A Method of Coherent Group Selection for Power System Reduction

A Method of Coherent Group Selection for Power System Reduction

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This paper describes a new method of selecting a coherent group[1] of generators for a power system reduction in stability studies. A factor analysis[2] using the data of voltage phase-angles of generators is applied to the selection of coherent groups. The appropriateness of this method is discussed by comparison with the stability measure[3] of the reduced system and that of the original system, and is illustrated by some numerical examples using the data obtained by means of the simulation with a digital computer.

1. Introduction

A rational system reduction is a useful and effective means of evaluating system states for on-line preventive control of a large interconnected power system. The possibility of the reduction of a power system depends on the existence of generators which swing together under the system disturbance. This paper presents a new method of selecting such a group of generators, namely the coherent group[1], and describes a method of construction of the reduced system based on this selection. The coherent groups of generators are classified by using the factor analysis[2] using the data of voltage phase-angles of generators under the variable operating modes.

In this paper, some examples using the data obtained by means of the simulation are given and the appropriateness of this method is discussed by comparison with the stability measure[3] based on the Liapunov stability theory of the reduced system and that of the original system.

2. Theory[4]

A theory is based on the statistical method using the sampled data of each generator's voltage phase-angles obtained by off-line simulation studies under the following assumptions:

The assumptions are: 1) internal voltage behind transient reactance is constant, 2) mechanical torque is constant, 3) impedance load is constant and 4) control system is not taken into account.

(1) Correlation of voltage phase-angles

The voltage phase-angle variation of each generator for small disturbances in the

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system operating stages, such as the continual changes in load or generation and the switching out of lines, shows approximately a linear property. In this case, the relation between voltage phase-angles of arbitary generators can be given approximately by the following regression equation:

$$\delta_{jk} = \frac{\sigma_j}{\sigma_i}\cdot\Phi_{ij}(\delta_{ik}-m_i)+m_j\,, \tag{1}$$

$$\Phi_{ij} = \frac{1}{M\sigma_i\sigma_j}\sum_{k=1}^{M}(\delta_{ik}-m_i)(\delta_{jk}-m_j)\,,$$

$$(i, j = 1, 2, \cdots, N)$$

where

δ_{ik}, δ_{jk}: voltage phase-angles of generators No. i and No. j at the time k,

σ_i, σ_j: standard deviations of voltage phase-angles of generators No. i and No. j,

Φ_{ij}: correlation coefficient between voltage phase-angles of generators No. i and No. j,

m_i, m_j: mean values of voltage phase-angles of generators No. i and No. j,

M: measuring counts.

(2) Coherent coefficient

According to Ref. (1), two generators No. i and No. j are said to be coherent, if there is a constant C_{ij} with respect to time, such that $\delta_i(t)-\delta_j(t)\simeq C_{ij}$. A group of generators in which any pair of generators is coherent, can be approximately replaced by one equivalent generator. Then, as a basis of judgement of coherency, the following two factors are adopted:

Factor 1: the gradient of an estimated linear equation (1),

$$\frac{\sigma_j}{\sigma_i}\cdot\Phi_{ij}\,.$$

This value shows 1 and $C_{ij}=m_i-m_j$ for generators which indicate perfectly coherent behavior for small changes in the system.

Factor 2: the mean value, $|m_i-m_j|$ of phase angle difference.

This value may be considered to show a degree of synchronizing force, and its low value corresponds to a large synchronizing force between generators. For each relative evaluation of these factors, the coherent rate is considered as follows:

In factor 1, the gradient with respect to the axis which coincides with an ideal line indicating perfectly coherent behavior, is adapted in place of that in Eq. (1). That is, the coherent rate g_{ij} is given as

$$g_{ij} = \left(1-\frac{|\sigma_i-\sigma_j|}{\sigma_i+\sigma_j}\right)\cdot\Phi_{ij}\,. \tag{2}$$

The value shows 1 for perfectly coherent behavior, too. The second term in parentheses of the above formula gives the gradient for $\Phi_{ij}=1$ measured from the above ideal line axis.

