Optimum Configuration of Vibration Reducers for Beam Systems with Random Excitations

Optimum Configuration of Vibration Reducers for Beam Systems with Random Excitations

Naofumi FUJIWARA* and Yoshisada MUROTSU**

(Received June 15, 1973)

A new method for finding optimum configuration of vibration reducers for elastic systems with random excitations is discussed. Optimum control theory in frequency domain is applied to this problem. The optimization technique is illustrated on a beam supported at both ends. It is found that for white noise excitation, the optimum vibration reducer for the first mode vibration of a beam can be mechanized by spring-dashpot element and can be really effective for vibrations with whole modes of the beam.

1. Introduction

The problems of designing vibration reducers for suppressing magnitude of response of elastic systems to random excitations have broad applications in the mechanical engineering, civil engineering and architecture. Many techniques for solving the problems were considered. One of those is a technique that adds a localized spring and mass[1] or dashpot[2] to the elastic systems and determines its position and parameter values so as to obtain the desired performance of the systems. It is, however, not always certified that the configurations of vibration reducers determined by the techniques mentioned above are the best.

In this paper, a new method for finding optimum configurations of vibration reducers for elastic systems with random excitations is proposed. The method makes use of the optimum control theory which is recently developed in the control engineering. The optimization technique is described for a general beam system subjected to random foundation excitations and illustrated on typical simple systems. Numerical examples are presented for simply-supported uniform beam excited by white noise or a typical random foundation motion. While the technique is basically equal to that for multi-degree-of-freedom systems[3)4)], the satisfactory results are obtained for elastic systems.

2. Optimizing procedure

A beam supported at both ends and having n vibration reducers is considered as shown in Fig. 1. The characteristics of the vibration reducers are assumed to

* Department of Aeronautical Engineering, College of Engineering.
** Department of Naval Architecture, College of Engineering.

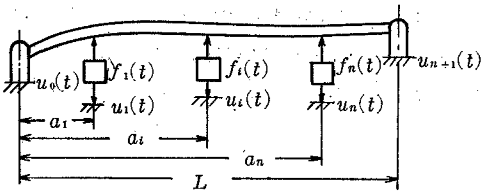

Fig. 1. Schematic diagram of a general beam system with n vibration reducers.

be black boxes which generate the undetermined forces. The wave equation that describes the transverse vibration of the beam may be written as follows;

$$\rho(x)\frac{\partial^2 u(x,t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}\right)=\sum_{i=1}^{n}f_i(t)\delta(x-a_i) \tag{1}$$

where

$u(x,t)$: transverse deflection of the beam

$$=y(x,t)+(1-x/L)u_0(t)+(x/L)u_{n+1}(t) \tag{2}$$

$\rho(x)$: mass per unit length

$EI(x)$: flexural rigidity

$y(x,t)$: elastic deformation

$f_i(t)$: force generated by the i-th vibration reducer

$u_i(t)$: deflection of foundation at $x=a_i$

a_i : length from left end of the beam to the i-th vibration reducer

$\delta(x)$: delta-function.

Problem will be stated as follows: When the foundations of the system shown in Fig. 1. are excited by the zero-mean stationary Gaussian random forces, design the optimum vibration reducers so that the elastic deformation of the beam may be minimized with specified constraints on the forces of the vibration reducers. This problem is formulated as an optimum control problem: Under the constraints

$$\langle f_i(t)^2\rangle \leqq M_i \quad (i=1,2,\cdots,n) \tag{3}$$

design the linear vibration reducers to minimize the performance index

$$I=\overline{(\langle y(x,t)^2\rangle)_w} \tag{4}$$

where

M_i : given constraint value for the i-th vibration reducer

$\langle\ \rangle$: time average

$\overline{(\cdot)_w}$: weighted mean over the beam $=\frac{1}{L}\int_0^L(\cdot)w(x)\mathrm{d}x$

$w(x)$: weighting function.

Following the Lagrange's method of undetermined multipliers, the above design problem is reformulated as the problem to minimize the following penalty function

$$J=\overline{(\langle y(x,t)^2\rangle)_w}+\sum_{i=1}^{n}\lambda_i^2\langle f_i(t)^2\rangle \tag{5}$$

where

λ_i^2: Lagrange's undetermined multiplier for the i-th vibration reducer.

