Analog Computer Simulation of a Separately Excited D-C Motor Driven by Square Wave Voltage

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It is difficult to obtain analytically the transient characteristics of a separately excited d-c motor driven by square wave voltage whose duty factor is controlled to limit the armature current within the desired value.

This paper deals with analog computer simulation of a separately excited d-c motor driven by square wave voltage, emphases are given on the starting characteristics of this driving system where peak value of the armature current is restricted and mean value of the armature current is maintained constant through out the transitional period. Desired results are obtained and compared with normal d-c drive.

1. Introduction

Heretofor Ward-Leonard system has been used as typical speed control of a separately excited d-c motor, (hereafter referred to as d-c motor simply), however with the development of thyristor, static Leonard system or chopper control method has now the place in the control system. Because these devices are superior to Ward-Leonard system in efficiency and compact.

Wave form of the supply voltage to d-c motor may be generally the square wave on chopper control or on-off control method. On this time, if some parameters (i. e., the period of square wave and the duty factor) are constant, the characteristics of d-c motor can be obtained analytically[1)2)3)4)]. But they are usually complicated form and in case these parameters are changed with various controls, it is difficult to obtain them.

For above mentioned reasons, analog computer simulation of d-c motor square wave voltage control system is required. Operational mode of this drive is divide into three modes according to the armature current flow. Analog computer simulation is carried out by the system equation on each mode. And the starting characteristic of a separately excited d-c motor square wave voltage system, where (1) peak value of the armature current is restricted, (2) average value of the armature current is held constant through out the transitional period, are obtained.

2. Analog Simulation of D-C Motor System

Driving circuit of d-c motor is shown in Fig. 1. The system equation which

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has switching elements is generally expressd as eq. (1).

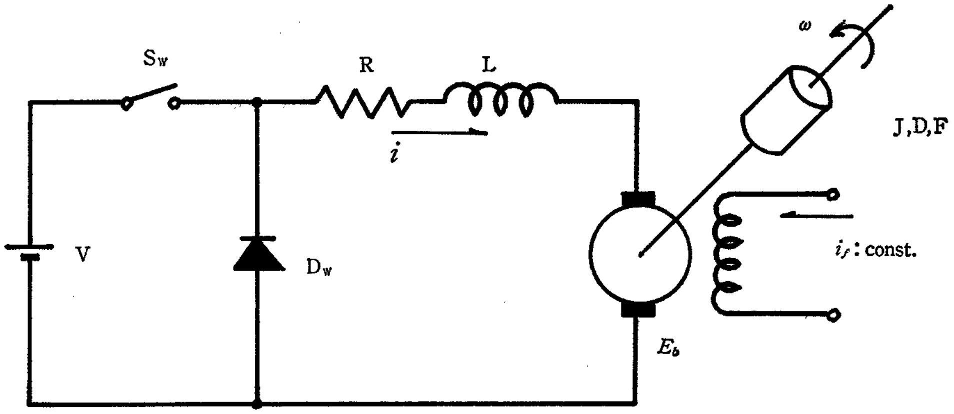


Fig. 1. Driving circuit of separately excited D-C motor.

$$\dot{x}=f_i\ (x,\ u) \tag{1}$$

where

x: independent state variables.

v: control forces.

i: number of modes which is of combination of on-off of switching elements.

The system in eq. (1) is nonlinear, however if system parameters (such as electric time constant and mechanical time constants) are constant, the system equation can be expressed by the linear differental equation.

2.1. Establishment of Mathematical Model of D-C Motor System

On the driving d-c motor shown in Fig. 1, three operational modes occur according to the armature current flows as following.

Mode I : The constant voltage is supplied to d-c motor (Sw-on) and the armature current flow from the source.

Mode II : The supply voltage to d-c motor is null (Sw-off) and the armature current flows through the fly-wheel diod Dw (Dw-on).

Mode III : Both the supply voltage and armature current are zero (Sw-off). On this mode d-c motor is coasting.

Normalized system equation on each mode is as follows.

$$\text{Mode I}: \frac{di_n(t)}{dt}=\frac{1}{\tau_e}\left(1-i_n(\mathrm{t})-\omega_n(\mathrm{t})-E_{bn}\right) \tag{2}$$

$$\frac{d\omega_n(t)}{\mathrm{dt}}=\frac{1}{\tau_{m1}}\left(1-\frac{\tau_{m1}}{\tau_{m2}}\right)\left(i_n(\mathrm{t})-\frac{\tau_{m1}}{\tau_{m2}-\tau_{m1}}\omega_n(\mathrm{t})-F_n\right) \tag{3}$$

$$\text{Mode II}: \frac{di_n(t)}{dt}=\frac{1}{\tau_e}(-i_n(\mathrm{t})-\omega_n(\mathrm{t})-E_{bn}) \tag{4}$$

