f/(2B)]$,
$\S_3(f)=\exp[\lambda(2B+f)/(2B)]$, $\S_4(f)=\exp[\lambda(4B-f)/(2B)]$.