# Equivalent Circuit for Fault Analysis of Power System Containing an Unsymmetrical Double-Circuit Line

# Equivalent Circuit for Fault Analysis of Power System Containing an Unsymmetrical Double-Circuit Line

Tsuneo TANIGUCHI*

(Received November 15, 1975)

In this paper, an equivalent circuit of a power system containing unsymmetrical double-circuit line is presented for fault analysis.

First, two types of basic circuits A and B, both having a mutual impedance between two lines and these equivalent circuits are exhibited.

These equivalent circuits are not of new type, but by using these, the equivalent circuits of symmetrical components of the above mentioned system can be easily composed. Therefore, the problems not only of fault analysis but also of constructing power system networks, are easily examined, by using these equivalent circuits.

## 1. Introduction

In power transmission lines, tappings are recently made from a halfway of a double-circuit line, to utilize existing lines.

In an industrial power system, also, tappings ($\pi$-form branch) are made from a main-line of a loop primary circuit to improve of the reliability.

These two lines, tapped off from the main-line, form themselves a double-circuit line with a small scale.

In this paper, such a double-circuit line is named unsymmetrical double-circuit line. A method of symmetrical tranformation of two vectors is useful for an analysis of faults on a symmetrical double-circuit line, by using jointly with symmetrical components, because its zero phase sequence network is divided into two networks which are independent of each other.

However, in the zero phase sequence network of the unsymmetrical double-circuit line, its divided two networks are not independent of each other.

For the fault of a power system including such an unsymmetrical double-circuit line, a numerical calculation[1] with a digital computer can be systematically carried out by using the graph theory.

However, an analyzing method using an equivalent network, also, is useful and convenient to make clear not only the whole aspect of the system, but also the effect of each component of the system.

Especially, in the studies of middle scale systems such as industrial power systems,

* Department of Electrical Engineering, College of Engineering.

distribution systems and reduced systems for analysis of transient stability of large systems, containing unsymmetrical circuits, the above analyzing method shows its merit.

In this paper, two types of basic circuits A and B, both having a mutual impedance between two lines and these equivalent circuits[2)] are exhibited.

These equivalent circuits are not of new type, but by using these, the equivalent circuits of the above mentioned unsymmetrical networks can be easily composed and, by this, the field engineers will be able to easily examine thire practical problems, without a deep knowledge of graph theory.

## 2. Basic Circuit

In this section, two types of basic circuits A and B are considered.

Type A circuit is shown in Fig. 1. This circuit consists of four nodes and two branches of which each branch has a series impedance and a mutual impedance between the branches, in essence.

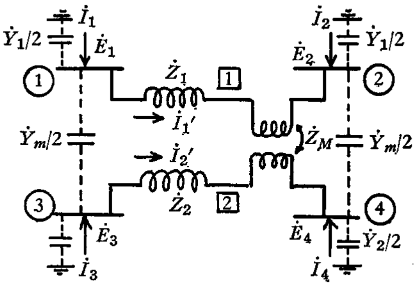



Fig. 1. Type A circuit

Shunt capacitances are divided into two equal parts, each of which is set at both ends of branches in accordance with $\pi$-circuit form.

First, consider an equivalent circuit of type A.

Here, the circuit now considered is only parts shown in Fig. 1 by solid lines, because all of capacitances are not directly concerned with this problem.

In this circuit:

$$\left.\begin{aligned} \dot{E}_1-\dot{E}_2 &= \dot{Z}_1\dot{I}_1'+\dot{Z}_M\dot{I}_2', \\ \dot{E}_3-\dot{E}_4 &= \dot{Z}_M\dot{I}_1'+\dot{Z}_2\dot{I}_2' \end{aligned}\right\} \quad (1)$$

where $\dot{E}_i$ : phase voltage of node ⓘ ($i=1, 2, 3, 4$),

$\dot{Z}_j$ : self impedance of branch $\boxed{j}$ ($j=1, 2$),

$\dot{Z}_M$ : mutual impedance between two branches $\boxed{1}$ and $\boxed{2}$,

$\dot{I}_j'$ : current of branch $\boxed{j}$ ($j=1, 2$).

