A systematic Description of 3-Workpiece Lapping Process

A Systematic Description of 3-Workpiece Lapping Process

Yoshiro SAKAI*, Yoshio TANAKA** and Mamoru IDO**

(Received June 14, 1975)

Abstract

3-workpiece lapping process is simulated on a digital computer and the principal characteristics of the profile change in the process are studied by making a qualitative comparison between the results of simulated and actual ones. Based upon the characteristics, a linear vector differential equation is derived to describe the process mathematically. The theory of dynamic programming together with the notion of a multistage process is applied to the problem of attaining the prescribed tolerance of flatness.

1. Introduction

3-workpiece lapping consists of a succession of abrading pairs of an upper work and a lower one chosen from among the prepared three workpieces. The process is a version of so-called Whitworth's lapping in the sense that the present one is mechanically performed, while Whitworth's manually. The process is still important as a precision machining to get flat surfaces. The characteristics of the process, however, seem so far not to have been investigated.

The former half of this paper is devoted to getting the principal characteristics which govern the change in surface form in the abrasion process, and to the derivation of an equation to describe the behavior. A simulation program was written to compare the simulated result with the actual process.

In the latter half, the construction of 3-workpiece lapping is discussed. That is, with the notion of a system, each abrasion process in the lapping can be regarded as a subsystem of the full system of the lapping. Then the equation obtained in the former half plays the role of the state equation of each subsystem. The convergence to flat surfaces in the process, however, depends on the length of time for which the workpieces are abraded in each subsystem. Hence we introduce the conception of a multistage decision problem. In other words, the system of 3-workpiece lapping consists of a sequence of such subsystems, and since the key to the convergence is the lapped time (or distance) in each subsystem as stated above, the decision of the length of lapping time must be made in each subsystem. Thus the theory of dynamic programming is applied to seek the optimal policy in the sense of the minimum time to attain the prescribed tolerance of flatness.

---

* Graduate Student, Department of Mechanical Engineering, College of Engineering.
** Department of Mechanical Engineering, College of Engineering.

2. Simulation of the abrasion process and postulates for it

The simulater was designed to simulate the change in cross-sectional forms of the pair of workpieces in the direction of the reciprocating motion of the lapping apparatus.

The postulates for the simulation are as follows.

1. The probability of the existence of a grain is equivalent everywhere on the surface. The distribution of grain size is gaussian.
2. When a grain is to cut the material, the depth of cut is proportional to the depth of penetration into the surface.
3. Abrasives are rigid and do not fracture.

The flow diagram is shown in Fig. 1. The initial profiles are given by (parabola +gaussian random numbers), and after each prescribed lapping distance profiles are measured and approximated by parabolas employing the method of least squares.

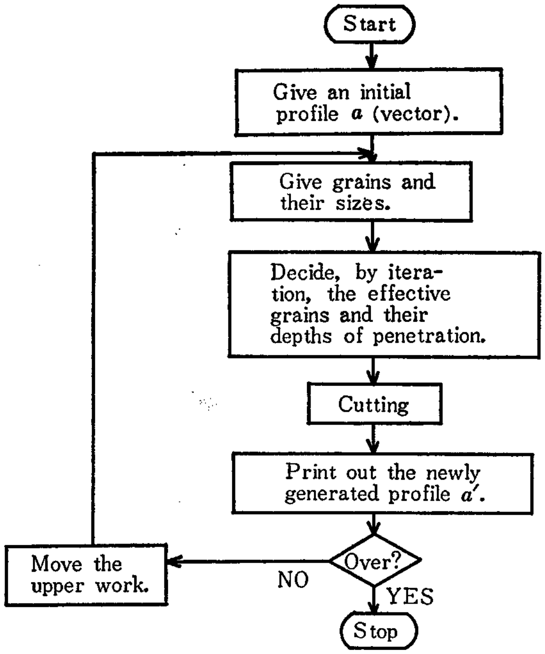

Fig. 1. Flow diagram for simulation

3. Experimental procedure

Conditions

dimensions of surfaces: 40 mm (length) × 20 mm (width)
stroke of reciprocating motion: 20 mm
lapping distance per minute: 2 m/min
abrasive: GC ♯400

vehicle: rape oil

volume ratio of grains to vehicle: 30%

The main part of the apparatus is shown in Fig. 2. The apparatus is designed to have three degrees of freedom with ball bearings numbered ① and ②, and a shaft ③ in the figure. That is, the weight ⑦ can move up and down, and rotate around the x axis, and the upper workpiece ④ held by its holder ⑧ is connected to the weight through the shaft ③, which allows the workholder to freely rotate around. Such conditions are similar to those in the simulation.