$$\frac{d\omega_n(t)}{dt}=\frac{1}{\tau_{m1}}\left(1-\frac{\tau_{m1}}{\tau_{m2}}\right)\left(i_n(t)-\frac{\tau_{m1}}{\tau_{m2}-\tau_{m1}}\omega_n(t)-F_n\right) \tag{5}$$

$$\text{Mode III: } \frac{d\omega_n(t)}{dt} = \frac{1}{\tau_{m1}}\left(1 - \frac{\tau_{m1}}{\tau_{m2}}\right)\left(-\frac{\tau_{m1}}{\tau_{m2} - \tau_{m1}}\omega_n(t) - F_n\right) \tag{6}$$

where

$\omega_n(t) = K_V\omega(t)/V$, $i_n(t) = Ri(t)/V$, $E_{bn} = E_b/V$, $F_n = RF/K_TV$

$\tau_{m1} = JR/(DR + K_TK_V)$: starting time constat of the motor system.

$\tau_{m2} = J/D$: coasting time constant of the motor system.

$\tau_e = L/R$: electric time constant.

K_V: E. M. F. constant.

K_V : torque constant.

R : resistance of armature eircuit.

L : inductance of armature circuit.

D : coefficient of viscous friction.

J : moment of inertia.

V : crest value of square wave voltage.

E_b : brush voltage drop.

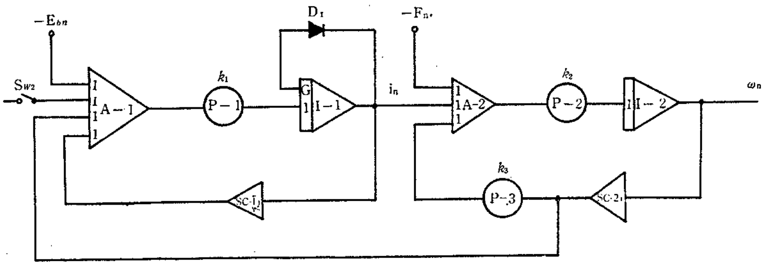
F : frictional torque.

$\omega(t)$: d-c motor speed.

$i(t)$: armature current.

2.2. Analog Simulation of D-C Motor System

Simulation circuit of d-c motor system in Fig. 1 is shown in Fig. 2. Diod D_1 is connected parallel to feedback condenser of integrator I-1 and carries out the purpose of D_W in Fig. 1. On the mode I, in-put of I-1 is negative, then D_1 is noncoducive. It operates as integrator. And eq. (2) is represented by adders A-1 and SC-I, and integrator I-1 ; eq. (3), by A-2, SC-2 and I-2. On the mode II, anode-



$$k_1 = \frac{1}{\alpha_t \tau_e},\quad k_2 = \frac{1}{\alpha_t \tau_{m1}}\left(1 - \frac{\tau_{m1}}{\tau_{m2}}\right),\quad k_3 = \frac{\tau_{m1}}{\tau_{m2} - \tau_{m1}}$$

α_t : time scale factor

Fig. 2. Simulation circuit of separately excited D-C motor.

cathod voltage of D_1 is negative with the aid of feedback condenser of I-1. And eq. (4) is represented by A-1, SC-1 and I-1; eq. (5), by A-2, SC-2 and I-2. Soon after the charged voltage of feedback condenser of I-1 becomes null, mode II turns to mode III, and the state is held by D_1 (the armature current is zero in this perid). Then eq. (6) is represented by A-2, SC-2 and I-2.

Thus three system equations are expressed by one simulation circuit shown in Fig. 2.

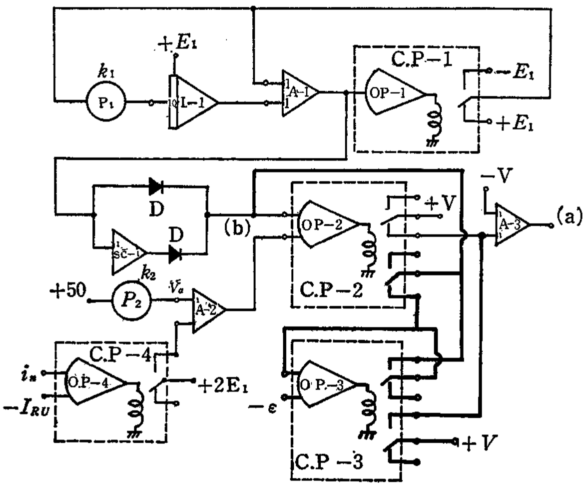
3. Analog Simulation of the Control System

On the driving the motor by square wave voltage, rush armature current that is rich in ripple flows into at starting. This causes excessive temparature rise more than normal drive, for the same out-put power. Then it is desired to have some current restriction at the frequent application of start and stop.