In factor 2, positive synchronizing force between two generators exists in a region of $0\sim\pi/2$ radians of voltage phase-angle difference. Then, this coherent rate S_{ij} is given as

$$S_{ij} = \left(1-H\left[\frac{|m_i-m_j|}{C}\right]\right) \tag{3}$$

where

$$|m_i-m_j|/C \leqq 1:\ H\left[\frac{|m_i-m_j|}{C}\right] = |m_i-m_j|/C\,,$$

$$|m_i-m_j|/C > 1:\ H\left[\frac{|m_i-m_j|}{C}\right] = 1\,.$$

(C: critical value, $\pi/2$ radians in this paper)

This value shows 1 for perfect synchronism. Considering above two coherent rates, a coherent coefficient ξ_{ij} is defined as follows:

$$\begin{aligned}\xi_{ij} &= g_{ij}\cdot S_{ij}\\ &= \Phi_{ij}\left(1-\frac{|\sigma_i-\sigma_j|}{\sigma_i+\sigma_j}\right)\left(1-H\left[\frac{|m_i-m_j|}{C}\right]\right).\end{aligned} \tag{4}$$

$$(i, j = 1, 2, \cdots, N)$$

The coherent coefficient shows 1 for perfect coherency, 0 for estrangement and -1 for contrariety in swing behaviors of generators No. i and No. j. Fig. 1 shows the relation ξ_{ij} versus the gradient with the parameter $|m_i-m_j|$.

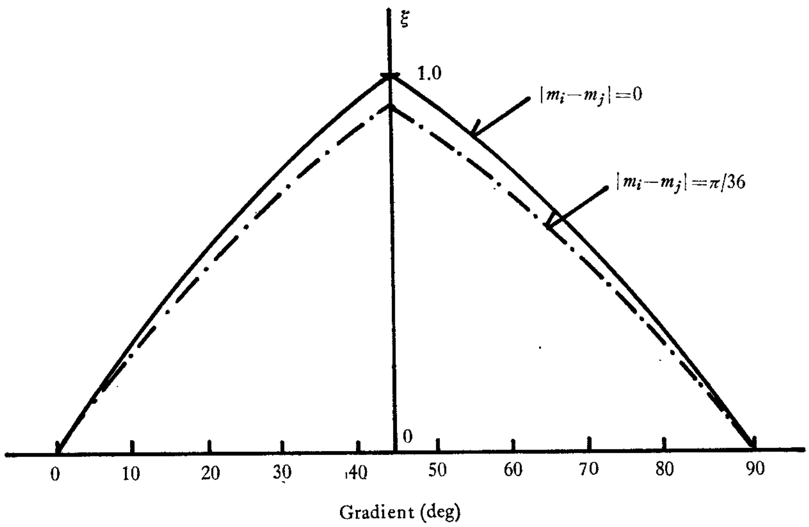


Fig. 1. Coherent coefficient vs. gradient relation.

(3) Coherent group coefficient and grouping method

Using the coherent coefficient of each pair of generators obtained from Eq. (4), a method of the selection of coherent groups is proposed. As mentioned above, a coherent group consists of a group of generators in which any pair of generators is coherent. For finding such a group, it is convenient to make use of the factor analysis.[2)] Then, a coherent group coefficient J is defined as follows:

$$J=\frac{\hat{S}}{\hat{T}}=\frac{2S}{T}\cdot\frac{N-g}{g-1}, \tag{5}$$

where

N: total number of generators,
g: total number of generators in the group A,
S: sum of coherent coefficients in the group A,
T: total number of coherent coefficients between generators in the group A and generators not belonging to the group A,
$\hat{S}$: mean value of S,
$\hat{T}$: mean value of T.

The process of grouping is as follows[2)]:

The coherent coefficients of each pair of generators are arranged in a matrix form of which (i,j) element is ξ_{ij} and the grouping is begun by selecting the pair of generators which has the largest coherent coefficient in all elements except diagonal elements of this matrix. Then, the first group A in which the above mentioned pair of generators —for example, generators No. e and No. f—becomes the origin, is formed, if the value ξ_{ef} exceeds a limiting value ξ_{base} ($\xi_{base}=0.9$ in this paper) specified in advance. If ξ_{ef} does not exceed the value ξ_{base}, it means non-existence of any coherent group in the system. Thus, the origin of the group A is formed by generators No. e and No. f.