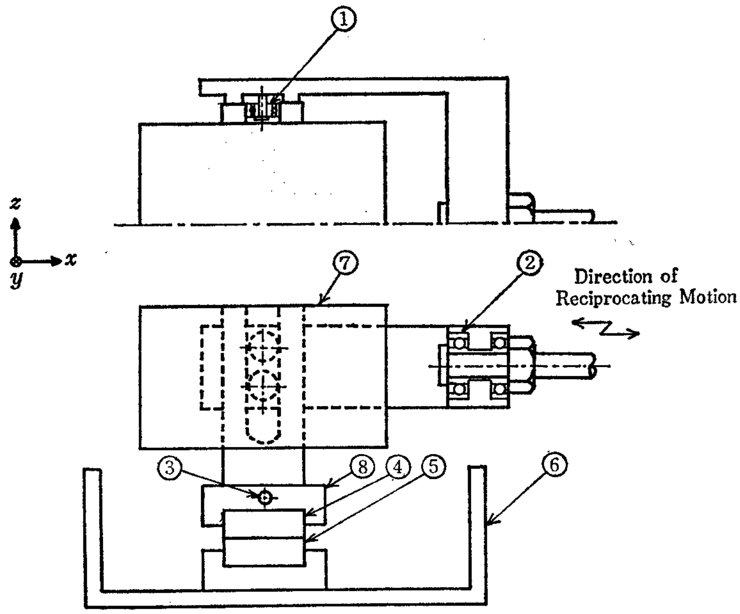

Fig. 2. Experimental Apparatus

Lapping is performed in a mixture of oil and abrasives, i.e., a kind of so-called wet lapping. As we have mentioned with regard to the simulation in the previous section, the profiles are measured and approximated by parabolas employing the method of least squares after each 10 meter lapping.

The results so got are shown in Fig. 3 (load: 5.2 kg). In the figure a_1 and a_2 are for upper and lower specimens, respectively. a's are the coefficients of the parabolas, $z=a(x-L/2)^2+c$ used for an approximation of profiles. In order to facilitate abrasives' getting into between the specimens, the weight together with the upper workpiece is lifted and released from the lower work at regular interval. In the case of Fig. 3, the interval is 4 meters. Setting it at 1 meter Fig. 4 is obtained.

4. Discussion on the characteristics of the profile change

The experimental results show that the profile (in vector form) tends to gradually approach an asymptote whose position depends on some conditions, mainly on

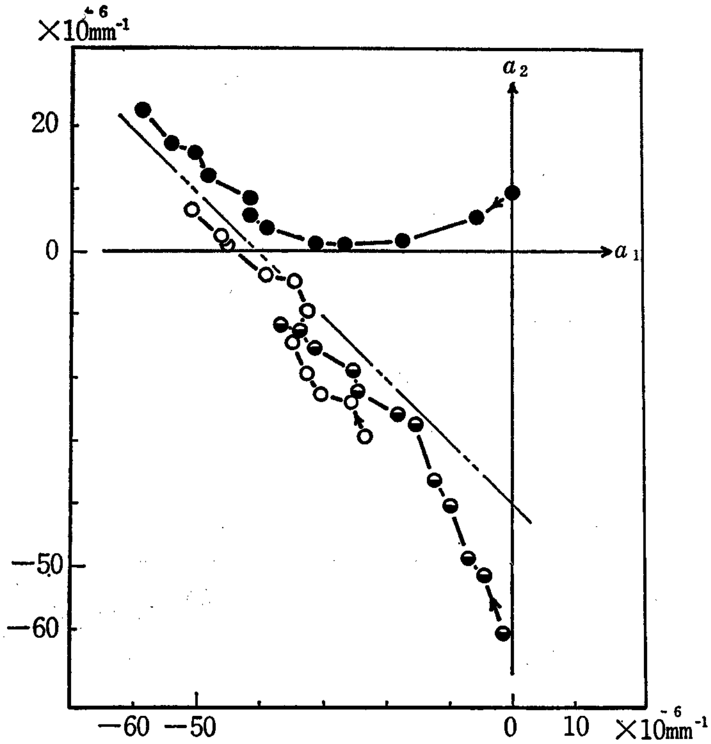

Fig. 3. Experimental results
(load=5.2 kg, interval=120 sec.s)