From Eq. (1), branch currents are

$$\left.\begin{aligned}\dot{I}_1' &= \frac{1}{\varDelta}(\dot{Z}_2\dot{E}_1-\dot{Z}_2\dot{E}_2-\dot{Z}_M\dot{E}_3+\dot{Z}_M\dot{E}_4)\,,\\ \dot{I}_2' &= \frac{1}{\varDelta}(-\dot{Z}_M\dot{E}_1+\dot{Z}_M\dot{E}_2+\dot{Z}_1\dot{E}_3-\dot{Z}_1\dot{E}_4)\end{aligned}\right\} \qquad (2)$$

where $\varDelta=\dot{Z}_1\dot{Z}_2-\dot{Z}_M{}^2$
and nodal currents are

$$\left.\begin{aligned}\dot{I}_1 &= \dot{I}_1'\,, & \dot{I}_2 &= -\dot{I}_1'\,,\\ \dot{I}_3 &= \dot{I}_2'\,, & \dot{I}_4 &= -\dot{I}_2'\,.\end{aligned}\right\} \qquad (3)$$

From Eqs. (2) and (3), nodal currents are given as follows:

$$\left.\begin{aligned}\dot{I}_1(=\dot{I}_1') &= \frac{1}{\varDelta}\{\dot{Z}_2(\dot{E}_1-\dot{E}_2)+\dot{Z}_M(\dot{E}_1-\dot{E}_3)-\dot{Z}_M(\dot{E}_1-\dot{E}_4\}\,,\\ \dot{I}_2(=-\dot{I}_1') &= \frac{1}{\varDelta}\{\dot{Z}_2(\dot{E}_2-\dot{E}_1)-\dot{Z}_M(\dot{E}_2-\dot{E}_3)+\dot{Z}_M(\dot{E}_2-\dot{E}_4)\}\,,\\ \dot{I}_3(=\dot{I}_2') &= \frac{1}{\varDelta}\{\dot{Z}_1(\dot{E}_3-\dot{E}_4)+\dot{Z}_M(\dot{E}_3-\dot{E}_1)-\dot{Z}_M(\dot{E}_3-\dot{E}_2)\}\,,\\ \dot{I}_4(=-\dot{I}_2') &= \frac{1}{\varDelta}\{\dot{Z}_1(\dot{E}_4-\dot{E}_3)-\dot{Z}_M(\dot{E}_4-\dot{E}_1)+\dot{Z}_M(\dot{E}_4-\dot{E}_2)\}\,.\end{aligned}\right\} \qquad (4)$$

Eq. (4) shows that $\dot{Z}_1/\varDelta$, $\dot{Z}_2/\varDelta$, $\dot{Z}_M/\varDelta$ and $-\dot{Z}_M/\varDelta$ are admittances between nodes, respectively.

Thus, an equivalent circuit of type A is shown in Fig. 2.

Also, shunt capacitances and capacitances between branches are shown in Fig. 2 by dotted lines and also, these are not directly concerned with this problem.

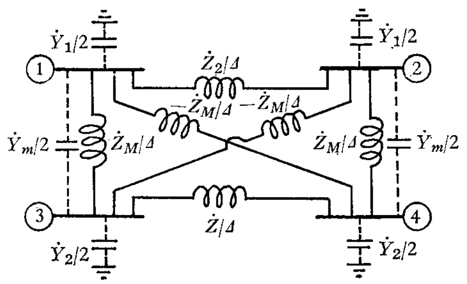



Fig. 2. Equivalent circuit of type A

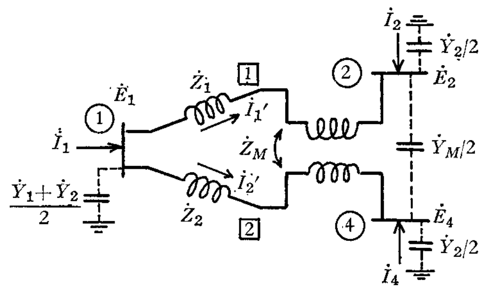



Fig. 3. Type B circuit

Next, type B circuit is shown in Fig. 3. This circuit is a modified circuit of type A in which node ③ is connected to node ①.