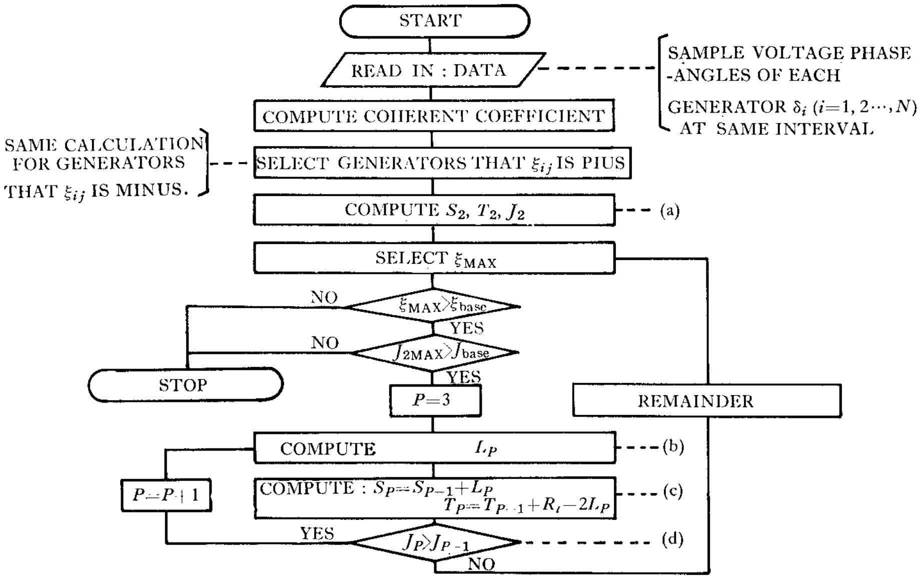
The successive procedure is that to the group A is added any generator k having coherent coefficient ξ_{ek} or ξ_{fk} for which S with the preceding is the highest. This process is continued until the value of J starts to decrease. When this starts in the value of J, the last generator added is withdrawn from this group. The coherent group is decided in this way. The other coherent groups are selected by the same manner, except already selected generators.

This process is shown in Table. 1.

In Table. 1, the meaning of the symbols is:

P: number of generators in the group A,
L: sum of coherent coefficients between a generator to be added and the generators already selected in the group A,
R: sum of coherent coefficients of the generator to be added in the group A,
S, T, J: the same symbols as in Eq. (5).

Table. 1. Computer program flow chart for grouping method.



Note: $\xi_{base}=0.9$, $J_{base}=1.0$ in this study.

3. Power system reduction[5)]

(1) The reduced nodes-admittance matrix

The original network equation is as follows:

$$\dot{I} = \dot{Y} \cdot \dot{E}, \tag{6}$$

where

$\dot{E}, \dot{I}$: column vectors ($N \times 1$) of internal voltage and current for generators,

$\dot{Y}$: nodes admittance matrix ($N \times N$),

N: number of generators.

Suppose that the original system can be divided into coherent groups A, B, $\cdots$, M. The voltages of generators in each coherent group are given as follows:

$$\dot{E}^K = \exp(j\bar{\delta}_{K0}) \cdot x^K, \qquad K = A, B, \cdots, M \tag{7}$$

where

$$x^K = [E_r \exp[j(\delta_{r,0} - \bar{\delta}_{K0})],\ E_{r+1} \exp[j(\delta_{r+1,0} - \bar{\delta}_{K0})],\ \cdots]^T,$$

$\bar{\delta}_{K0} = \dfrac{\sum_i M_i \delta_i}{\sum_i M_i}$: voltage phase-angle of inertia center of the group K,

$E_r, E_{r+1}, \cdots$: voltages of generators in the group K,

$\delta_{r,0}, \delta_{r+1,0}, \cdots$: initial voltage phase-angles of generators in the group K.

The above equation has been derived under the following assumption. That is, each generator in the same group swings together with constant voltage phase-angle differences from voltage phase-angle of each group's inertia center during the system disturbances.

The power equation of the group K is given as follows:

$$\begin{aligned} P^K &= Real\,([\bar{\dot{E}}^K]^T \cdot \dot{I}^K) \\ &= Real\,[\exp(-j\bar{\delta}_{K0})\,[\bar{x}^K]^T \{\dot{Y}^{KA} x^A \exp(j\bar{\delta}_{A0}) \\ &\quad + \cdots + \dot{Y}^{KM} x^M \exp(j\bar{\delta}_{M0})\}] \\ &= Real\,[\exp(-j\bar{\delta}_{K0})\{[\bar{x}^K]^T \cdot \dot{Y}^{KA} \cdot x^A \exp(j\bar{\delta}_{A0}) \\ &\quad + \cdots + [\bar{x}^K]^T \cdot \dot{Y}^{KM} \cdot x^M \exp(j\bar{\delta}_{M0})\}]\,, \end{aligned} \tag{8}$$

where

$\dot{Y}^{KK}$: nodes-admittance of only the group K and square matrix composed of the group K of $\dot{Y}$ in Eq. (6),

