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|  | 作成者：Irie，Hisaichi，Fujii，Tomoo，Ishizaki，Takemitsu <br> メールアドレス： <br>  <br>  <br> 所属： |
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# Characteristics of Separately Excited D-C Motor Driven by Bilateral Chopper 

Hisaichi Irie* Tomoo FujiI* and Takemitsu Ishizaki*

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#### Abstract

A bilateral chopper may be considered as the combination of a voltage step-down chopper and a voltage step-up chopper, and it functions as a change over switch. Using it for the control of the d-c machines, regenerative braking is accomplished automatically by feed-back of power to the supply power-source. The characteristic equations of the motor driven by the bilateral chopper are derived and the test results from an experimental chopper-motor system are presented in this paper.


## 1. Introduction

Thyristor choppers being developed and used in a variety of d-c motor drive applications. For d-c motor control, several papers ${ }^{122)}$ dealt with bilateral chopper which could provide not only efficient motor running but also provide regenerative braking without reconnection by return of energy to the d-c power-source. However, very few work has been done to estimate the characterirtics of the d-c motor controlled by this bilateral chopper.

Previously, we reported on the thyristor chopper for separately excited d-c motor control ${ }^{3}$. In the paper, was proposed a method that is useful in designing a motor control system driven by a chopper. In the present paper, we will discuss the cotrol characteristics of the d-c motor driven by the bilateral chopper.

The analysis of the characteristics is made under the following assumptions: (1) the pulsation of the motor speed in a chopper cycle and the voltage drop in semiconductors are negligible, (2) the supply voltage to the motor is of perfect square waveform, (3) the magnitude of the supply voltage is constant.

## 2. Fundamental Circuit

### 2.1 Bilateral Chopper Circuit for D-C Motor Control

When a separately excited $d-c$ motor is driven by a fixed excitation, the counter EMF $E_{c}$ induced in the armature is proportional to the speed $N$ of the motor. $E_{c}$ is given by

$$
\begin{equation*}
E_{c}=k_{e} N \tag{1}
\end{equation*}
$$

where $k_{e}$ is the counter EMF coefficient. Then, the equivalent circuit of the separately excited d-c motor in its running condition is a series circuit consisting of a battery $E_{c}$, a resistance $R$ and a inductance $L$ of the armature circuit.

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Fig. 1. Bilateral chopper circuit for separately excited d-c motor control.
Fig. 1 shows the circuit of the separately excited d-c motor control used in the present experiment. The bilateral chopper, which is interposed between a powersource $E_{s}$ and an armature of the motor, may be considered as the combination of a voltage step-down chopper and a voltage step-up chopper. The former consists of a main thyristor $T h_{1}$ with its commutating components $T h_{1 A}, L_{c_{1}}, C_{c}$, and a diode $D_{2}$ together with the inductance $L$, while the latter consists of a main thyristor $T h_{2}$ with its commutating components $T h_{2 A}, L_{c 2}, C_{c}$, and a diode $D_{1}$ with $L$.

If the pair of main thyristors may be gated on alternately and the turn-on of one thyristor can be ensured by the turn-off of the other, then the thyristors $T h_{1}$ and $T h_{2}$ may be replaced by the switch $s_{1}$ and $s_{2}$ which are to function as a change over switch. Thus, neglecting the commutating components of the main thyristors, Fig. 1 can be simplified to Fig. 2, where the motor is shown in its equivalent circuit of $E_{c}-R-L$.


Fig. 2. Equivalent circuit of bilateral chopper and separately excited d-c motor.

### 2.2. Voltage and Current Waveforms

When the switch contact alternates between $s_{1}$ and $s_{2}$ in the constant period $T_{r}$, denoting the contacting time of $s_{1}$ by $t_{w}$, then the motor voltage $v$ and its average
value $V$ can be expressed by

$$
\left.\begin{array}{rll}
v=E_{s} & & \left(0<t<t_{w}\right) \\
=0 & & \left(t_{w}<t<T_{r}\right) \tag{3}
\end{array}\right\}
$$

where $d_{F}=t_{w} / T_{r}$ is the duty factor of the chopper.