For simplification of mathematical manipulations, Eq. (5) is converted into the frequency domain. First, consider the functional relation between the force of the i-th vibration reducer and the foundation motion $u_k(t)$. Since the candidate vibration reducers being seeked are of linear type as assumed, $f_i(t)$ is characterized by the relation[3)]

$$f_i(s)=\sum_{k=0}^{n+1}F_{ik}(s)\ddot{u}_k(s) \tag{6}$$

where $F_{ik}(s)$ is a transfer function which will be determined. In Eq. (6), the following notations are introduced;

s : Laplace transform variable

$\cdot$: time derivative

$f_i(s)$: $\mathscr{L}\{f_i(t)\}$

$\ddot{u}_k(s)$: $\mathscr{L}\{\ddot{u}_k(t)\}$.

Next, the relation between $y(x,t)$ and $\ddot{u}_i(t)$ is considered. Substituting Eq. (2) into Eq. (1) leads to

$$\rho(x)\frac{\partial^2 y(x,t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left\{EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}\right\}=\sum_{i=1}^{n}f_i(t)\delta(x-a_i)$$
$$-\rho(x)(1-x/L)\ddot{u}_0(t)-\rho(x)(x/L)\ddot{u}_{n+1}(t) \tag{7}$$

The solution to Eq. (7) can be represented in terms of all the normal modes $Y_j(x)$ and all the normal coordinates $q_j(t)$, and thus $y(x,t)$ is expressed as

$$y(x,t)=\sum_{j=1}^{\infty}Y_j(x)q_j(t) \tag{8}$$

where $Y_j(x)$ is determined by specifying the boundary conditions. Substituting Eq. (8) into Eq. (7) yields

$$\sum_{j=1}^{\infty}\rho(x)Y_j(x)\ddot{q}_j(t)+\sum_{j=1}^{\infty}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left\{EI(x)\frac{\mathrm{d}^2Y_j(x)}{\mathrm{d}x^2}\right\}q_j(t)$$
$$=\sum_{i=1}^{n}f_i(t)\delta(x-a_i)-\rho(x)\left(1-\frac{x}{L}\right)\ddot{u}_0(t)-\frac{\rho(x)}{L}x\ddot{u}_{n+1}(t) \tag{9}$$

Multiplying Eq. (9) by $Y_i(x)$ and integrating over the beam, the following equation is obtained;

$$M_j\ddot{q}_j(t)+K_jq_j(t)=\sum_{i=1}^{n}Y_j(a_i)f_i(t)-b_j\ddot{u}_0(t)-c_j\ddot{u}_{n+1}(t) \tag{10}$$

where

$$M_j=\int_0^L\rho(x)Y_j(x)^2\mathrm{d}x$$

$$K_j=\int_0^L\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left\{EI(x)\frac{\mathrm{d}^2Y_j(x)}{\mathrm{d}x^2}\right\}Y_j(x)\mathrm{d}x$$

$$b_j=\int_0^L\rho(x)\left(1-\frac{x}{L}\right)Y_j(x)\mathrm{d}x$$

$$c_j=\int_0^L \frac{x}{L}\rho(x)Y_j(x)\mathrm{d}x$$

By taking the Laplace transform of Eq. (10) and substituting Eq. (6) into it, the normal coordinate is obtained as follows:

$$q_j(s)=\sum_{k=0}^{n+1}G_{kj}(s)\ddot{u}_k(s) \tag{11}$$

where

$$G_{kj}(s)=\frac{\sum_{i=1}^{n}Y_j(a_i)F_{ik}(s)-p_{kj}}{M_js^2+K_j}=\frac{q_j(s)}{\ddot{u}_k(s)} \tag{12}$$

$$p_{kj}=\begin{cases} b_j & \text{for } k=0 \\ 0 & \text{for } k=1,2,\cdots,n \\ c_j & \text{for } k=n+1 \end{cases}$$