$\dot{Y}^{KL}$: matrix composed of all the mutual admittances between the group K and the group L of $\dot{Y}$ in Eq. (6).

and these admittances are given as follows:

$$\begin{aligned} \dot{Y}^{KK} &= [\bar{x}^K]^T \cdot \dot{Y}^{KK} \cdot x^K\,, \\ \dot{Y}^{KL} &= \dot{Y}^{LK} \simeq \{[\bar{x}^K]^T \cdot \dot{Y}^{KL} \cdot x^L + [\bar{x}^L]^T \cdot [\dot{Y}^{KL}]^T \cdot x^K\}/2\,. \end{aligned}$$

(2) Mechanical power and inertia moment of coherent group for the reduced system

Mechanical power P_m^K of the group K is given as follows:

$$P_m^K = \sum_J Y^{KJ} \cos(\delta_{K0} - \delta_{J0} - \varphi_{KJ})\,. \tag{9}$$

where

Y: absolute value of the reduced admittance,

φ: admittance angle of the reduced admittance.

Inertia moment of the group K is given as follows:

$$M^K = \sum_j M_j\,. \tag{10}$$

4. Numerical examples

The 5-machine and 10-machine system are shown in Figs. 2 and 7, respectively. Line impedances, constants of apparatus and the operating conditions of these systems are shown in Tables. 2, 3, 5 and 6. The system disturbance is assumed a three phase short circuit fault on a transmission line (see the fault point F in Fig. 2 or Fig. 7), and after the removal of the disturbance, the system returns to that before the disturbance.

(a) 5-machine system

Fig. 3 shows the numerical results of the coherent group coefficient applying the grouping method in section 2 to a 5-machine system. From the results, generators Nos.

1, 2 and 3 can be treated as one coherent group. Generators Nos. 4 and 5 must be treated individually, because the coherent coefficients, except those corresponding already selected generators, do not exceed the limiting value ξ_{base}. For the original and the reduced system, voltage phase-angles vs. time relations are shown in Figs. 4 and 5, and the critical switching times in Fig. 6, respectively. From these figures, it will be clear that very similar results are obtained for the original and the reduced system.

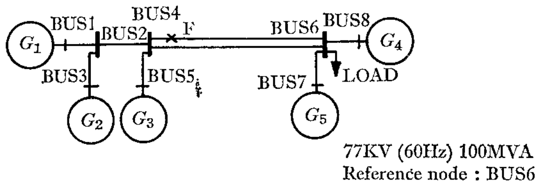


Fig. 2. 5-machine power system.

Table. 2. Impedances. (p.u.)

BUS-BUS	Impedance	BUS-BUS	Impedance
1–2	j 1.3566	4–6	j 0.3015
2–3	j 0.9001	6–7	j 0.7755
2–4	j 0.0123	6–8	j 0.2170
4–5	j 0.6752	Load	p.f. 0.9 171KW~200KW

Table. 3. Generator constants.

Gen.	E (p.u.)	M (sec)	P_m (p.u.)
G_1	1.2257	2.6	0.25
G_2	1.2378	3.9	0.40
G_3	1.2398	5.2	0.55
G_4	1.0723	13.75	0.80
G_5	1.5429	2.25	0.0

Table. 4. Coherent coefficients.

	G_1	G_2	G_3	G_4	G_5
G_1	1.0000	0.9896	0.9856	0.1620	0.0165
G_2	0.9896	1.0000	0.9926	0.1596	0.0162
G_3	0.9856	0.9926	1.0000	0.1609	0.0164
G_4	0.1620	0.1596	0.1609	1.0000	0.1732
G_5	0.0165	0.0162	0.0164	0.1732	1.0000

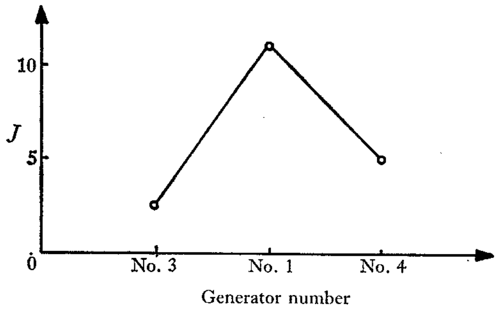


Fig. 3. Coherent group coefficient referred to No. 2 generator.

