Maximum Energy Efficiency of Linear Induction Motor in Accelerating Period

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Abstract

This paper deals theoretically with the maximum energy efficiency of a linear induction motor in an accelerating period, for a constant and an adjustable frequency operation. Those conditions relating to the secondary resistance are derived. On the condition for the valuable secondary resistance, the resistance is adjusted in proportion to the slip. On the other condition for the constant secondary resistance, the resistance is set as a function of the final slip in the accelerating period. By adjusting the resistance, the maximum energy efficiency becomes considerably high at the small final slip, especially on the linear motor with the small magnetizing reactance. On the other hand, the effect of adjustable frequency operation is evident. But, with the decreasing the magnetizing reactance, that effect decreases, and can not be expected too much in the linear induction motor.

1. Introduction

All the world is interested in an application of linear induction motors to a high speed train, and a very important problem is the efficiency of the motor in an accelerating period because a great deal of supplied power is consumed in that period to drive a train trailing many carriages. Though edge effects of the motor in the high-speed and small-slip steady-state operation have been studied by many authors,[1),2),3)] studies on the energy efficiency in the period are very few in spite of its importance. As an exciting current of conventional rotating induction motors is negligible, the energy efficiency in the accelerating period from standstill to synchronous speed is theoretically 50% and has no relation to the primary and the secondary resistances. But, in linear induction motors the exciting current may not be negligible, because the air-gap length is extremely long and the magnetizing reactance is small compared with those of conventional motors. The accelerating energy efficiency of linear motors, therefore, is fairly poor and is influenced by the secondary resistance like the driving force is.

In this paper, the accelerating energy efficiencies of linear motors for the constant and the variable frequency supplies are examined, and the optimum conditions of the maximum efficiency relating to the secondary resistance are derived.

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2. Power Efficiencies of Induction Motor

When the characteristics of linear motors are analyzed, a serious consideration shoud be given to the edge effect. Especially, in the high-speed and small slip operation, the driving force and the power factor are markedly reduced, resulting in a poor power efficiency. But, in the accelerating period in which the slip is much than 0.2, the edge effect usually has only a minor influence on the motor performance. The present analyses and discussions on the accelerating performance, therefore, assume that the theory of conventional motors is applicable to linear motors.

2.1 Conventional Induction Motor

The equivalent circuit for one phase of a conventional three-phase induction motor is shown in Fig. 1(a). From this equivalent circuit, the following input and output equations are obtained:

$$P_i=3V^2[(r_2+sr_m)A+(x_2+x_m)Bs]/(A^2+B^2)\,, \tag{2.1.1}$$

$$P_0=3V^2(1-s)sr_2(r_m^2+x_m^2)/(A^2+B^2)\,, \tag{2.1.2}$$

where $A=(r_1+r_m)r_2+s(r_1r_m-x_1x_m-x_1x_2-x_2x_m)$,

$B=(x_1+x_m)r_2+s(r_1x_m+r_mx_1+r_1x_2+r_mx_2)$,

and V: the supply voltage,

r_1 and r_2: the primary and the secondary resistances,

x_1 and x_2: the primary and the secondary leakage reactances,

r_m and x_m: the magnetizing resistance and reactance,

s: the slip.

From Eqs. (2.1.1) and (2.1.2), the power efficiency is given by

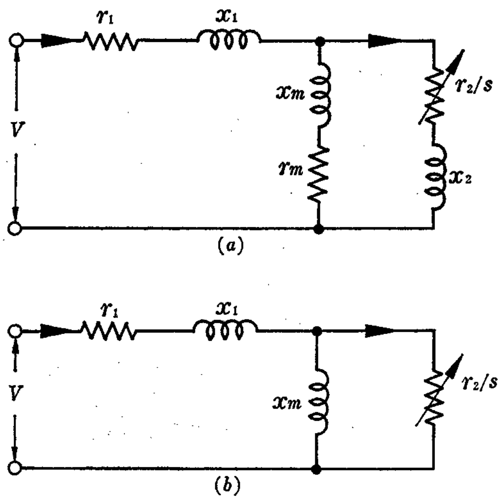

Fig. 1. The equivalent circuits;
(a) conventional induction motor.
(b) linear induction motor.

$$\eta_p = \frac{P_0}{P_i} = \frac{(1-s)sr_2(r_m^2+x_m^2)}{(r_2+sr_m)A+(x_2+x_m)Bs} . \tag{2.1.3}$$