In this case,

$$\left.\begin{aligned}\dot{E}_1 &= \dot{E}_3\,, \qquad \dot{I}_3=0\,,\\ \dot{I}_1 &= \dot{I}_1'+\dot{I}_2'\end{aligned}\right\} \qquad (5)$$

and a capacitance $\dot{Y}_M/2$ between node ① and node ③ in type A circuit is neglected.

Referring to Eqs. (4) and (5), nodal currents are

$$\left.\begin{aligned} \dot{I}_1 &= \frac{1}{\Delta}\{(\dot{Z}_1-\dot{Z}_M)(\dot{E}_1-\dot{E}_4)+(\dot{Z}_2-\dot{Z}_M)(\dot{E}_1-\dot{E}_2)\}\ , \\ \dot{I}_2 &= \frac{1}{\Delta}\{(\dot{Z}_2-\dot{Z}_M)(\dot{E}_2-\dot{E}_1)+\dot{Z}_M(\dot{E}_2-\dot{E}_4)\}\ , \\ \dot{I}_4 &= \frac{1}{\Delta}\{(\dot{Z}_1-\dot{Z}_M)(\dot{E}_4-\dot{E}_1)+\dot{Z}_M(\dot{E}_4-\dot{E}_2)\} \end{aligned}\right\} \quad (6)$$

where $\Delta=\dot{Z}_1\dot{Z}_2-\dot{Z}_M^2$

An equivalent circuit of type B shown in Fig. 4.

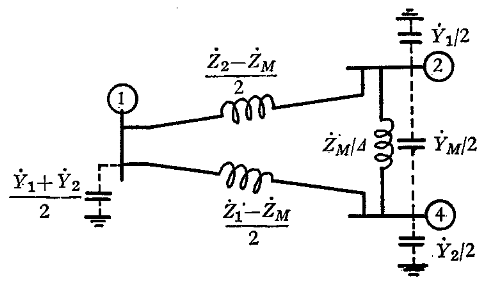


Fig. 4. Equivalent circuit of Type B

## 3. Equivalent Circuit for Fault Aanlysis on Unsymmetrical Double-Circuit Line

In this section, for the fault calculation of an unsymmetrical double-circuit line, an equivalent sequence networks of symmetrical components are introduced, based on two basic circuits.

A sample system network is shown in Fig. 5. In Fig. 5, both No. 1 route and No. 2 route are the double-circuit lines.

Each line has different line constants and capacitanes of lines are neglected for simplicity.

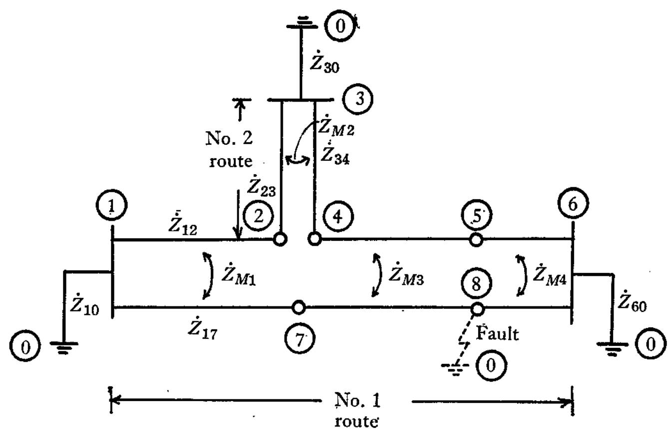


Fig. 5. Sample system network

The system forms an unsymmetrical circuit as described in the introduction. In Fig. 5,

$\dot{Z}_{ij}$ : series impedance between node ⓘ and node ⓙ,

$\dot{Z}_{i0}$ : impedance viewed from node ⓘ towards reference node ⓪,

$Z_{Mi}$: mutual impedance between parallel lines,

node ② and ④: tapped off nodes for $\pi$ branch,

node ⑦: node on opposite line corresponding to nodes ② and ④,

node ⑤: node on opposite line corresponding to node ⑧.

This system is divided into four parts with type A and type B circuits as shown in Fig. 6.