Substituting Eq. (11) into the Laplace transform of Eq. (8) leads to

$$y(x,s)=\sum_{k=0}^{n+1}H_k(x,s)\ddot{u}_k(s) \tag{13}$$

where

$$H_k(x,s)=\sum_{j=1}^{\infty}Y_j(x)G_{kj}(s) \tag{14}$$

Using the well-known relationship between the mean-square value and power spectral density, the mean-square value of deformation at point x of the beam is

$$<y(x,t)^2>=\frac{1}{2\pi j}\int_{j-\infty}^{j\infty}\sum_{i=0}^{n+1}\sum_{k=0}^{n+1}H_i(x,s)H_k(x,-s)\phi_{\ddot{u}_{ik}}\mathrm{d}s \tag{15}$$

where $\phi_{\ddot{u}_{ik}}$ is the cross power spectral density between $\ddot{u}_i(t)$ and $\ddot{u}_k(t)$. Thus, the weighted mean of $<y(x,t)^2>$ over the beam is

$$\begin{aligned}\overline{(<y(x,t)^2>)_w}&=\frac{1}{L}\int_0^L<y(x,t)^2>\cdot w(x)\mathrm{d}x\\ &=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\phi_y\mathrm{d}s\end{aligned} \tag{16}$$

where

$$\phi_y=\sum_{i=0}^{n+1}\sum_{k=0}^{n+1}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{L}\int_0^L w(x)Y_l(x)Y_m(x)\mathrm{d}x\cdot G_{il}(s)G_{km}(-s)\phi_{\ddot{u}_{ik}} \tag{17}$$

From the relation Eq. (6), the mean-square value of the i-th reducing force is written as follows;

$$<f_i(t)^2>=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\phi_{f_i}\mathrm{d}s \tag{18}$$

where

$$\phi_{f_i}=\sum_{l=0}^{n+1}\sum_{k=0}^{n+1}F_{ik}(s)F_{il}(-s)\phi_{u_{kl}} \tag{19}$$

Substituting Eqs. (16) and (18) into Eq. (5), the following relation in frequency domain is obtained

$$J=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left(\phi_y+\sum_{i=1}^{n}\lambda_i^2\phi_{f_i}\right)\mathrm{d}s \tag{20}$$

Applying the frequency domain optimization technique by Chang[5], the condition for the penalty function J to be minimized is

$$\frac{\partial\phi_y}{\partial F_{ij}(-s)}+\sum_{k=1}^{n}\lambda^2{}_k\frac{\partial\phi_{f_k}}{\partial F_{ij}(-s)}=R_{ij}(s) \tag{21}$$

$$(i=1,2,\cdots,\ n;\ j=0,1,\cdots,\ n+1)$$

where $R_{ij}(s)$ is a function which does not possess any pole in the LHP (left-half-plane). The equation (21) yields a set of linear simultaneous equations for $F_{ij}(s)$. By solving the Eq. (21) the optimum transfer functions $F_{ij}{}^0(s)$ are determined. The procedure of determining $F_{ij}{}^0(s)$ will be illustrated on the next chapter.

In addition to the above problem, some other problems to minimize the acceleration $\overline{(\langle\ddot{y}(x,t)^2\rangle)_w}$ or the moment $\overline{(\langle m(x,t)^2\rangle)_w}$ of the beam with specified constraints on the forces $\langle f_i(t)^2\rangle$ or the relative displacements $\langle y_{r_i}(a_i,t)^2\rangle$ of the vibration reducers are considered. For such problems, the formulations are done in the same way as that of the above problem by replacing $\overline{(\langle\ddot{y}(x,t)^2\rangle)_w}$ or $\overline{(\langle m(x,t)^2\rangle)_w}$ with $\overline{(\langle y(x,t)^2\rangle)_w}$ and replacing $\langle y_{r_i}(a_i,t)^2\rangle$ with $\langle f_i(t)^2\rangle$.