2.2 Linear Induction Motor

As the secondary side of linear motors is usually made of a metalic sheet and the air-gap length is extremely long compared with that of conventional motors, the secondary leakage reactance and the magnetizing resistance are negligible, and the equivalent circuit can be simplified as shown in Fig. 1(b).[4] Therefore, the following power and efficiency equations may be obtained by putting x_2 and r_m in Eqs. (2.1.1) to (2.1.3) equal to zero:

$$P_i = \frac{3V^2(r_1r_2^2+sr_2x_m^2+s^2r_1x_m^2)}{[r_1^2+(x_1+x_m)^2]r_2^2+2sr_1r_2x_m^2+s^2(r_1^2+x_1^2)x_m^2} , \tag{2.2.1}$$

$$P_0 = \frac{3V^2(1-s)sr_2x_m^2}{[r_1^2+(x_1+x_m)^2]r_2^2+2sr_1r_2x_m^2+s^2(r_1^2+x_1^2)x_m^2} , \tag{2.2.2}$$

$$\eta_p = \frac{1-s}{1+sr_1/r_2+r_1r_2/(sx_m^2)} . \tag{2.2.3}$$

3. Maximum Energy Efficiencies during Acceleration for Constant Frequency Operation

3.1 Adjustable Secondary Resistance Operation

At first, the maximum energy efficiency during acceleration for the constant frequency operation on condition that the secondary resistance is changed with the slip is examined.[5] Differentiating Eq. (2.1.3) with respect to the secondary resistance, and equating to zero, an expression is obtained for the maximum power efficiency of conventional induction motors at any slip:

$$A(r_2+sr_m)+Bs(x_2+x_m) \\ = r_2[(r_2+sr_m)\partial A/\partial r_2+s(x_2+x_m)\partial B/\partial r_2+A] .$$

Solving the above equation with respect to r_2, the secondary resistance which maximize the power efficiency is given by

$$[r_2]_{\eta m} = s\sqrt{x_2^2+r_1(r_m^2+x_m^2+2x_2x_m)/(r_1+r_m)} . \tag{3.1.1}$$

For linear motors, by putting r_m and x_2 equal to zero, the following equation is obtained:

$$[r_2]_{\eta m} = sx_m . \tag{3.1.2}$$

From Eqs. (3.1.1) and (3.1.2), it is seen that the secondary resistance for the maximum power efficiency is proportional to the slip. In linear motors, the ratio of the secondary resistance to the slip is decided by only the magnetizing reactance which is smaller than that of conventional motors owing to the large air-gap. On substituting Eq. (3.1.2) in Eq. (2.2.3), the maximum power efficiency of linear motors is obtained as

$$\eta_{pm}=\frac{1-s}{1+2r_1/x_m}\ . \tag{3.1.3}$$

Now, the input energy in the accelerating period from standstill to final speed v_e can be derived as follows:

$$W_i=\int_0^{t_e} P_i dt=\int_0^{t_e}\frac{P_0}{\eta_p}dt=M\int_0^{v_e}\frac{v}{\eta_p}dv\ , \tag{3.1.4}$$

where v: the speed,

t_e: the accelerating time for v_e,

M: the mass of accelerated object.

As linear motors are accelerated satisfying the condition given by Eq. (3.1.2), the minimum input energy in the accelerating period may be brought about. On substituting η_{pm} of Eq. (3.1.3) instead of η_p in Eq. (3.1.4), the accelerating minimum input energy is obtained as

$$W_{im}=\frac{Mv_e^2}{1-s_e}\left(1+2\frac{r_1}{x_m}\right)\ , \tag{3.1.5}$$

where s_e is the final slip.

On the other hand, the output energy, that is the kinetic energy of accelerated objects, is $W_0=Mv_e^2/2$. Thus, the maximum energy efficiency during acceleration for the constant frequency and adjustable secondary resistance operation is given by

$$\eta_{em}=\frac{W_0}{W_{im}}=\frac{1-s_e}{2(1+2r_1/x_m)}\ . \tag{3.1.6}$$

In Fig. 2, those values are plotted against r_1/x_m, for a number of final slips. Those