Two kinds of armature current restriction methods are described below.

3.1. Peak Armature Current Restriction Control

Simulation circuit is shown in Fig. 3. And the operational relation of comparators and out-put wave forms in each part are shown in Fig. 4.



i_n : Armature current in Fig. 2

I_{RU} : Reference peak current

Fig. 3. Square wave voltage generation circuit with peak current restriction.

Operational principle is as follows.

Comparator C.P-1 operates in the period in odd number. Sawtooth wave is generated at point (a). It is expressed as repeated function $2E_1(1-t/T)$, $(0 \leq t \leq T)$. This period is defined by $1/k_1$ (k_1 : coefficient set by potentiometer P-1). Operation of comparators C.P-2 and C.P-3 has following two cases.

1) When comparator C.P-4 does not operate for a period T. While the input $2E_1(1-t/T)-V\alpha$ of C.P-2 is possitive, C.P-2 operates (i. e., during αT in Fig. 4). Comparator C.P-3 operates at αT and this state is held for the period βT (for this period, its in-put is $2E_1\ (1-t/T)-\varepsilon>0$). Then it is reset at $(\alpha+\beta)T$ and held this state for γT. And so at the point (a) in Fig. 3, square wave form with period T and duty factor α is generated.

2) When C.P-4 operates at $\alpha_1 T$, C.P-2 is reset and C.P-3 operates. Because C.P-4 is reset at instant (i. e., that is due to electric time constant of armature circuit), C.P-2 operates again till αT. But C.P-3 is held this state till $(\alpha+\beta)T$

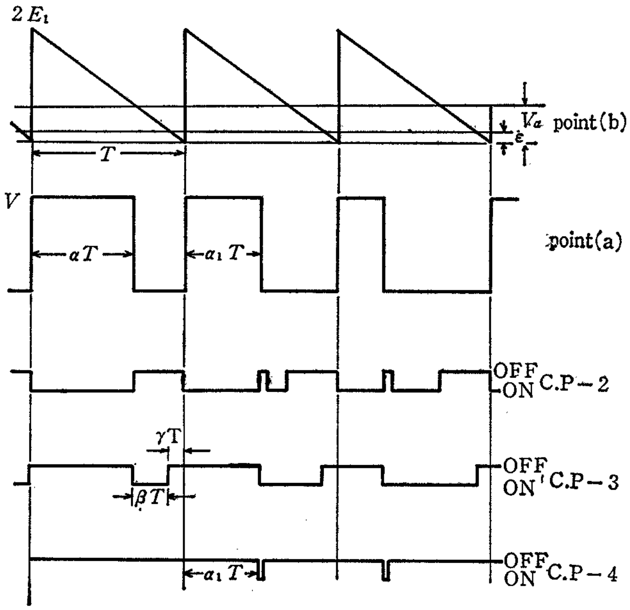


Fig. 4. Relation of the operation of comparators and out put wave from in each part.

by the circuit shown with thick line in Fig. 3. And C.P-3 is reset for the time γT, on this period its in-put is $2E_1(1-t/T)-\varepsilon>0$.

Then at the point (*a*) in Fig. 3, square wave with period T and duty factor α_1 is generated.

3.2. Constant Armature Current Control

Simulation circuit and its operational relation are shown in Fig. 5. I_{RU} is

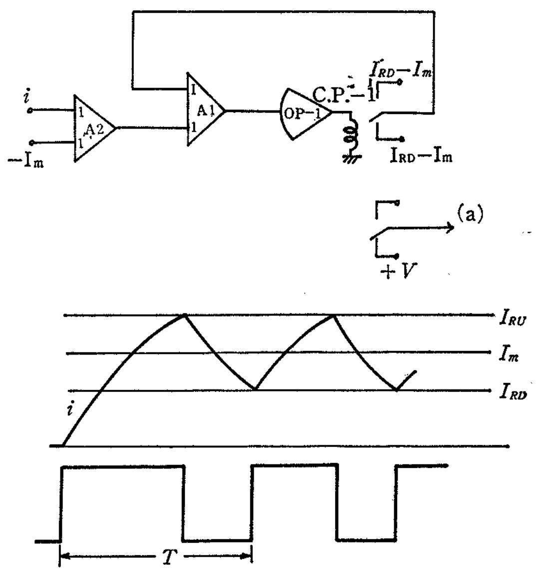


Fig. 5. Constant current circuit and wave form at point (a).

upper reference current, I_{RD} is lower reference current and mean current $I_m = (I_{RU} + I_{RD})/2$. At starting, C.P-1 does not operate for the armature current $i < I_{RU}$. Voltage V is impressed on the motor. When the armature current is $i > I_{RU}$, C.P-1 operates and the motor supply voltage is zero. Then the armature current becomes $i \geq I_{RD}$, and voltage V is impressed on the motor again. Repeating these operations, constant current control may be done.