Next, we get the expression of the quantities mentioned above. Since $\ddot{y}(x,t)$ and $m(x,t)$ are the acceleration and the moment at point x of the beam, respectively, they are represented by following relations

$$\ddot{y}(x,t)=\sum_{i=0}^{\infty}Y_i(x)q_i(t) \tag{22}$$

$$m(x,t)=-EI(x)\frac{\partial^2 y(x,t)}{\partial x^2} \tag{23}$$

Taking the Laplace transform of Eqs. (22) and (23) and substituting Eq. (11) into them, the acceleration and the moment of the beam are written in frequency domain as follows:

$$\ddot{y}(x,s)=\sum_{k=0}^{n+1}P_k(x,s)\ddot{u}_k(s) \tag{24}$$

$$m(x,s)=\sum_{k=0}^{n+1}Q_k(x,s)\ddot{u}_k(s) \tag{25}$$

where

$$P_k(x,s)=\sum_{i=0}^{\infty}Y_i(x)s^2G_{ki}(s) \tag{26}$$

$$Q_k(x,s)=-\sum_{i=0}^{\infty}EI(x)\frac{\mathrm{d}^2Y_i(x)}{\mathrm{d}x^2}G_{ki}(s) \tag{27}$$

The equations (24) and (25) are of the same type as the equation (13). Therefore, $\overline{(\langle\ddot{y}(x,t)^2\rangle)_w}$ and $\overline{(\langle m(x,t)^2\rangle)_w}$ can be calculated in the same manner as $\overline{(\langle y(x,t)^2\rangle)_w}$.

The relative displacement of the i-th vibration reducer is expressed as

$$y_{r_i}(a_i,t)=u(a_i,t)-u_i(t)$$
$$=y(a_i,t)+(1-a_i/L)u_0(t)+(a_i/L)u_{n+1}(t)-u_i(t) \tag{28}$$

Taking the Laplace transform of Eq. (28) leads to

$$y_{ri}(a_i,s)=\sum_{k=0}^{n+1}H_k'(a_i,s)\ddot{u}_k(s) \tag{29}$$

where

$$H_k'(a_i,s)=H_k(a_i,s)+A_k(a_i,s) \tag{30}$$

$$A_k(a_i,s)=\begin{cases}(1-a_i/L)/s^2 & \text{for } k=0\\ 0 & \text{for } k=1,2,\cdots,\ n\ (k\neq i)\\ -1/s^2 & \text{for } k=i\\ a_i/Ls^2 & \text{for } k=n+1\end{cases}$$

Then, the mean-square value of the relative displacement of the i-th vibration reducer is expressed as

$$\langle y_{r_i}(a_i,t)^2\rangle=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\sum_{k=0}^{n+1}\sum_{l=0}^{n+1}H_k'(a_i,s)H_l'(a_i,-s)\phi_{\ddot{u}_{kl}}ds \tag{31}$$

3. Minimum-deformation problem

Consider a system with a vibration reducer as shown in Fig. 2. In this chapter, it is assumed that the random foundation motion is uniform, i.e.,

$$u_0(t)=u_1(t)=u_2(t) \tag{32}$$

and that the weighting function $w(x)$ is proportional to $\rho(x)$:

$$w(x)=(L/M_1)\rho(x) \tag{33}$$

Using Eqs. (6) and (32), the following relation is obtained

$$f(s)=F(s)\ddot{u}_0(s) \tag{34}$$

where $f(s)=f_1(s)$ and $F(s)=F_{10}(s)+F_{11}(s)+F_{12}(s)$.
Substituting the above relation into Eq. (11) leads to

$$\frac{q_j(s)}{\ddot{u}_0(s)}=\frac{Y_j(a_1)F(s)-b_j-c_j}{M_js^2+K_j}=G_j(s) \tag{35}$$
$$(j=1,2,\cdots,\ \infty)$$

Substituting Eq. (33) into Eq. (17) and using the well-known orthogonality relation between the normal modes, the following relation is obtained

$$\phi_y=\sum_{j=1}^{\infty}G_j(s)G_j(-s)\phi_{\ddot{u}_0} \tag{36}$$

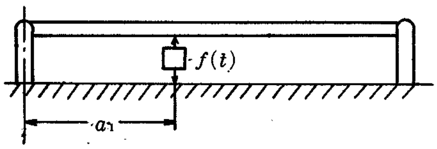

Fig. 2. Schematic diagram of a beam supported at both ends and equipped with a vibration reducer.