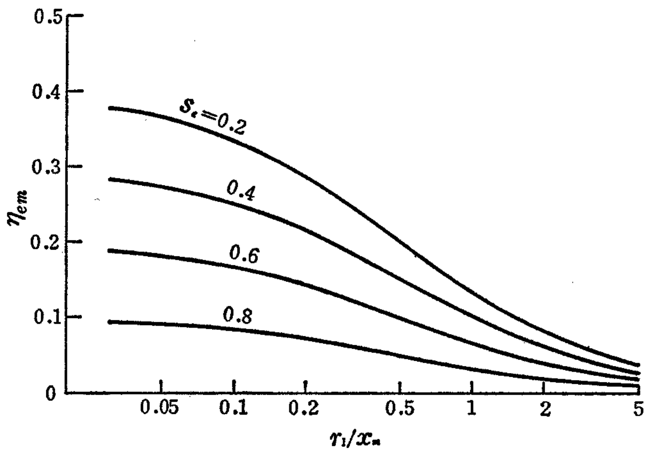

Fig. 2. The maximum energy efficiencies for the constant frequency and adjustable secondary resistance operations.

results show that the maximum energy efficiency decreases as the final slip and r_1/x_m are increased. In linear motors, the value of r_1/x_m is so large[6] that the energy efficiency may be poor.

3.2 Constant Secondary Resistance Operation

In the preceding section, the characteristics on condition that the secondary resistance is adjustable have been reviewed. However, in practice, it is difficult to adjust the resistance of the secondary sheet conductor according to the slip. In this section, the maximum energy efficiency during acceleration on the constant secondary resistance operation is dealt with. The secondary resistance is denoted by a constant value R_2, and the following power efficiency is given from Eq. (2.2.3):

$$[\eta_P]_{R_2}=\frac{1-s}{1+sr_1/R_2+r_1R_2/(sx_m^2)}\,. \tag{3.2.1}$$

The input energy is derived as

$$\begin{aligned}[W_i]_{R_2}&=M\int_0^{v_e}\frac{v}{[\eta_P]_{R_2}}dv\\ &=\frac{Mv_e^2}{1-s_e}\left[1+\frac{(1+s_e)r_1}{2R_2}-\frac{r_1R_2}{(1-s_e)x_m^2}\ln s_e\right].\end{aligned} \tag{3.2.2}$$

The condition of the maximum energy efficiency is equivalent to that of the minimum input energy. From Eq. (3.2.2), therefore, the constant secondary resistance on that condition is given by

$$R_2=Gx_m\,, \tag{3.2.3}$$

where, $$G=\sqrt{\frac{1-s_e^2}{2\ln 1/s_e}}\,.$$

From Eq. (3.2.2), the energy efficiency for the constant secondary resistance operation is

$$[\eta_e]_{R_2}=\frac{W_0}{[W_i]_{R_2}}=\frac{1-s_e}{2[1+(1+s_e)r_1/2R_2-r_1R_2\ln s_e/(1-s_e)x_m^2]}\,. \tag{3.2.4}$$

On substituting Eq. (3.2.3) in Eq. (3.2.4), the maximum energy efficiency during acceleration for the constant frequency and secondary resistance operation is obtained as

$$[\eta_{em}]_{R_2}=\frac{1-s_e}{2(1+2Hr_1/x_m)}\,, \tag{3.2.5}$$

where

$$H=\sqrt{\frac{(1+s_e)\ln 1/s_e}{2(1-s_e)}}\,.$$

Comparing Eq. (3.2.5) with Eq. (3.1.6), the following equation is given:

$$\frac{[\eta_{em}]_{R_2}}{\eta_{em}}=\frac{1+2r_1/x_m}{1+2Hr_1/x_m}\ . \tag{3.2.6}$$

Fig. 3 shows the theoretical maximum energy efficiencies for the constant frequency and secondary resistance operation. Those for the adjustable secondary resistance operation are also included in Fig. 3 for comparison. In the region of small slip driving, the difference between the former and the latter is considerably recognized, especially for the large values of r_1/x_m. In Fig. 4 the calculated values of G and H are plotted against the final slip. A decrease in the value of G with decreasing the final slip shows that the small secondary resistance is required on the small final slip operation as given in Eq. (3.2.3). The serious increase in the value of H at the final slip less than 0.2 stands in a causal relation to the difference mentioned above.