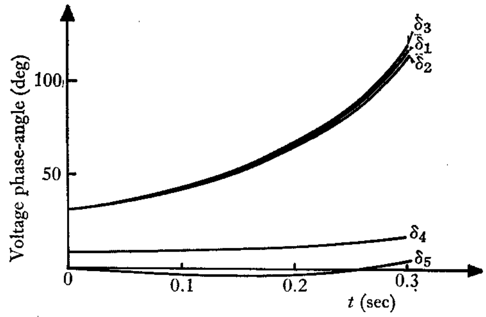


Fig. 4. Voltage phase-angles vs. time relation for the original system.

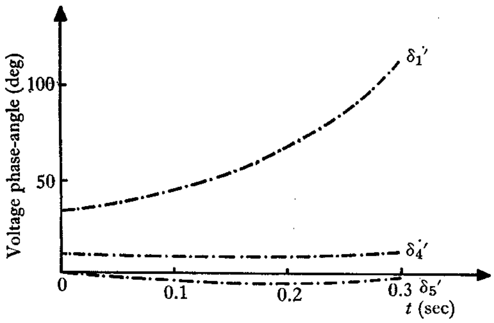


Fig. 5. Voltage phase-angles vs. time relation for the reduced system.

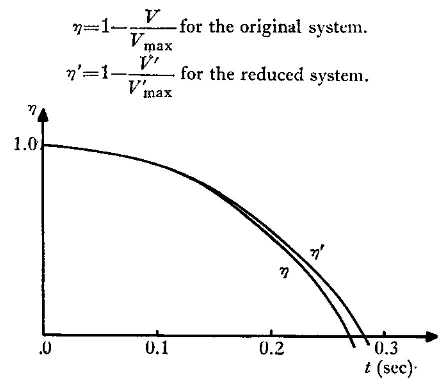


Fig. 6. Critical switching times for the original system and the reduced system.

(b) 10-machine system

Fig. 8 shows the numerical results of the coherent group coefficient for this system. From the results, generators Nos. 1, 2, 3, 4 and 5 can be treated as one coherent group. Generators Nos. 6, 7, 8, 9 and 10 must be treated individually. For the original and the reduced system, voltage phase-angles vs. time relations are shown in Figs. 9 and 10, and critical switching times in Fig. 11, respectively. For this system, the same results as those in a 5-machine system are obtained, too.

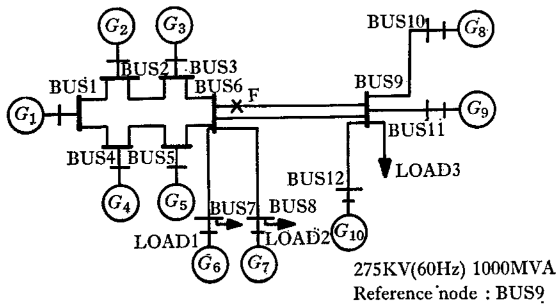


Fig. 7. 10-machine power system.

Table. 5. Impedances. (p.u.)

BUS-BUS	Impedance	BUS-BUS	Impedance
1–2	j 0.03	6–7	j 0.72
1–4	j 0.09	6–8	j 0.76
2–3	j 0.02	6–9	j 0.60
3–6	j 0.78	9–10	j 0.34
4–5	j 0.04	9–11	j 0.60
5–6	j 0.87	9–12	j 0.02
LOAD 1	p.f.0.98: 371 MW ~400 MW	LOAD 3	p.f.0.98: 1942 MW ~2000 MW
LOAD 2	p.f.0.98: 1442 MW ~1500 MW		

Table. 6. Generator constants.

Gen.	X' (p.u.)	X_t (p.u.)	E (p.u.)	M (sec)	P_m (p.u.)
G_1	0.725	0.248	1.117	4.20	0.240
G_2	5.250	1.700	1.323	0.47	0.036
G_3	3.420	1.300	1.285	0.57	0.048
G_4	3.060	1.030	1.252	0.97	0.060
G_5	2.410	0.980	1.275	1.32	0.051
G_6	0.175	—	1.335	9.68	0.498
G_7	0.562	—	1.226	8.32	0.655
G_8	0.020	0.010	1.286	17.25	0.829
G_9	0.420	0.250	1.380	14.31	0.206
G_{10}	0.670	0.010	1.072	2.25	0.261

X': transient reactance X_t: transformer reactance

Table. 7. Coherent coefficients.