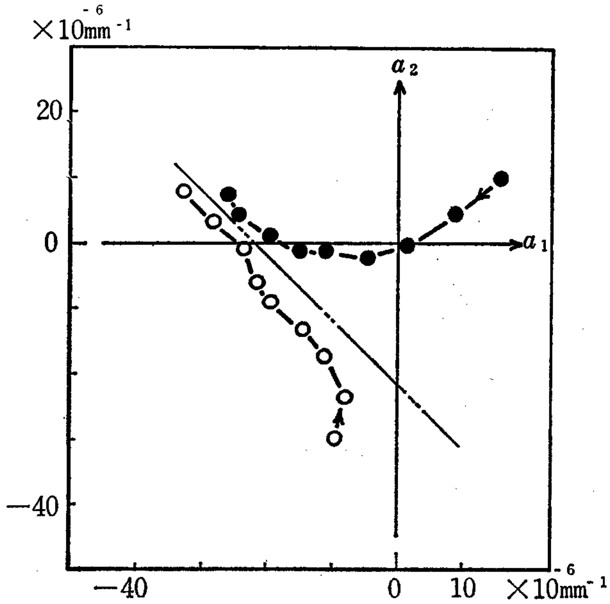

Fig. 4. Experimental results
(load=5.2 kg, interval=30 sec.s)

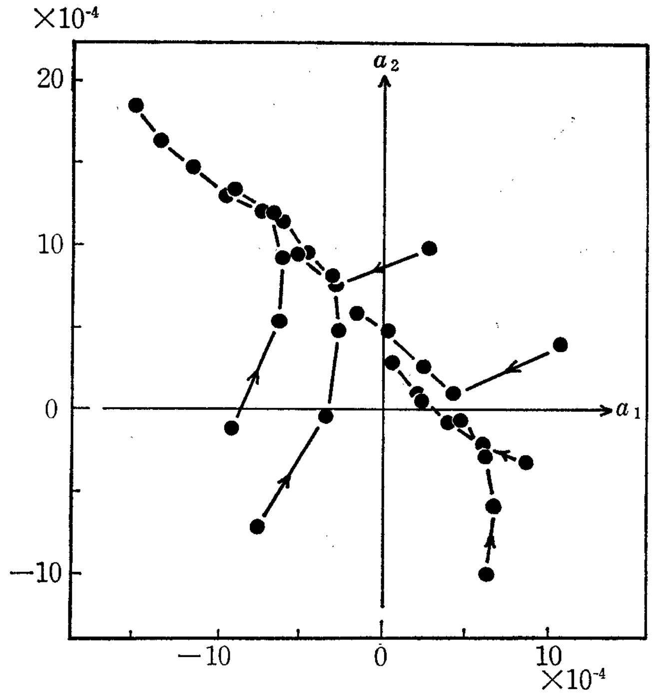

Fig. 5. Simulation results

load and on interval at which the weight is lifted. The slope of the asymptote is considered to always be -45 deg.. Such a situation also holds true in the simulation, the results of which are shown in Fig. 5. The velocity of the change in profile is represented as the resultant of two vectors $T(a)$ and v_0 which are perpendicular to and in the direction of the asymptote, respectively (refer to Fig. 6). The former component is considered to be a characteristic by which the profile tends to approach the asymptote. For simplicity, assume that the asymptote passes through the origin of (a_1, a_2) plane, then its equation is given by

$$a_1 + a_2 = 0 \qquad \text{..............(1)}$$

and this means the absolute values of a_1 and a_2 are equal to each other when on the asymptote, i.e., they are completely coincident with each other. In both the actual and the simulated processes, however, the asymptotes lie a little away from the origin and the value of the right-hand side of Eq. (1) is, say, b and never 0*. The sign of b is negative in the actual process and positive in the simulation. Such a difference in the sign of b comes from the fact that in the simulation abrasives are distributed uniformly on the surface, but such is not the case in the experiment, i.e., abrasives are to be supplied from both ends of the surface of the lower specimen, so they rest more at the ends than at the inner part. This implies the necessity of

* 'Never zero' is for the present cases and at least theoretically b can take the value zero. This will be mentioned in Section 7.

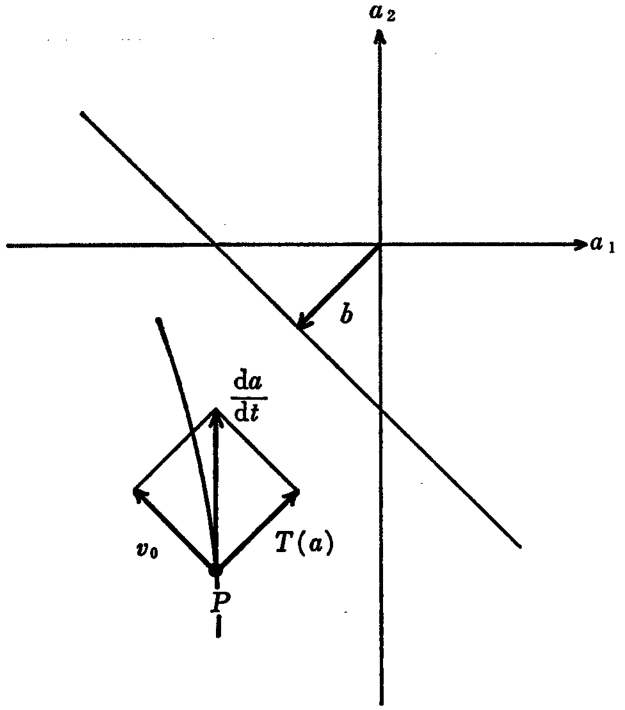

Fig. 6. Illustration of velocity da/dt

lifting up the weight.