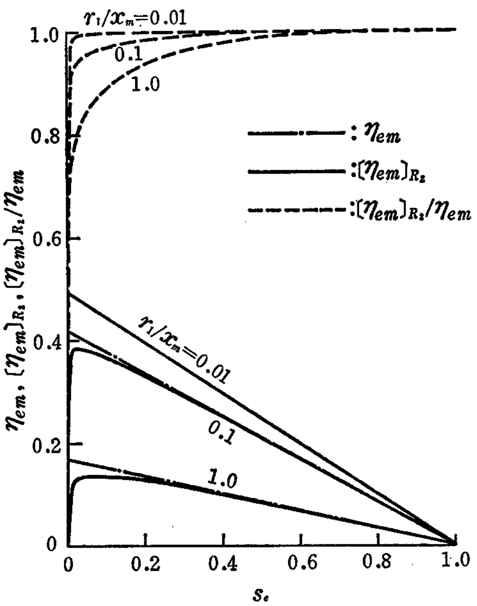

Fig. 3. The maximum energy efficiencies for the constant frequency operation.

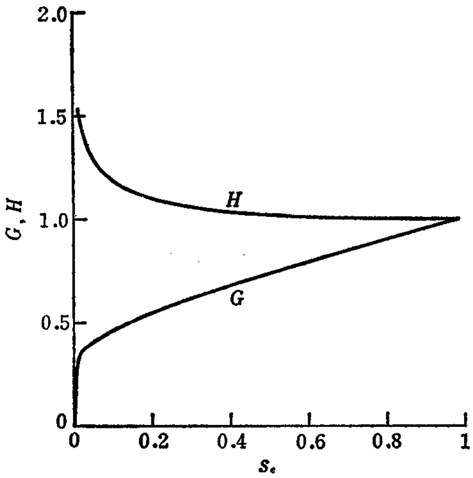

Fig. 4. The values of G and H.

4. Maximum Energy Efficiencies during Acceleration for Adjustable Frequency Operation

In this chapter, the motor performances on a variable-frequency supply are examined. Since the supply frequency f is adjusted in proportion to the speed, the slip is constant. Also the supply voltage must be changed proportionately to the speed in order to maintain the proper magnetic conditions in the core. This mode of operation is known as constant Volts/Hz. However, the starting on this method is impossible because the voltage and frequency at starting are equal to zero. Then, as shown in Fig. 5, in the first accelerating period from standstill to speed kv_e the voltage and frequency are maintained at constant values, respectively, and in the

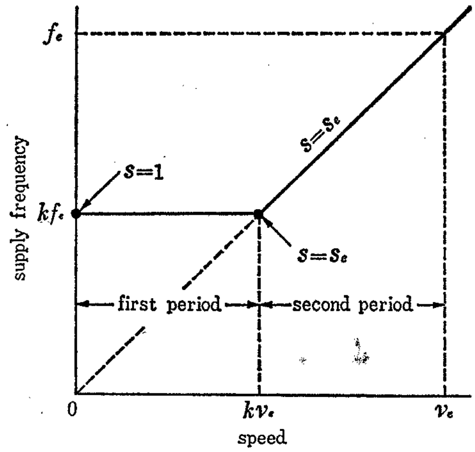

Fig. 5. The mode of operation for the adjustable frequency.

second accelerating period from speed kv_e to final speed v_e, they are changed according to the above-mentioned method.

4.1 Adjustable Secondary Resistance Operation

In the first period, the minimum input energy is obtained by substituting kv_e and kx_{me}, instead of v_e and x_m, in Eq. (3.1.5):

$$W'_{im}=\frac{M(kv_e)^2}{1-s_e}\left(1+2\frac{r_1}{kx_{me}}\right), \tag{4.1.1}$$

where x_{me} is the magnetizing reactance at the frequency f_e. In the second period, the maximum power efficiency at any frequency corresponding to Eq. (3.1.3) is given by

$$\eta''_{pm}=\frac{1-s_e}{1+2(f_e/f)r_1/x_{me}}\ . \tag{4.1.2}$$

On combining Eq. (4.1.2) with Eqs. (3.1.4) and (3.1.5), the following equation of the minimum input energy for the second period is obtained:

$$\begin{aligned} W''_{im}&=M\int_{kv_e}^{v_e}\frac{v}{\eta''_{pm}}\,dv \\ &=\frac{Mv_e^2}{2(1-s_e)}\left[(1-k^2)+4(1-k)\frac{r_1}{x_{me}}\right]. \end{aligned} \tag{4.1.3}$$