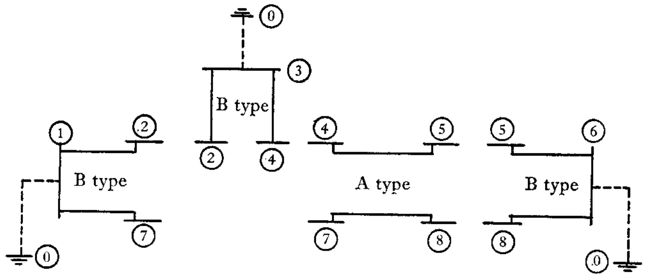
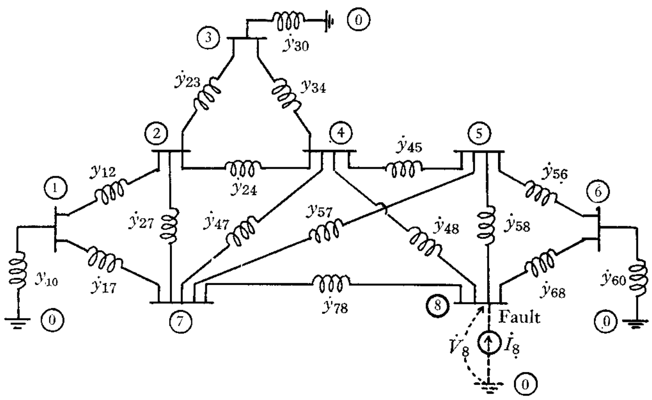



Fig. 6. Division of network

Then, as the equivalent circuit of each part is given by Fig. 2 or Fig. 4, an equivalent circuit of the system can be easily composed by connecting the nodes of the same numbers of these equivalent circuits as shown in Fig. 7.




Fgi. 7. Equivalent network of sample system

The positive, negative and zero sequence networks of the system shown in Fig. 5 will be represented by the above equivalent circuit which has corresponding component line constants.

The line constants are shown in Table 1.

Table 1. Line Constants (without ground wires)

| | Zero sequence circuit | Positive and negative sequence circuit |
|---|---|---|
| $\dot{Z}_i$ | $r_i+j\omega(l_{si}+2_{mi})$ | $r_i+j\omega(l_{si}-l_{mi})$ |
| $\dot{Z}_M$ | $j\omega(3l_m')$ | 0 |
| $\dot{Y}_i$ | $j\omega C_{si}$ | $j\omega(C_{si}+3C_{mi}+3C_m')$ |
| $\dot{Y}_M$ | $-j\omega(3C_m')$ | 0 |

Simbol;

$r_i$: resistance of No. i line per phase,
$l_{si}$: self inductance of No. i line per phase,
$l_{mi}$: mutual inductance between conductors of No. i line per phase,
$l_m'$: mutual inductance between parallel lines per phase,
$C_{si}$: capacitance between conductor of No. i and ground per phase,
$C_{mi}$: capacitance between conductors of No. i line per phase,
$C_m'$: capacticance between parallel lines per phase.

Moreover, $\dot{Z}_{i0}$ in Fig. 5 is an impedance between the node ⓘ and the neutral node in the positive or negative sequence network and in the zero sequence network, $\dot{Z}_{i0}$ is three times as much as the impedance of the earth return path between the node ⓘ and the fault point.[2),3)] Each of them is inserted between the node ⓘ and the reference node ⓪.

In unsymmetrical fault studies of a power system such as line-to-line or line-to ground, the impedance or the admittance of each sequence network viewed from the point of fault to its network is required. For example, in case that the fault occurs at the node ⑧ in Fig. 5, the admittance $\dot{Y}_f$ of an arbitary sequence network is given by using the corresponding equivalent circuit as follows (see the dotted part in Fig. 7):

$$\dot{Y}_f = (\dot{I}_8/\dot{V}_8)_{at\,\dot{I}_i=0} \qquad (i=1, \cdots, 7) \tag{7}$$

were $\dot{I}_8$: injection current directed towards the node ⑧,
$\dot{V}_8$: voltage of the node ⑧.

$\dot{Y}_f$ can be also given by using the nodal equation (see appendix).

## 4. Conclusion

An equivalent circuit of a power system with unsymmetrical double-circuit line is described.

Two basic circuits presented are convenient to compose equivalent circuits of symmetrical components of an unsymmetrical system.