Fig. 3. Voltage and current waveforms in Fig. 2. ( $S_{F}=2$ )
Fig. 3 shows voltage and current waveforms for one cycle in steady state running condition. The operation of the chopper-motor circuit can be devided into three modes as follows.

Mode I; The step-down chopper operates but the step-up chopper does not, then the forward positive current flows in the motor. Voltage and current waveforms in this mode is shown in Fig. 3 (a). As the result, the power is supplied to
the motor from the power-source $E_{s}$.
Mode If; The step-up chopper operates but the step-down chopper does not, then the backward negative current flows in the motor as shown in Fig. 3 (b). As the result, the power is returned to the power-source $E_{s}$ from the counter EMF $E_{c}$ induced in the motor.

Mode III ; The step-down and step-up choppers operate alternately, then the motor current flows in the positive and negative direction during one eycle as shown in Fig. 3 (c).

## 3. Calculation of Characteristics

### 3.1. Current

From Fig. 2, the equation for the motor circuit is expressed by

$$
\begin{array}{ll}
L \frac{d i_{1}}{d t}+R i_{1}=E_{s}-E_{c} & \left(0<t<t_{w}\right) \\
L \frac{d i_{2}}{d t}+R i_{2}=-E_{c} & \left(t_{w}<t<T_{r}\right) \tag{4}
\end{array}
$$

Since $\left.i_{1}\right|_{t=0}=\left.i_{2}\right|_{t=T r}$ and $\left.i_{1}\right|_{t=t_{w}}=\left.i_{2}\right|_{t=t_{w}}$, we get

$$
\begin{align*}
& i_{1}=\frac{E_{s}-E_{c}}{R}-\frac{E_{s}}{R} \frac{1-e^{-S_{F}\left(1-d_{F}\right)}}{1-e^{-S_{F}}} e^{-\frac{1}{T_{e}} t} \\
& i_{2}=-\frac{E_{c}}{R}-\frac{E_{s}}{R} \frac{1-e-S_{F d_{F}}}{1-e^{-S_{F}}} e^{-\frac{1}{T_{e}}\left(t-t_{w}\right)} \tag{5}
\end{align*}
$$

where $T_{e}=L / R$ is the time constant of the motor circuit and $S_{F}=T_{r} / T_{e}$ is the smoothing factor.

The average value $I$ and the effective value $I_{e}$ of the motor current in one cycle are calculated as follows,

$$
\begin{align*}
I & =\frac{1}{T_{r}}\left\{\int_{0}^{t_{w}} i_{1} d t+\int_{t_{w}}^{T_{r}} i_{2} d t\right\}=\frac{V-E_{c}}{R}  \tag{6}\\
I_{e} & =\sqrt{\frac{1}{T_{r}}\left\{\int_{0}^{t_{w} i_{1} 2} d t+\int_{t_{w}}^{T_{r} i_{2}{ }^{2} d t}\right\}} \\
& =\sqrt{I^{2}+\left(\frac{E_{s}}{R}\right)^{2}\left\{d_{F}\left(1-d_{F}\right)-\frac{\left(1-e^{-S_{F} d_{F}}\right)\left(1-e^{-S_{F}\left(1-d_{F}\right)}\right)}{S_{F}\left(1-e^{-S_{F}}\right)}\right\}} \tag{7}
\end{align*}
$$

From Eqs. (6) and (7), a-c component $I_{a c}$ of the motor current can be obtained as

$$
\begin{align*}
I_{a c} & =\sqrt{I_{e}^{2}-I^{2}} \\
& =\frac{E_{S}}{R} \sqrt{d_{F}\left(1-d_{F}\right)-\frac{\left(1-e^{-S_{F} d_{F}}\right)\left(1-e^{-S_{F}\left(1-d_{F}\right)}\right)}{S_{F}\left(1-e^{-S_{F}}\right)}} \tag{8}
\end{align*}
$$

Eq. (6) shows that, when the motor is driven by a bilateral chopper, the same
relation holds among $V, R, I$ and $E_{c}$ as when the motor is driven by the constant voltage d-c power-source such as a battery or a d-c generator.