where $\phi_{\ddot{u}_0}$ represents the power spectral density of the acceleration of the foundation motion $\ddot{u}_0(t)$. From Eqs. (19) and (34), we have

$$\phi_f = F(s)F(-s)\phi_{\ddot{u}_0} \tag{37}$$

The further assumption that the vibration of the beam consists of the only first mode is introduced. Then,

$$\phi_y = \frac{Y_1(a_1)F(s)-d_1}{M_1(s^2+\omega_1^2)} \cdot \frac{Y_1(a_1)F(-s)-d_1}{M_1(s^2+\omega_1^2)}\phi_{\ddot{u}_0} \tag{38}$$

where $d_1 = b_1 + c_1$ and $\omega_1^2 = K_1/M_1$.

Substituting Eqs. (37) and (38) into Eq. (21) leads to

$$\frac{\lambda^2\phi_{\ddot{u}_0}}{(s^2+\omega_1^2)^2}\left[\left\{(s^2+\omega_1^2)^2+\beta^2\right\}F(s)-\frac{d_1}{Y_1(a_1)}\beta^2\right] = R(s) \tag{39}$$

where $\beta^2 = Y_1(a_1)^2/\lambda^2 M_1$. Giving the functional form of $\phi_{\ddot{u}_0}$, $F(s)$ can determined by the following procedure.

3.1. White noise excitation

First, consider the case of white noise excitation;

$$\phi_{\ddot{u}_0} = S_0 = \text{const.} \tag{40}$$

Then, $F(s)$ must have the form

$$F(s) = \frac{A_1 s + A_0}{(s+\xi+i\eta)(s+\xi-i\eta)} \tag{41}$$

where A_0 and A_1 are undetermined coefficients and $s = -\xi \pm i\eta$ are the LHP roots in Eq. (42)

$$(s^2+\omega_1^2)^2 + \beta^2 = 0 \tag{42}$$

thus,

$$\left.\begin{aligned} \xi^2 &= (\sqrt{\omega_1^4+\beta^2}-\omega_1^2)/2 \\ \eta^2 &= \xi^2+\omega_1^2 \end{aligned}\right\} \tag{43}$$

Substituting Eqs. (40) and (41) into Eq. (39) leads to

$$\frac{\lambda^2 S_0}{(s^2+\omega_1^2)^2}\left[(s^2-2\xi s+\xi^2+\eta^2)(A_1 s+A_0)-\frac{d_1}{Y_1(a_1)}\beta^2\right] = R(s) \tag{44}$$

Since $R(s)$ does not possess any pole in the LHP, the LHP poles in Eq. (44) must be cancelled out by the zeros. Hence,

$$\left.\begin{aligned} A_0 &= 2\xi^2 d_1/Y_1(a_1) \\ A_1 &= 2\xi d_1/Y_1(a_1) \end{aligned}\right\} \tag{45}$$

Substituting Eq. (45) into Eq. (41), the optimum transfer function $F_0(s)$ is obtained. Therefore, the optimum vibration reducing force $f_0(s)$ is

$$f_0(s) = \frac{2d_1\xi(s+\xi)}{Y_1(a_1)(s^2+2\xi s+\xi^2+\eta^2)}\ddot{u}_0(s) \tag{46}$$

and substituting Eq. (41) into Eq. (35) leads to

$$q_1(s) = -\frac{d_1/M_1}{s^2+2\xi s+\xi^2+\eta^2}\ddot{u}_0(s) \tag{47}$$

Cancelling out $\ddot{u}_0(s)$ from Eqs. (46) and (47) yields

$$\begin{aligned} f_0(s) &= -\{2M_1\xi/Y_1(a_1)\}(s+\xi)q_1(s) \\ &= -(c_0 s+k_0)y_r(a_1,s) \end{aligned} \tag{48}$$

where

$$\left.\begin{aligned} c_0 &= 2M_1\xi/Y_1(a_1)^2 \\ k_0 &= 2M_1\xi^2/Y_1(a_1)^2 \end{aligned}\right\} \tag{49}$$

and $y_r(a_1,t)$ is the relative displacement of the vibration reducer, i.e., $y_r(a_1,t)=Y_1(a_1)q_1(t)$. From Eq. (48), the optimum vibration reducer can be mechanized by a spring with spring constant k_0 and a dashpot with damping coefficient c_0 as shown in Fig. 3. For general discussion, the following variables are introduced;

$$\left.\begin{aligned} \omega_0 &= \sqrt{k_0/M_1} \\ \zeta_0 &= c_0/2\sqrt{M_1k_0} \end{aligned}\right\} \tag{50}$$