Therfore, the problems not only of fault analysis, but also of constructing system networks, are easily examined.

## Acknowledgement

The author wishes to express his gratitude to Professor Masao Nakamura of Univ. of Osaka Prefecture for helpful discussion.

## References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2) E. Clarke, Circuit Analysis of A-C Power System, Vol 1, New York, John Wiely & Sons (1950)
3) B.M. Weedy, Electric Power Systems, New York, John Wiely & Sons (1967)


## Appendix

The nodal equation of an equivalent circuit shown in Fig. 7 can be expressed as follows[3]:

$$\begin{pmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_8 \end{pmatrix} = \begin{pmatrix} \dot{Y}_{11} & \dot{Y}_{12} \cdots & \dot{Y}_{18} \\ \dot{Y}_{21} & \dot{Y}_{22} \cdots\cdots & \\ \vdots & & \vdots \\ \dot{Y}_{81} \cdots\cdots & \cdots\cdots & \dot{Y}_{88} \end{pmatrix} \cdot \begin{pmatrix} \dot{E}_1 \\ \dot{E}_2 \\ \vdots \\ \dot{E}_8 \end{pmatrix} \tag{A-1}$$

where $\dot{I}_i$ : nodal current of the node ⓘ,

$\dot{E}_i$ : nodal voltage of the node ⓘ,

$\dot{Y}_{ii}$ : self admittance of the node ⓘ and the value is equal to the sum of all the admittances connected to this node,

$\dot{Y}_{ij}$: mutual admittance between the nodes ⓘ and ⓙ, and the value is equal to the negative of the sum of the admittances connected between the nodes ⓘ and ⓙ.

With partitioning method, Eq. (A-1) is partitioned into the following two:

$$\dot{\boldsymbol{I}}_1 = \dot{\boldsymbol{Y}}_{11}\dot{\boldsymbol{E}}_1 + \dot{\boldsymbol{Y}}_{12}\dot{E}_8 , \tag{A-2}$$

$$\dot{I}_8 = \dot{\boldsymbol{Y}}_{12}^T\dot{\boldsymbol{E}}_1 + \dot{Y}_{88}\dot{E}_8 \tag{A-3}$$

where

$$\dot{\boldsymbol{I}}_1 = \begin{pmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_7 \end{pmatrix}, \quad \dot{\boldsymbol{E}}_1 = \begin{pmatrix} \dot{E}_1 \\ \dot{E}_2 \\ \vdots \\ \dot{E}_7 \end{pmatrix}, \quad \dot{\boldsymbol{Y}}_{12} = \begin{pmatrix} \dot{Y}_{18} \\ \dot{Y}_{28} \\ \vdots \\ \dot{Y}_{78} \end{pmatrix},$$

$$\dot{\boldsymbol{Y}}_{11} = \begin{pmatrix} \dot{Y}_{11} \cdots & \dot{Y}_{17} \\ \vdots & \vdots \\ \dot{Y}_{71} \cdots & \dot{Y}_{77} \end{pmatrix},$$

$\dot{\boldsymbol{Y}}_{12}^T$: transposed matrix of $\boldsymbol{Y}_{12}$.

Now, since all elements of $\dot{\boldsymbol{I}}_1$ are set zero, in Eq. (A-2)

$$\dot{\boldsymbol{E}}_1 = -\dot{\boldsymbol{Y}}_{11}^{-1}\dot{\boldsymbol{Y}}_{12}\dot{E}_8 . \tag{A-4}$$

Substituting this expression for $\dot{\boldsymbol{E}}_1$ into Eq. (A-3) yields

$$\dot{I}_8 = (\dot{Y}_{88} - \dot{\boldsymbol{Y}}_{12}^T\dot{\boldsymbol{Y}}_{11}^{-1}\dot{\boldsymbol{Y}}_{12})\dot{E}_8 . \tag{A-5}$$

Thus, $\dot{Y}_f$ in Eq. (7) is given by the following equation;

$$\dot{Y}_f = \dot{Y}_{88} - \dot{\boldsymbol{Y}}_{12}^T\dot{\boldsymbol{Y}}_{11}^{-1}\dot{\boldsymbol{Y}}_{12} . \tag{A-6}$$