The minimum total input energy during acceleration is given by adding Eq. (4.1.1) to (4.1.3). Thus,

$$\begin{aligned} [W_{im}]_f&=W'_{im}+W''_{im} \\ &=\frac{Mv_e^2}{2(1-s_e)}\left[(1+k^2)+4\frac{r_1}{x_{me}}\right]. \end{aligned} \tag{4.1.4}$$

By Eq. (4.1.4), the maximum energy efficiency during acceleration for the adjustable frequency and secondary resistance operation is obtained:

$$[\eta_{em}]_f=\frac{W_0}{[W_{im}]_f}=\frac{1-s_e}{(1+k^2)+4r_1/x_{me}} . \tag{4.1.5}$$

Comparing Eq. (3.1.6) with Eq. (4.1.5), the following ratio is given:

$$\frac{[\eta_{em}]_f}{\eta_{em}}=\frac{2(1+2r_1/x_{me})}{(1+k^2)+4r_1/x_{me}} . \tag{4.1.6}$$

The calculated results of Eqs. (4.1.5) and (4.1.6) are shown in Fig. 6. Increasing the values of r_1/x_{me}, the effects of adjustable frequency operation decrease and can not be expected too much in the linear motors with large r_1/x_{me}. For the adjustable frequency operation, the quantity V/f is constant. However, at low frequency the primary resistive potential difference becomes significant compared with the induced e.m.f. of the primary winding. This causes a reduction in the air-gap flux and motor characteristics, especially for the motors with large r_1/x_m. In order to improve the characteristics for the adjustable frequency operation, the Volts/Hz must be increased at low frequency. In Fig. 7, above efficiencies and ratios are shown as a function of the coefficient k. The effects of the adjustable frequency operation increase with decreasing the coefficient k.

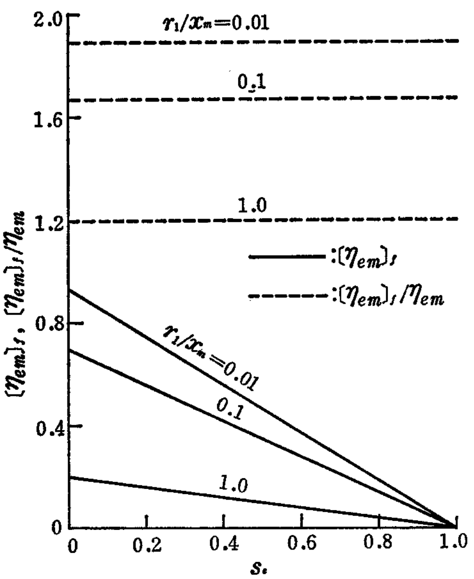

Fig. 6. The maximum energy efficiencies for the adjustable frequency and secondary operations. (k=0.2)

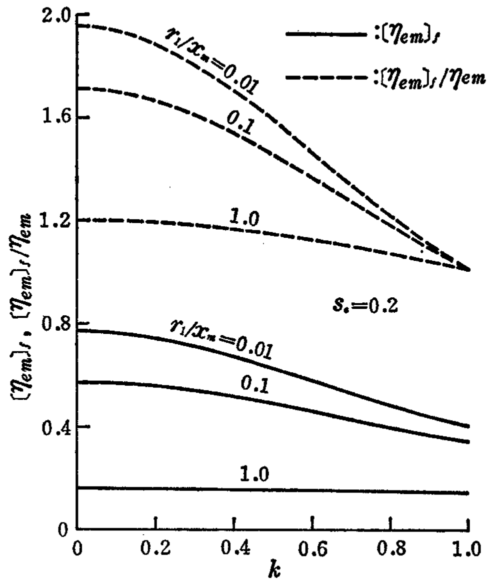

Fig. 7 The maximum energy efficiencies for the adjustable frequency and secondary operations. (Se=0.2)

4.2 Constant Secondary Resistance Operation

As stated in the preceding section, the input energy is given by adding the energies in the first and the second period. The input energy for the constant secondary resistance operation is therefore given by

$$[W_i]_{R_2f}=M\int_0^{kv_e}\frac{v}{[\eta_p]'_{R_2}}\,dv+M\int_{kv_e}^{v_e}\frac{v}{[\eta_p]''_{R_2f}}\,dv$$
$$=\frac{Mv_e^2}{2(1-s_e)}\Big[1+k^2+\frac{r_1}{R_2}(k^2+s_e)$$
$$-\frac{2r_1R_2}{x_{me}^2}\Big(\frac{1}{s_e}\ln k+\frac{1}{1-s_e}\ln s_e\Big)\Big], \qquad (4.2.1)$$

where $[\eta_p]'_{R_2}$: the power efficiency given by substituting kx_{me} instead of x_m in Eq. (3.2.1),

$[\eta_p]''_{R_2f}$: the power efficiency given by substituting $(f/f_e)x_{me}$ and s_e instead of x_m and s in Eq. (3.2.1).