The velocity component in the direction parallel to the asymptote is considered to be the immediate consequence of reciprocating motion. In other words, the power supply in the form of reciprocating motion with the vertical load applied involves such a velocity component and the other one mentioned above is an indirect effect, or the interaction between the profiles of sepecimens.[1]

5. Description of the process by a differential equation

The characteristics are the following two, as stated in the previous section:

1) The inherent property in the lapping method.
2) The interaction between the profiles.

Assuming the linearity of the velocity components due to the above two properties, the following differential equation is obtained.

$$\frac{da}{dt} = -\xi(K_0 a - b) + v_0 , \qquad \text{..............(2)}$$

where

$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$,

$K_0 = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$: projection operator of a on the direction perpendicular to the asymptote, singular matrix,

v: constant vector in the direction of the asymptote,

$b=\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$: distance between the origin and the asymptote,

ξ: positive number.

The first term of the right-hand member is for the characteristic 1) and the second for the characteristic 2).

Putting $K_1=-\xi K_0$, $v_1=v_0+\xi b$, Eq. (2) yields

$$\frac{\mathrm{d}a}{\mathrm{d}t}=K_1 a+v_1 . \qquad \text{(3)}$$

As is well known, the solution of the above equation is of the form,

$$a(t)=\mathrm{e}^{K_1 t}+\int_0^t \mathrm{e}^{K_1(t-s)}\, v\mathrm{d}s . \qquad \text{(4)}$$

On introducing new coordinates φ such that

$$\varphi=Ha,\ \varphi=\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix},\ H=\begin{pmatrix} \cos(-\pi/4) & \sin(-\pi/4) \\ -\sin(-\pi/4) & \cos(-\pi/4) \end{pmatrix},$$

we have

$$\varphi(t)=\mathrm{e}^{HK_1H^{-1}t}\varphi(0)+\int_0^t \mathrm{e}^{HK_1H^{-1}(t-s)}Hv\mathrm{d}s . \qquad \text{(5)}$$

After integrating the second term of the right-hand side of Eq. (5) we see that

$$\varphi(t)=\begin{pmatrix} \varphi_1(0)+v_{\varphi_1}t \\ \mathrm{e}^{2kt}\varphi_2(0)+\frac{1}{2k}(\mathrm{e}^{2kt}-1)v_{\varphi_2} \end{pmatrix}, \qquad \text{(6)}$$

where

$$K_1=k\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},\ k\text{: negative constant},\ Hv=\begin{pmatrix} v_{\varphi_1} \\ v_{\varphi_2} \end{pmatrix}\text{: constant vector.}$$

Eliminating the parameter t yields

$$\varphi_2(0)=\frac{\varphi_2(0)+\frac{v_{\varphi_2}}{2k}}{\mathrm{e}^{\frac{2k}{v_{\varphi_1}}\varphi_1(0)}}\,\mathrm{e}^{\frac{2k}{v_{\varphi_1}}\varphi_1(t)}-\frac{v_{\varphi_2}}{2k} . \qquad \text{(7)}$$

The directions of φ_1 and φ_2 coincide with those of v_0 and $T(a)$, respectively.

6. 3-workpiece lapping process

We shall call each abrasion process 'a stage' after the term 'multistage process'. Then the solution of Eq. (2) predicts the profile $a(t)$ after any t meters of lapping distance, provided the initial profile $a(0)$ at a stage is given. At each stage any two of the prepared three workpieces are abraded with each other, as already mentioned.

At each stage there exists a third specimen not abraded. To include it in the system equation (2), we introduce R^3 (R: reals). An element of R^3 is denoted by $\boldsymbol{a}$. Thus the extended equation can be written in the same form as Eq. (2),

$$\frac{\mathrm{d}\boldsymbol{a}}{\mathrm{d}t}=\boldsymbol{K}\boldsymbol{a}+\boldsymbol{v}, \qquad \text{..............(8)}$$

where

$$\boldsymbol{a}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix},\ \boldsymbol{K}=\begin{pmatrix}K & \\ & 0\end{pmatrix},\ \boldsymbol{v}=\begin{pmatrix}v_1\\0\end{pmatrix},$$

and the subscript 3 is for the specimen not concerned in abrasion.