Substituting Eq. (41) into Eqs. (16) and (18) leads to

$$\frac{\overline{(\langle y(x,t)^2\rangle_w}}{S_0/\omega_1^3}=\frac{(d_1/M_1)^2\omega_1^3}{4\xi(\xi^2+\eta^2)} \tag{51}$$

and

$$\frac{\langle f(t)^2\rangle}{S_0d_1^2\omega_1}=\frac{\xi(2\xi^2+\eta^2)}{\omega_1Y_1(a_1)(\xi^2+\eta^2)} \tag{52}$$

For example, consider the case of the simply-supported uniform beam as shown in Fig. 3. Then,

$$\left.\begin{aligned} Y_1(x) &= \sin(\pi x/L) \\ \rho(x) &= \rho = \text{const.} \\ EI(x) &= EI = \text{const.} \end{aligned}\right\} \tag{53}$$

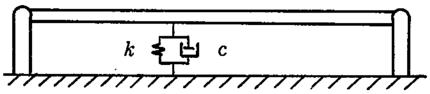

Fig. 3. A beam supported at both ends and equipped with a spring-dashpot vibration reducer.

The values of ω_0 and ζ_0 for some positions of the optimum vibration reducer are calculated by the use of Eq. (50) and shown in Fig. 4. It is found from Fig. 4 that ζ_0 is constant for the specified value of a_1 and takes the minimum value $1/\sqrt{2}$ at $a_1=L/2$ and ω_0 is proportional to the value of $\langle f(t)^2\rangle$. Fig. 5 shows the value of ζ_0 versus the position of the vibration reducer. The relation between $\overline{(\langle y(x,t)^2\rangle)_w}$ and $\langle f(t)^2\rangle$ is shown by solid lines for specified positions of the vibration reducer in Fig. 6. It is found that if one chooses the value of $\langle f(t)^2\rangle$, the deformation of the beam will be obtained and the minimum deformation is obtained at $a_1=L/2$. Although higher modes of the beam are not considered in the above description, influence of the higher mode vibrations against the vibration containing the whole modes of the beam must be considered in practice. The sum of the responses of the first, second and third modes is shown by dotted lines in Fig. 6. From Fig. 6, the responses of the higher modes are smaller than that of the first mode and the whole response of the beam is nearly equal to that of the

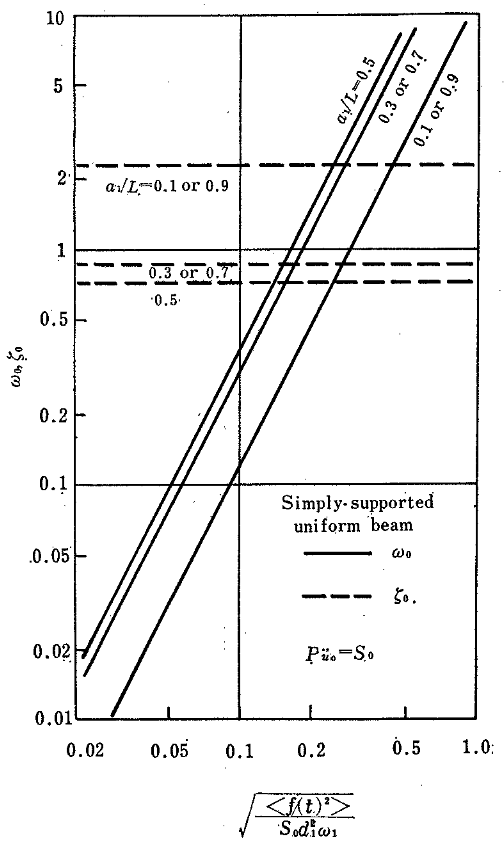

Fig. 4. Plots of ω_0 and ζ_0 versus vibration reducing force.