The energy efficiency is

$$[\eta_e]_{R_2f}=\frac{W_0}{[W_i]_{R_2f}}$$
$$=\frac{1-s_e}{1+k^2+(k^2+s_e)r_1/R_2+(1/s_e\ln 1/k+1/(1-s_e)\ln 1/s_e)2r_1R_2/x_{me}^2}\,. \qquad (4.2.2)$$

The following constant secondary resistance for the maximum energy efficiency during acceleration is given by Eq. (4.2.2):

$$[R_2]_f=I\,x_{me}\,, \qquad (4.2.3)$$

where

$$I=\sqrt{\frac{k^2+s_e}{2[1/s_e\ln 1/k+1/(1-s_e)\ln 1/s_e]}}\,.$$

From Eqs. (4.2.2) and (4.2.3), the maximum energy efficiency during acceleration for the adjustable frequency and constant secondary resistance operation is obtained as follows:

$$[\eta_{em}]_{R_2f}=\frac{1-s_e}{1+k^2+4Jr_1/x_{me}}\,, \qquad (4.2.4)$$

where

$$J=\sqrt{\frac{k^2+s_e}{2}\Big(\frac{1}{s_e}\ln\frac{1}{k}+\frac{1}{1-s_e}\ln\frac{1}{s_e}\Big)}\,.$$

Comparing Eq. (3.2.5) with Eq. (4.2.4) and Eq. (4.1.5) with Eq. (4.2.4), the following ratios are obtained, respectively,

$$\frac{[\eta_{em}]_{R_2f}}{[\eta_{em}]_{R_2}}=\frac{2+4Hr_1/x_{me}}{1+k^2+4Jr_1/x_{me}}\,, \qquad (4.2.5)$$

$$\frac{[\eta_{em}]_{R_2f}}{[\eta_{em}]_f}=\frac{1+k^2+4r_1/x_{me}}{1+k^2+4Jr_1/x_{me}}\,. \qquad (4.2.6)$$

The calculated results of Eq. (4.2.4) are shown in Figs. 8 and 9. The ratios of Eq. (4.2.5) are included to show the effect of the adjustable frequency operation. The effect is slightly less than that for the motor with the adjustable secondary resistance, especially for large r_1/x_m. The calculated values of I and J are plotted against the

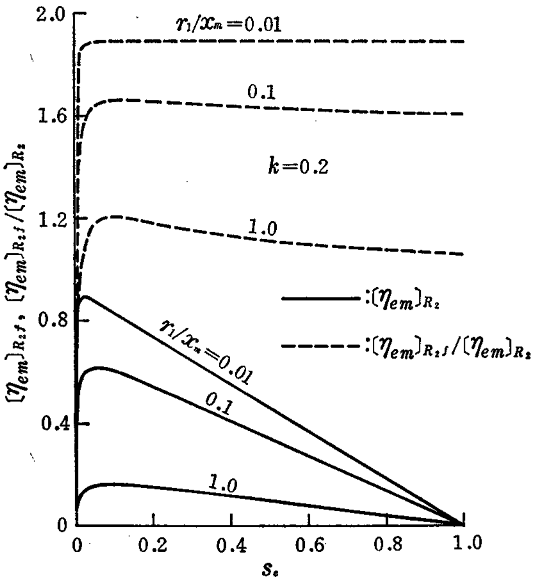

Fig. 8. The maximum energy efficiencies for the adjustable frequency and constant secondary resistance operations. ($k=0.2$)