Let A, B, C be the names of the prepared three workpieces. Eq. (3) is for the representation of the trajectory of $\boldsymbol{a}$ in R^3, the upper and lower workpieces being selected from among the A, B, C. Hence, unless the correspondence between two triples (A, B, C) and (1, 2, 3) is given, Eq. (3) has no meaning. To give the correspondence, the concept of permutation is useful.

The procedure is as follows. Let S denote the set $\{1, 2, 3\}$ which is the index set of a_i 's, $i=1, 2, 3$ and $\mathfrak{S}(S)$ the set of all permutations on S. Let a mapping $\gamma: S\rightarrow U$, ($U=\{A, B, C\}$) be such that $\pi\epsilon\mathfrak{S}(S)$,

$$\gamma(\pi(1)) = A,$$
$$\gamma(\pi(2)) = B,$$
$$\gamma(\pi(3)) = C.$$

Ths means that if ε, the identity element of $\mathfrak{S}(S)$, is selected, then $(1, 2, 3)=(A, B, C)$. $\tau=\gamma\circ\pi$ is one-to-one, so τ^{-1}, the inverse of τ, exists. Thus the coordinate transformation $\boldsymbol{P}=(p_{ij})$: $V\rightarrow W$, corresponding to τ, can be constructed as follows.

$$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}=\begin{pmatrix}p_{11} & p_{12} & p_{13}\\ p_{21} & p_{22} & p_{23}\\ p_{31} & p_{32} & p_{33}\end{pmatrix}\begin{pmatrix}a_A\\a_B\\a_C\end{pmatrix},\ \tilde{\boldsymbol{a}}=\begin{pmatrix}a_A\\a_B\\a_C\end{pmatrix}\epsilon V,$$

where p_{ij}'s, the elements of , are such that

$$p_{ij}=\begin{cases}1, \text{ if } \pi^{-1}(i)=j\\ 0, \text{ otherwise.}\end{cases}$$

This is just the ordinary procedure for numbering objects and transforming coordinates.

If the trajectories each of which is the solution of Eq. (3) at the n-th stage ($n=1, 2,\ldots, N$) are drawn in W, then they are generally disconnected with one another, since $\boldsymbol{a}_n(t_n)\neq\boldsymbol{a}_{n+1}(0)$ (t_n: the lapped time at the n-th stage) in general. In V, on the contrary, the change in profile through all the stages can be drawn as a single trajectory. In this aspect it is convenient to describe the process of the change in profile in V. The coordinate transformation $\boldsymbol{P}$ is nonsingular and so it has the inverse $\boldsymbol{P}^{-1}$ which transforms $\boldsymbol{a}$ back to $\tilde{\boldsymbol{a}}$. Thus Eq. (3) has the following form in V.

$$\frac{d\tilde{\boldsymbol{a}}}{dt}=\boldsymbol{P}^{-1}\boldsymbol{K}\boldsymbol{P}\tilde{\boldsymbol{a}}+\boldsymbol{P}^{-1}\tilde{\boldsymbol{v}}\,, \qquad \text{...............(9)}$$

or,

$$\frac{d\tilde{\boldsymbol{a}}}{dt}=\tilde{\boldsymbol{K}}\tilde{\boldsymbol{a}}+\tilde{\boldsymbol{v}}\,, \qquad \text{...............(10)}$$

where $\tilde{\boldsymbol{K}}=\boldsymbol{P}^{-1}\boldsymbol{K}\boldsymbol{P}$ and $\tilde{\boldsymbol{v}}=\boldsymbol{P}^{-1}\boldsymbol{v}$.

3-workpiece lapping process as a multistage process is schematically shown in Fig. 7. In the figure each t_k denotes the length of lapping time (some t_k's might be zero) and each subscript k is the number counted backward, i.e., k is connected to n so that $k=N-n+1$, since a functional equation of backward type will be employed.

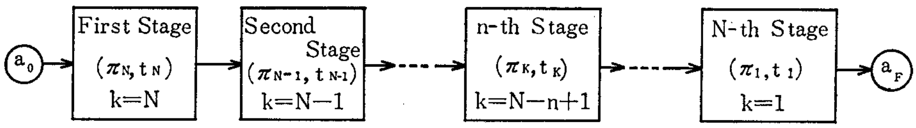

Fig. 7. Scheme of 3-workpiece lapping process as a multistage process

Now let us consider an N-stage process and the problem of improving the flatness (straightness) of the final profile $\tilde{\boldsymbol{a}}_F$, or the profile when the process terminates, within the prescribed tolerance of flatness, say a_f (a real number), which is supposed to be the absolute value allowed equally for each component of $\tilde{\boldsymbol{a}}_F$.

Such a question arises that under what condition $\tilde{\boldsymbol{a}}(0)$, some initial profile of the process can attain the region $O(a_f)$ restricted by $|a_A|$, $|a_B|$, $|a_C|\leq a_f$. For example, which subset of V is the set G whose elements are reachable to $O(a_f)$, given a_f or, conversely, what condition on a_f is required in order that $G=V$. Required here is the answer to the latter, since the purpose of 3-workpiece lapping is to flatten any provided surface profile.