4. Result

The result of the simulation of starting characteristics of d-c motor speed and current without restriction is shown in Fig. 6. At starting, the armature current over 200% of rated armature current flows (generally normalized rated armature current is 0.3). The ratio of the starting time constant shown in Fig. 6 to normal d-c motor time constant τ_{m1} is 1.1. The result of the simulation with peak armature current restriction control is shown in Fig. 7. At this time, peak armature

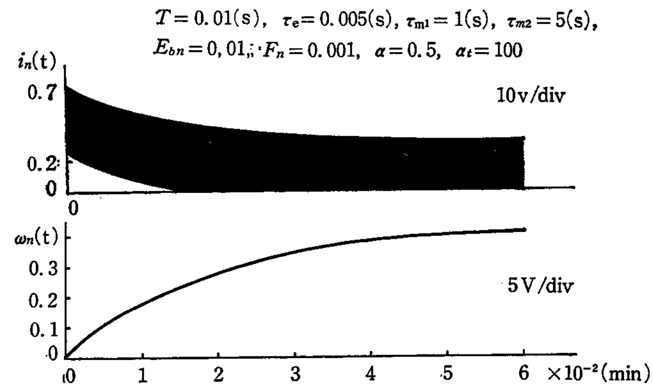


Fig. 6. Oscillograms of starting characteristies of speed and current without restriction.

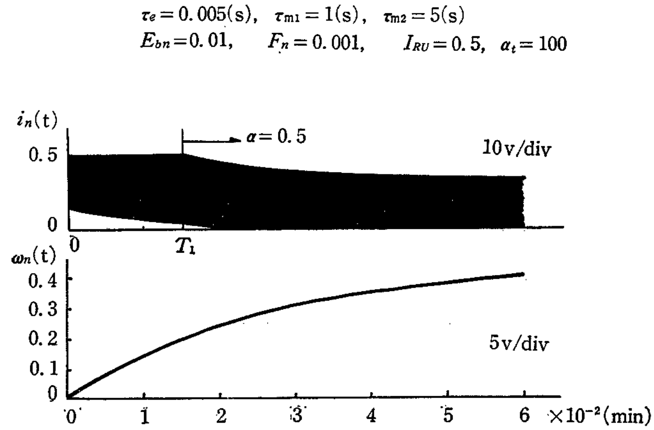


Fig. 7. Oscillograms of starting characteristics of speed and current with peak current control.

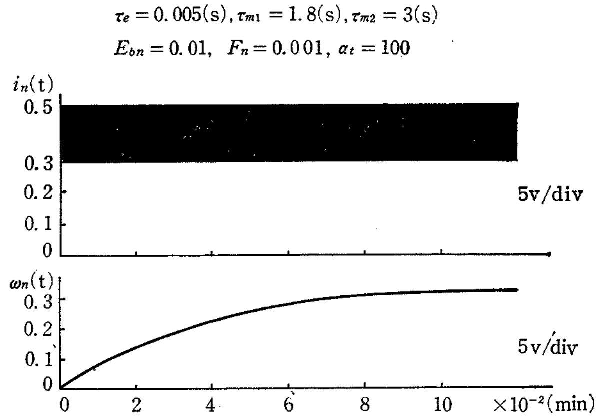


Fig. 8. **Oscillograms of starting characteristics of speed and current with constant current control.**

current is limited in 0.5 for $0 \sim T_1$ in Fig. 7. After the time T_1, simulation is carried out with $\alpha=0.5$, and period T is 0.01 (s) on any conditions. The ratio of the starting time constant in Fig. 7 to τ_{m1} is 1.31. In these simulation, motor and circuit constants are elected as follows ; $\tau_e=0.005$ (s), $\tau_{m1}=1.0$ (s) $\tau_{m2}=5.0$ (s), $E_{bn}=0.01$, $F_n=0.001$, in Fig. 6 $T=0.01$(s), $\alpha=0.5$ and in Fig. 7 $I_{RU}=0.5$. And the result of the simulation with constant armature current control which has normalized references (i.e., upper reference current 0.5 and lower reference current 0.3) is shown in Fig. 8. The ratio of starting time constant to τ_{m2} is nearly equal to 1.0. This is the reason why the armature current is constant on this control. In this simulation, motor constants are $\tau_{m1}=1.8$ (s), $\tau_{m2}=3.0$ (s).

5. Conclusion

Even if it is trouble to obtain analytically d-c motor characteristics, it can be given by expressing d-c motor system and its control systems on patch board. Also using this method, we get insight into behavior of d-c motor characteristics on influence of change in the system parameters (such as, starting time constant, coasting time canstant and electric time constant). So that it is serviceable for the optimal design of square wave voltage d-c motor drive system.

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