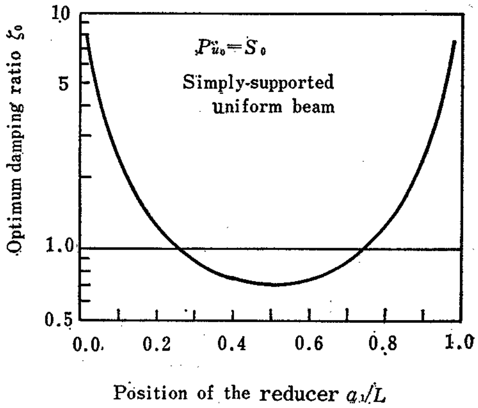

Fig. 5. Relation of ζ_0 to a_1/L.

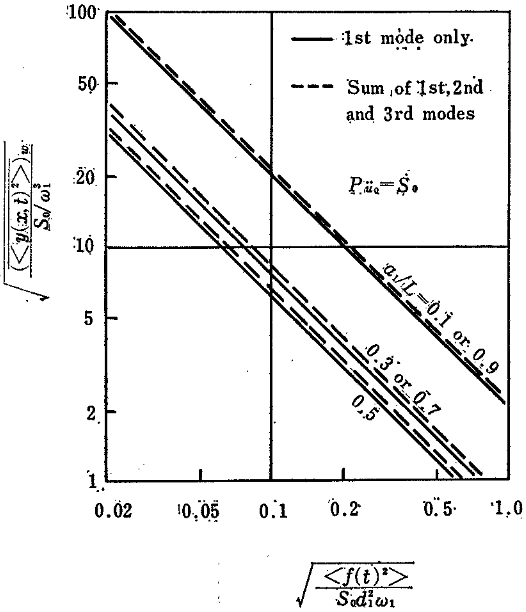

Fig. 6. Performance of a simply-supported and uniform beam with the optimum vibration reducer for white noise excitation.

first mode. Therefore, it is found that the optimum vibration reducer for the first mode vibration can be really effective for vibrations with the whole modes of the beam.

3.2. A typical random excitation

Consider the case of a typical random excitation whose power spectral density is given as

$$\phi_{\ddot{u}_0}=\frac{S_0\omega_0^2}{-s^2+\omega_0^2} \tag{54}$$

where S_0 is a constant. Then, $F(s)$ must have the form

$$F(s)=\frac{B_2s^2+B_1s+B_0}{(s+\xi+i\eta)(s+\xi-i\eta)} \tag{55}$$

where B_0, B_1 and B_2 are undetermined coefficients, and ξ and η are constants given in Eq. (43). Substituting Eq. (55) into Eq. (21) leads to

$$\frac{\lambda^2 S_0\omega_0^2}{(-s^2+\omega_0^2)(s^2+\omega_1^2)^2}\left\{(s^2-2\xi s+\xi^2+\eta^2)(B_2s^2+B_1s+B_0)-\frac{d_1}{Y_1(a_1)}\beta^2\right\}=R(s) \tag{56}$$

The LHP poles in Eq. (56) must be cancelled out by the zeros. Hence,

$$\left.\begin{aligned} B_2 &= \frac{2\xi(\omega_0+\xi)d_1}{(\omega_0^2+2\xi\omega_0+\xi^2+\eta^2)Y_1(a_1)} \\ B_1 &= 2\xi d_1/Y_1(a_1) \\ B_0 &= \omega_1^2B_2+\xi B_1 \end{aligned}\right\} \tag{57}$$