Let us define R, U, I, O_W, G as follows.

$R(\tilde{\boldsymbol{a}})$: region attainable in finite time from $\tilde{\boldsymbol{a}}$.

$U(b)$: $\{\boldsymbol{a}: \boldsymbol{a} \in W$ and satisfies $a_1+a_2+b<0.\}$.

$I(\tilde{\boldsymbol{a}})$: $R(\boldsymbol{a}) \cap O(a_f)$.

$O_W(\boldsymbol{a})$: $\{\boldsymbol{a}: \boldsymbol{a} \in W$ and satisfies $|a_1|$, $|a_2|$, $|a_3|\leq a_f.\}$.

G: $\{\tilde{\boldsymbol{a}}: \tilde{\boldsymbol{a}} \in V$ and $I(\tilde{\boldsymbol{a}})\neq\emptyset\}$.

Then the following statement holds, as will be seen below.

If the number of stages $N\geq 3$ *and* a_f *is determined so that* $O_w(a_f)\cap U(b)\neq\emptyset$, *then* $G=V$. ....(A)

If the identity element ε is taken, trajectories are in the plane a_C=const. and are as shown in Fig. 8(a). The case of

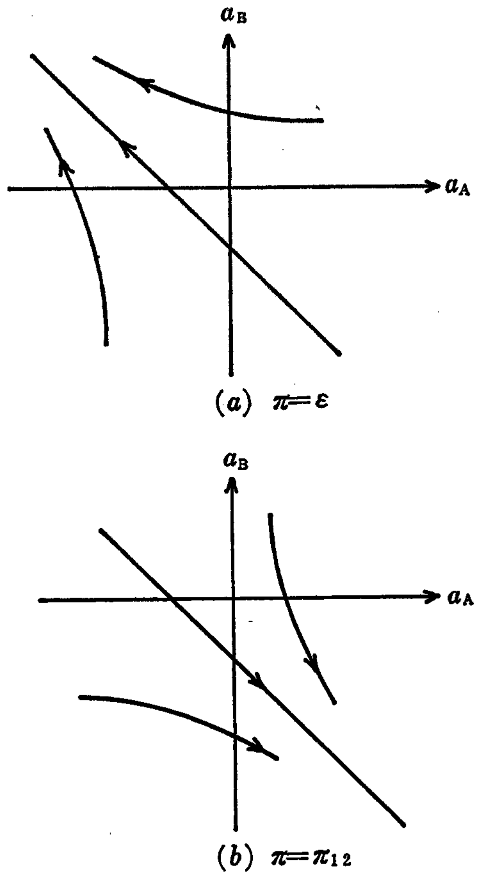

Fig. 8. Direction fields

$$\pi_{12}=\begin{pmatrix}123\\213\end{pmatrix}$$

is shown in Fig. 8(b).

Thus if the third component a_C of $\tilde{a}$ has already satisfied the condition that $|a_C|\leq a_f$, then it follows that through at most two stages $\tilde{a}$ can attain $O(a_f)$ as shown in Fig. 9, i.e., if an initial profile $\tilde{a}$ ($|a_C|\leq a_f$) is a point in the shaded region, it needs only a single stage to reach $O(a_f)$, otherwise two stages. In case of $|a_C|>a_f$, if $|a_C|$ can be decreased so that $|a_C|\leq a_f$ $\tilde{a}$ has a path to $O(a_f)$ by the subsequent procedure discussed above. Hence it suffices to show that it is able to decrease $|a_C|$ less than a_f. If $a_C>a_f$, take

$$\pi_{13}=\begin{pmatrix}123\\321\end{pmatrix}$$

for instance. The semitrajectory ($t\geq 0$) crosses the region $|a_C|\leq a_f$. It is obvious from Fig. 8(a), substituting a_A by a_C. In like manner one can take