Fig. 4. Component circuit of the chopper-motor circuit.
From Eq. (8), it is readily seen that $I_{a c}$ is independent of counter EMF $E_{c}$ and it is a function of $E_{s}, R, S_{F}$ and $d_{F}$ (i. e., $E_{s}, R, L, T_{r}$ and $t_{w}$ ). Fig. 4 shows $\mathrm{d}-\mathrm{c}$ and a-c component circuits of the chopper-motor circuit. In the d-c component circuit shown Fig. 4 (a), if the motor terminal voltage $V$ is greater than counter EMF $E_{c}$, then positive current flows in the motor and the motor is driven by the chopper. while, if $V$ is less than $E_{c}$, then the negative current flows and the motor is braked by the chopper.

Fig. 4 (b) shows the a-c component circuit consists of $v_{a c}, L$ and $R_{a c} . R_{a c}$ in this figure can be considered as an a-c resistance in the armature circuit and its value is not a constant but varies with the chopper period. The a-c voltage can be written by

$$
\left.\begin{array}{rlrl}
v_{a c} & =\left(1-d_{F}\right) E_{s} & & \left(0<t<t_{w}\right)  \tag{9}\\
& =-d_{F} E_{s} & & \left(t_{w}<t<T_{r}\right)
\end{array}\right\}
$$

The torque $\tau$ which is developed in the motor with constant excitation is proportional to the average value of the motor current $I$, then we have

$$
\begin{equation*}
\tau=k_{t} I \tag{10}
\end{equation*}
$$

where $k_{t}$ is the torque cofficient.

### 3.2. Power and Loss

The instantaneous power $p$ supplied to the motor from the power-source through the bilateral chopper can be written as follows,

$$
\begin{aligned}
p=p_{1}=E_{s} i_{1} & & \left(0<\mathrm{t}<\mathrm{t}_{w}\right) \\
=p_{2}=0 \cdot i_{2}=0 & & \left(t_{w}<t<T_{r}\right)
\end{aligned}
$$

The average power $P_{i}$ is calculated as

$$
\begin{align*}
P_{i} & =\frac{1}{T_{r}}\left\{\int_{0}^{\left.t_{w} p_{1} d t+\int_{t_{w}}^{T_{r}} p_{2} d t\right\}}\right. \\
& =E_{c} I+I^{2} R+I_{a c}{ }^{2} R \tag{11}
\end{align*}
$$

The output $P$, which is converted to mechanical power, and the power loss $W$ are written by

$$
\begin{align*}
& P=\frac{1}{T_{r}}\left\{\int_{0}^{t_{w}} E_{c} i_{1} d t+\int_{t_{w}}^{T_{r}} E_{c} i_{2} d t\right\}=E_{c} I  \tag{12}\\
& W=\frac{1}{T_{r}}\left\{\int_{0}^{t_{w}} i_{1}{ }^{2} R d t+\int_{t_{w}}^{T_{r}} i_{2}{ }^{2} R d t\right\}=I_{e}{ }^{2} R=I^{2} R+W_{a c} \tag{13}
\end{align*}
$$

where $W_{a c}=I_{a c}{ }^{2} R$.
From Eq. (13), it is easily seen that power loss of the motor driven by the chopper is larger than that of driven by the constant voltage $d-c$ power-supply because of the a-c ohmic loss of a-c component current.

### 3.3. Efficiency

In case of motoring operation that the motor is driven by the chopper, the efficiency is written as follows,

$$
\begin{equation*}
\eta_{M}=\frac{P}{P+W}=\frac{E_{c} I}{E_{c} I+I^{2} R+W_{a c}}=\frac{d_{F} E_{s} I-I^{2} R}{d_{F} E_{s} I+W_{a c}} \tag{14}
\end{equation*}
$$

while, in case of generatoring operation that the motor is braked by the chopper, the efficiency is rewritten as

$$
\begin{align*}
\eta_{G}=\frac{P+W}{P}=\frac{E_{c} I+I^{2} R+W_{a c}}{E_{c} I} & =\frac{d_{F} E_{s} I+W_{a c}}{d_{F} E_{s} I-I^{2} R} \\
(I & \left.<0 \text { and }\left|E_{c} I\right|>I^{2} R+W_{a c}\right) \tag{15}
\end{align*}
$$