Substituting Eq. (57) into Eq. (55), the optimum transfer function $F_0(s)$ is obtained. Therefore, the optimum-vibration-reducing-force is

$$f_0(s)=\frac{B_2s^2+B_1s+B_0}{s^2+2\xi s+\xi^2+\eta^2}\ddot{u}_0(s) \tag{58}$$

and the normal coordinate is expressed as

$$q_1(s)=-\frac{Cd_1/M_1}{s^2+2\xi s+\xi^2+\eta^2}\ddot{u}_0(s) \tag{59}$$

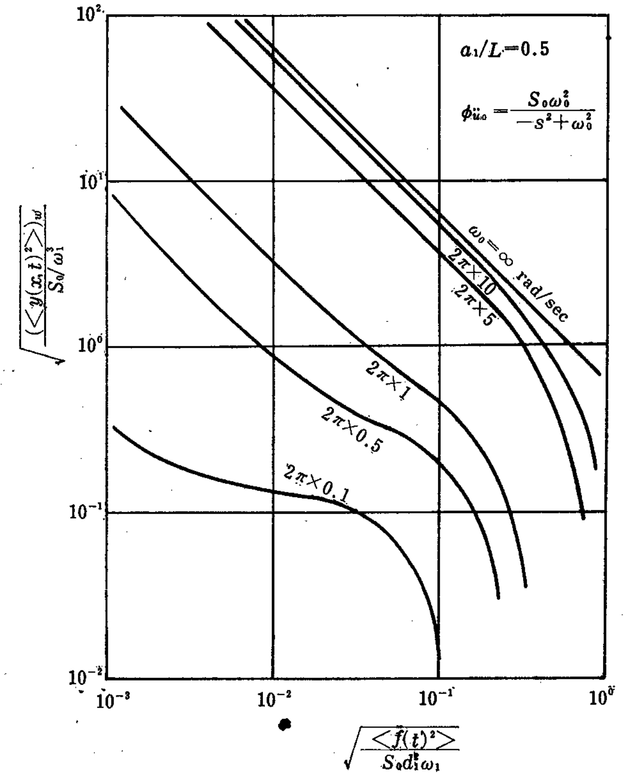

Fig. 7. Performance of a simply-spported and uniform beam with the optimum vibration reducer at $a_1/L=0.5$ for a typical noise excitation.

where

$$C=\frac{\omega_0^2+\omega_1^2}{\omega_0^2+2\xi\omega_0+\xi^2+\eta^2}$$

Cancelling out $\ddot{u}_0(s)$ from Eqs. (58) and (59) yields

$$f_0(s)=-\{Cd_1/M_1Y_1(a_1)\}(B_2s^2+B_1s+B_0)y_r(a_1,s) \quad (60)$$

Since $f_0(s)$ has the term of s^2, the optimum vibration reducer can only be mechanized by active elements. The relation between $\overline{(\langle y(x,t)^2\rangle)_w}$ and $\langle f(t)^2\rangle$ for a simply-supported beam is shown in Fig. 7. As seen from Fig. 7, when ω_0 is greater than 20π rad/sec, the performance is approximately equal to that for $\omega_0=\infty$, i.e., white noise excitation. Therefore, the vibration reducers for such a case may be mechanized by a spring-dashpot element as shown in section 3.1.

4. Conclusion

A method for finding the optimum configuration of vibration reducers for elastic systems with random foundation excitations is discussed. The optimization technique is illustrated on a beam supported at both ends and having a vibration reducer. As a numerical example, a simply-supported beam is considered for white noise or a typical random foundation excitation. It is concluded that for white noise excitation, the optimum vibration reducer for the first mode vibration of a beam supported at both ends can be mechanized by spring-dashpot elements and can be really effective for vibrations with the whole modes of the beam, but the mechanization of the optimum vibration reducers for the other systems can only be realized by the use of active elements.

Although no mention has been made of complex systems and of systems subjected to random force excitations, the proposed technique will be easily extended to those systems. Such studies are to be reported in near future.

Acknowledgement

The authors would like to express their appreciation to the SAKKOKAI FOUNDATION for the financial aid.

References

1) J.T. Weissenburger, J. Appl. Mech., **35**, 2 (1968).
2) T. Shimogo, KIKAI NO KENKYU (in Japanese), **23**, 11 (1971).
3) N. Fujiwara, et al., Proc. 18th Japan Nati. Cong. Appl. Mech. 1968 (1970); Bull. Japan Soc. Mech. Engr., **14**, 78 (1971).
4) E.K. Bender, AACC Joint Auto. Cont. Conf. (1967), p. 28.
5) S.S.L. Chang, Synthesis of Optimum Control Systems, p. 63, McGraw-Hill (1961).