$$\pi_{312}=\begin{pmatrix}123\\312\end{pmatrix}$$

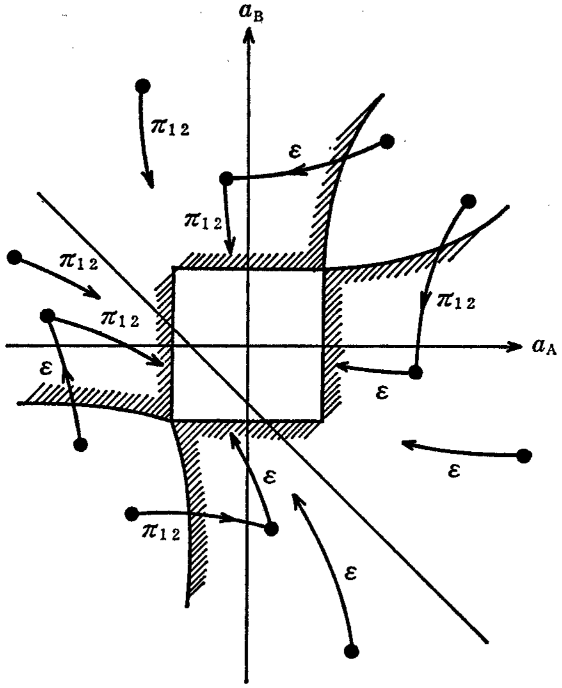

Fig. 9. Attainability through at most two stages (with a_C, $|a_C|<a_f$)

to let $|a_C|\leq a_f$ when $a_C<-a_f$.

According to the statement (A), the process of three stages is fundamental in the sense that if the process is allowed at most three stages, an arbitrary initial profile $\tilde{\boldsymbol{a}}\in V$ does have at least one path to $O(a_f)$.

Here we shall deal with the problem of minimizing the time required for $\tilde{\boldsymbol{a}}\in V$ to reach $O(a_f)$. Assume that at each stage any element of $\mathfrak{S}(S)$ can be taken. Suppose the same element of $\mathfrak{S}(S)$, say π, is taken at some successive two stages, then it is natural to regard them as a single stage π. Hence, in more detail, the permutation element must be chosen from

$$\mathfrak{S}_k(S)=\mathfrak{S}(S)/\{\pi_{k-1}\},$$

π_{k-1} : the element of $\mathfrak{S}(S)$ taken at the $(k-1)$st stage,

at each stage numbered k, $k\geq 2$.

Applying the principle of optimality in the theory of dynamic programming, the following functional equation is obtained.

$$F_k(\tilde{\boldsymbol{a}})=\min_{\pi,\,q}[t_k(q)+F_{k-1}(T(\tilde{\boldsymbol{a}},\pi,q)],\ k=1,2,\dots N, \qquad \text{..............(11)}$$

where $F_k(\tilde{\boldsymbol{a}})$: the minimum time required to reach $O(a_f)$ starting with a profile $\tilde{\boldsymbol{a}}$ at stage k and ending at stage 1 employing an optimal policy,

t_k: $t_k(q)=q\geq 0$, the lapping time at the stage k,

T: $V \times \mathfrak{S}(S) \times R \rightarrow V$, a transformation which gives the profile after being lapped for the time q and employing π started with $\tilde{\boldsymbol{a}}$, or the solution of Eq. (10), P determined.

At each stage a pair of parameters π, q must be determined. $F_N(\tilde{\boldsymbol{a}}_0)$ gives the minimum time for $\tilde{\boldsymbol{a}}_0$ to attain $O(a_f)$. $\tilde{\boldsymbol{a}}_0$ does not always require N stages to reach $O(a_f)$, so, in essence, the process may be of $n(<N)$ stages. This is also explained from the fact already mentioned that the successive two stages employing the same π should be regarded as a single stage.

Numerical calculation was made for the case $N=3$ (load$=5.2$ kg, interval$=30$ sec.s). The results are shown in Table 1. In the examples each component of the initial profiles are the same in absolute value. The total lapping distance of Example 1, however, is much less than that of Example 2. The difference in speed due to the characteristic 2) causes such an effect.

Table 1. Evaluation of optimal policies

unit: mm^{-1}

Ex.	Stage n	(1, 2, 3)	$^n\tilde{a}_i$ → n-th Stage → $^n\tilde{a}_o$ The profile at the beginnig of n-th stage.	The profile when through.	Lapped Distance (m)
1	1	(A, C, B)	$\tilde{a}_0=(50.0, 60.0, 80.0)$	(27.4, 60.0, 61.1)	5.7
	2	(A, B, C)	(27.4, 60.0, 61.1)	(15.0, 50.2, 61.1)	4.0
	3	(C, B, A)	(15.0, 50.2, 61.1)	$(15.0, 15.0, 15.0)=\tilde{a}_F$	16.5
				Total	26.2
2	1	(C, B, A)	$\tilde{a}_0=(-50.0, -60.0, -80.0)$	(−50.0, −15.0, −48.0)	21.6
	2	(A, C, B)	(−50.0, −15.0, −48.0)	(−14.5, −15.0, −27.0)	21.6
	3	(C, A, B)	(−14.5, −15.0, −27.0)	$(-15.0, -15.0, -15.0)=\tilde{a}_F$	25.0
				Total	68.2