### 3.4. Chracteristic Curves

To generalize the above equations, the following dimensionless factors are introduced;

$$
\begin{aligned}
& n_{F}=\frac{E_{c}}{E_{s}}=\frac{k_{e}}{E_{s}} N, \quad v_{F}=\frac{V}{E_{s}}=d_{F}, \quad i_{F}=\frac{R}{E_{s}} I, \quad \tau_{F}=\frac{R}{E_{s} k_{t}} \tau, i_{a c F}=\frac{R}{E_{s}} I_{a c}, \\
& p_{i F}=\frac{R}{E_{s}{ }^{2}} P_{i}, \quad p_{F}=\frac{R}{E_{s}{ }^{2}} P, w_{F}=\frac{R}{E_{s}^{2}} W
\end{aligned}
$$

where $n_{F}$ is denoted as speed factor, $v_{F}$ as voltage factor, $i_{F}$ as current factor, $\tau_{F}$ as torque factor, $i_{a c F}$ as a-c current factor. $p_{i F}$ as input factor, $p_{F}$ as output factor and $w_{F}$ as loss factor.

Using these dimensionless factor, Eqs. (6), (8), (11), (12), (14) and (15) are rewritten as follows;

$$
\begin{aligned}
& n_{F}=v_{F}-i_{F}=d_{F}-i_{F} \\
& i_{a c F}=\sqrt{d_{F}\left(1-d_{F}\right)-\frac{\left(1-e^{-s_{F} d_{F}}\right)\left(1-e^{-s_{F}\left(1-d_{F}\right)}\right)}{S_{F}\left(1-e^{-s_{F}}\right)}} \\
& p_{i F}=n_{F} i_{F}+i_{F}^{2}+i_{a c F}{ }^{2}=d_{F} i_{F}+i_{a c F}{ }^{2} \\
& p_{F}=n_{F} i_{F}=d_{F} i_{F}-i_{F}{ }^{2} \\
& \eta_{M}=\frac{p_{F}}{p_{i F}}=\frac{d_{F} i_{F}-i_{F}{ }^{2}}{d_{F} i_{F}+i_{a c F}} \quad\left(i_{F}>0\right)
\end{aligned}
$$

$$
\eta_{G}=\frac{p_{i F}}{p_{F}}=\frac{d_{F} i_{F}+i_{a c F^{2}}}{d_{F} i_{F}-i_{F}^{2}} \quad\left(i_{F}<-\frac{i_{a c F^{2}}}{d_{F}}\right)
$$

The calculated characteristics of the sparately excited d-c motor driven by the bilateral chopper may be obtained from the above eqnations.


Fig. 5. The a-c component current of the motor vs. duty factor of the chopper.

Fig. 5 shows the relation between $i_{a c F}$ and $d_{F}$ for various values of $S_{F}=T_{r} / T_{e}$ From this figure, it is easily seen that $i_{a c F}$ decrease with $S_{F}, i_{a c F}=0$ at $S_{F}=0$, and


Fig. 6. (a) Speed vs. average current of the motor.


Fig. 6. (b) Output vs. average current of the motor.
$i_{a c F}$ for a constant value of $S_{F}$ has a maximum value at $d_{F}=0.5$.
The relations between $n_{F}, p_{F}$ and $i_{F}$ are shown in Fig. 6. Fig. 6 (a) shows the characteristic curves of $n_{F}$ vs. $i_{F}$ for various of $d_{F}$, Fig. 6 (b) shows the curves of $p_{F}$ vs. $i_{F}$ for various values of $n_{F}$. From Fig. 6, it can be seen that the chracteristics $n_{F}$ and $p_{F}$ vs. $i_{F}$ of the motor driven by the chopper can be represented with the same straight lines as the characteristics of the motor driven by the constant voltage d-c power-supply.