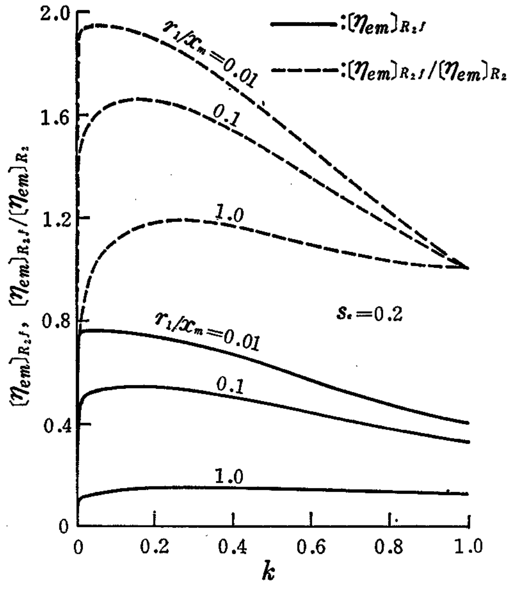

Fig. 9. The maximum energy efficiencies for the adjustable frequency and constant secondary resistance operations. ($S_e=0.2$)

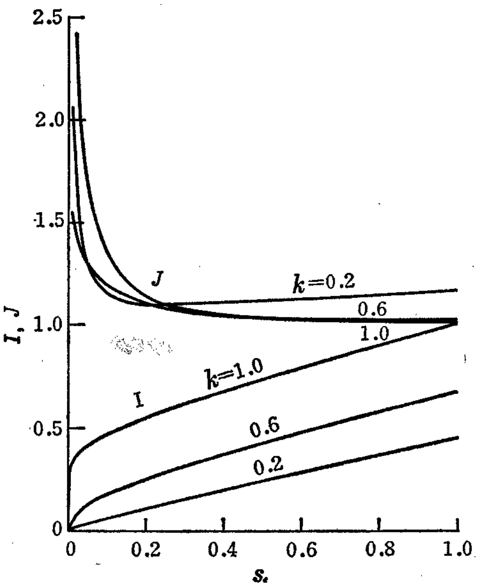

Fig. 10. The values of I and J.

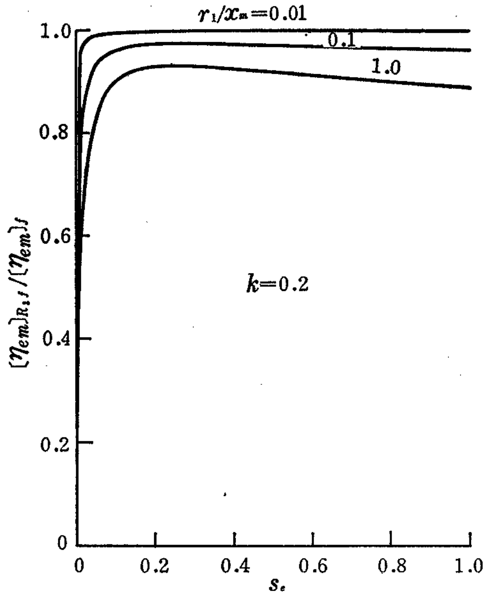

Fig. 11. The ratios of $[\eta_{em}]_{R_2f}$ and $[\eta_{em}]_f$. (k=0.2)

final slip in Fig. 10. When the coefficient k is unity, the values of I and J are equal to G and H shown in Fig. 4, respectively. The calculated values of Eq. (4.2.6) are shown in Fig. 11. Comparing with Fig. 3, it is seen that the effect of the adjustable secondary resistance is more remarkable for the adjustable frequency operation than for the constant operation, especially in the large final slip region.

5. Conclusion

The maximum energy efficiencies of linear induction motors in the accelerating period for various ways of operation have been analyzed and the optimum conditions of the maximum efficiency relating to the secondary resistance have been given for each operation. The results obtained are summarized as follows:

(1) The secondary resistance which corresponds to the maximum energy efficiency during acceleration must be adjusted with the slip following the equation:

$$[r_2]_{\eta m} = s x_m \, .$$

(2) With regard to the motor with the constant secondary resistance, the secondary resistance should satisfy the following equations. For the constant frequency operation

$$R_2 = G x_m \, ,$$

and for the adjustable frequency operation

$$[R_2]_f = I \, x_{me} \, ,$$

where the coefficients G and I are the functions of the final slip.

(3) The effect of the adjustable secondary resistance is considerable in a range of small final slip. Especially, in linear motors with large r_1/x_m, this effect is more marked.

(4) The effect of the adjustable frequency operation is evident. But, with an increase in r_1/x_m, the effect decreases, and can not be expected too much in linear motors.

(5) To improve the energy efficiency of linear motors with large r_1/x_m for the adjustable frequency operation, the supply voltage should be increased considerably above its frequency-proportional value.

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