$\tilde{a}_0$: initial profile $\tilde{a}_F$: final profile

Experiments were also designed and carried out according to the optimal policies previously determined for the initial values. The results are shown in Table 2. The difference of the actual final profile from the calculated one comes from the fact that the real process is a nonstationary stochastic process and the deviation from the expected value which is no other than the solution of Eq. (5) increases with time. Therefore some stochastic treatment is required to take into account such a fluctuation of $\tilde{\boldsymbol{a}}$.[2)]

7. Discussion on the case $b=0$

We shall be concerned with the case $b=0$ in Eq. (2). The major discrepancy between the results of Fig. 3 and Fig. 4 lies in the position of the asymptote, i.e., the distance between the asymptote and the origin $\|b\|$ is shorter in the case of

Table 2 Experimental results obtained employing optimal policies

(a)

unit : mm^{-1}

Stage n	(1,2,3)	${}^n\tilde{a}_i \longrightarrow$ n-th Stage $\longrightarrow {}^n\tilde{a}_o$ The profile at the beginnig of n-th stage.	The profile when through.	Lapped Distance (m)
1	(C, A, B)	$\tilde{a}_0=(-52.6, -66.7, -76.3)$	$(-20.0, -66.7, -53.0)$	14.7
		$\tilde{a}_0^E=(-52.6, -66.7, -76.3)$	$(-22.9, -66.7, -60.3)$	
2	(B, C, A)	$(-20.0, -66.7, -53.0)$	$(-20.0, -41.5, -15.0)$	20.3
		$(-22.9, -66.7, -60.3)$	$(-22.9, -51.5, -28.7)$	
3	(A, B, C)	$(-20.0, -41.5, -15.0)$	$(-15.0, -15.0, -15.0)=\tilde{a}_F$	35.0
		$(-22.9, -51.5, -28.7)$	$(-27.0, -13.0, -15.0)=\tilde{a}_F^E$	
			Total	70.0

(b)

unit : mm^{-1}

Stage n	(1,2,3)	${}^n\tilde{a}_i \longrightarrow$ n-th Stage $\longrightarrow {}^n\tilde{a}_o$ The profile at the beginnig of n-th stage.	The profile when through.	Lapped Distance (m)
1	(C, B, A)	$\tilde{a}_0=(54.3, -54.4, -73.3)$	$(54.3, 3.3, -40.4)$	17.2
		$\tilde{a}_0^E=(54.3, -54.4, -73.3)$	$(54.3, 4.7, -44.0)$	
2	(A, B, C)	$(54.3, 3.3, -40.4)$	$(15.0, -19.0, -40.4)$	11.9
		$(54.3, 4.7, -44.0)$	$(24.4, -19.5, -44.0)$	
3	(B, C, A)	$(15.0, -19.4, -40.4)$	$(15.0, -15.0, -15.0)=\tilde{a}_F$	14.9
		$(24.4, -19.5, -44.0)$	$(24.4, -11.8, -17.5)=\tilde{a}_F^E$	
			Total	44.0

$\tilde{a}_0$: initial profile $(=\tilde{a}_0^E)$ $\tilde{a}_F$: final profile (calculated)

$\tilde{a}_0^E$: initial profile (actual) $\tilde{a}_F^E$: final profile (actual)

Fig. 4 than in the case of Fig. 3. The probable explanation of this is that the rate of wear and fracture of abrasives decreases as the interval mentioned in Section 3 and the applied load decrease[3]. Since in the simulation Postulate (3) is assumed, the process is ideal and the point where the asymptote cuts the a_2 axis is positive. Consequently, b can take the value zero.

In such a situation profiles can be approached arbitrarily close to the origin in finite time by the 3-workpiece lapping process, but can not exactly reach the origin except when they happen to be on the asymptote which is the straight line through the origin.

The system of $b=0$ can also be obtained by coordinate transformation $\tilde{a}=a-b$. It is not, however, of $b=0$ in its own right at all, and is merely for convenience sake, since the origin in this case does not correspond to the flat surface. The situation stated in the previous paragraph also holds true. From this the condition $O_w(a_f) \cap U(a) \neq \emptyset$ is required to be $G=V$.

8. Conclusions

A deterministic treatment of 3-workpiece lapping process was given. The differential equation derived for the description of the change in profile was based upon the following characteristics:

1) The inherent property in the lapping method.

2) The interaction between profiles of workpieces.

It was shown that the process of three stages is a fundamental one in the sense that any profile approximated by parabola can be flattened to a good extent. Applying the principle of optimality an effective procedure for flattening was also discussed.

References

1) M. Ido, Y. Tanaka and Y. Sakai, J. Japan Society of Precision Engineering, 41, 978 (1975).
2) M. Ido, Y. Tanaka and Y. Sakai, The Oral Report at General Meeting of JSPE, Oct. 1974.
3) M. Ido, Y. Tanaka and Y. Sakai, The Oral Report at Local Meeting of JSPE (in Kansai District), June 1974.