Fig. 7. Motor input vs. average current for $S_{F}=10$.
Fig. 7 shows the relation between $p_{i F}$ and $i_{F}$ for various values of $d_{F}$ at the constant value of $S_{F}=10$. This relation is also represensed by the straight lines. The values of $p_{i F}$ at $i_{F}=0$ represent ohmic loss due to the a-c component of


Fig. 8. Efficiency of armature vs. average current.
motor current.
Fig. 8 shows the relation between $\eta$ and $i_{F}$ for various values of $S_{F}$. The value of $\eta_{G}$ for generatoring operation ( $i_{F}<0$ ) is different from $\eta_{M}$ for motoring operation ( $i_{F}>0$ ), and no power is feed-backed to the power-sourse for $-\frac{i_{a c F^{2}}}{d_{F}}<\mathrm{i}_{F}<0$ in spite of $p_{F}<0$, because of a-c ohmic loss $i_{a c F}{ }^{2}$.

## 4. Experimental Result

The experiment has been conducted by using a separately excited d-c motor which has a nominal rating of $100(\mathrm{~W}), 100(\mathrm{~V}), 2(\mathrm{~A})$ and $2000(\mathrm{rpm})$. The measured values of the armature resisistance and inductance were $5.45(\Omega)$ and $26(\mathrm{mH})$ respectively. The counter EMF and torque coefficient were about 0.042 ( $\mathrm{V} / \mathrm{rpm}$ ) and $0.26(\mathrm{~N}-\mathrm{m} / \mathrm{A})$.

A d-c machine is directly coupled to the motor. The machine is used as a generator for the motor load at the motoring operation, and is used to drive the motor at the generatoring operation.

Fig. 9 shows the relation between speed and current of d-c motor driven by the bilateral chopper for $E_{s}=100(\mathrm{~V}), T_{r}=5(\mathrm{~ms})$. For comparison, this relation of the motor driven by the d-c power-supply with the equivalent voltage $V=d_{F} E_{s}$ is illustrated by the dotted line. The measured relation between speed and current differs slightly from the calculated one. This deviation is becouse of the voltage drop in the semiconductors, the armature reaction and the increase of winding resistance due to temperature rise.

Fig. 10 shows the relation between motor input and current for $E_{s}=100(\mathrm{~V})$,


Fig. 9. Measured speed vs. current of the motor driven by the bilateral chopper or d-c power-supply with calculated ones.


Fig. 10. Measured motor input vs. current with calculated ones.
$d_{F}=0.5$. In calculation, it is assumed that $S_{F}=T_{r} / T_{e}=2.8,1.4,0.28$ for $T_{r}=10,5$, 1 (ms). The values of $P_{i}$ at $I=0$, in this figure, represent ohmic loss due to the a-c component of motor current. It is easily seen that a-c ohmic loss $W_{a c}$ increases


Fig. 11. Measured efficiency of armature vs. current with calcated ones.
with the chopper period. And the measured value of the armature resistance $R_{a c}$ for $60(\mathrm{~Hz})$ a-c sinusoidal waveform was $7.35(\Omega)$.

Fig. 11 shows the relation between efficiency and current of the motor driven by the bilateral chopper. It is seen from Figs. 10 and 11 the measured values agree fairly well with calculated ones, although several assumptions have been given in the analysis.

## 5. Conclusion

In the present analysis, the sepately excited d-c motor driven by the bilateral chopper was simply considered as the battery load $E_{c}-R-L$ of the chopper circuit shown in Fig. 2. Under this consideration, the chracteristic equations were derived. It was found that the chopper-motor circuit can be decomposed into two circuits i.e., a d-c and an a-c component circuits independent of each other. The former consists of a constant d-c voltage $V$, a counter EMF $E_{c}$ and a resistance $R$. While the latter consists of an a-c voltage $v_{a c}$, a inductace $L$ and an a-c resistance $R_{a c}$ which is not equal to $R$.

From the both component circuits, the expression for characteristics can be derived easily. It was conformed that the measured performance characteristics agree fairly well with the calculated ones although several assumptions were introduced in the analysis.

As results of the analysis and experiment, it was also found that the precise control of d-c motor obtained by the means of Ward-Leonard system can be easily realized by the bilateral chopper.

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[^0]:    * Department of Electrical Engineering, College of Engineering.