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}
G_1	1.00	0.92	0.88	0.95	0.89	0.56	0.43	0.68	0.61	0.52
G_2	0.92	1.00	0.97	0.96	0.85	0.63	0.48	0.75	0.59	0.58
G_3	0.88	0.97	1.00	0.93	0.82	0.65	0.51	0.78	0.59	0.60
G_4	0.95	0.96	0.93	1.00	0.89	0.60	0.46	0.72	0.61	0.55
G_4	0.89	0.85	0.82	0.89	1.00	0.51	0.39	0.63	0.70	0.48
G_6	0.56	0.63	0.65	0.60	0.51	1.00	0.80	0.69	0.44	0.72
G_7	0.43	0.48	0.51	0.46	0.39	0.80	1.00	0.67	0.35	0.81
G_8	0.68	0.75	0.78	0.72	0.63	0.69	0.67	1.00	0.58	0.80
G_9	0.61	0.59	0.59	0.61	0.70	0.44	0.35	0.58	1.00	0.44
G_{10}	0.52	0.58	0.60	0.55	0.48	0.72	0.81	0.80	0.44	1.00

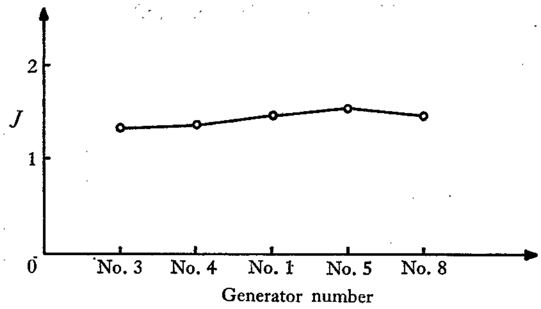


Fig. 8. Coherent group coefficient referred to No. 2 generator.

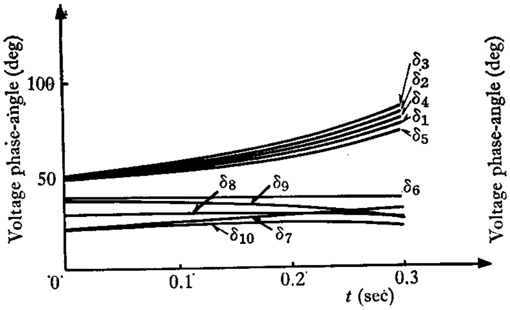


Fig. 9. Voltage phase-angles vs. time relation for the original system.

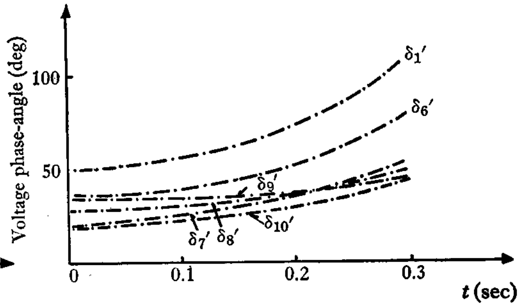


Fig. 10. Voltage phase-angles vs. time relation for the reduced system.

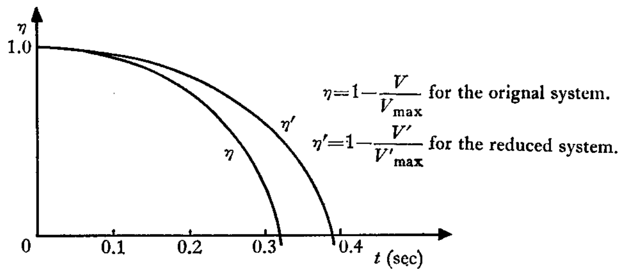


Fig. 11. Critical switching times for the original system and the reduced system.

5. Conclusion

A method of selection of the coherent group based on a factor analysis using the data of voltage phase-angles of generators has been described. This method gives a merit to be able to successively construct the reduced systems corresponding to every actual operating stage, for on-line preventive control. In general, as the on-line data are those at the terminals of generators, the data must be somewhat corrected. About this problem, the authors wish to discuss in the next opportunity.

Acknowledgement

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