Fundamental Studies on Dynamic Characteristics of Contact Surface in Machine Tool Structure

Fundamental Studies on Dynamic Characteristics of Contact Surface in Machine Tool Structure

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This paper discusses the transfer characteristics of the contact surface in a torsional vibration system using the forced vibration method. The dynamic characteristic of the coupling rig subject to the sinusoidal excitation is divided into sinusoidal and rectangular wave response. Our experimentation comes to a conclusion that the critical condition for the rectangular wave response can be obtained from the relationship between maximum static frictional torque and axial load.

1. Introduction

A machine tool which is used at high speed requires the high dynamic stiffness of the structure. No detailed analysis, however, has been tried on the dynamic characteristic of the contact surface in the machine tool structure. Few papers[1] treat the damping mechanism based on the difference between the dynamic behaviors of contact surface.

The critical condition is the focus of our study at which the response of the system through contact surface changes from a sinusoidal wave to a rectangular wave. Therefore, the critical condition for the slide by a coupling model is analyzed, and the critical condition for the rectangular wave response is compared with the results observed experimentally.

The practical mating surfaces consist usually of planes or cylinders for simplicity of the fabrication. Two mating parts of whichever element are contacted together under the clamping pressure provided either by an interference fit or an initial loading. Only two types of the relative motions on interface or mating surface are generally important factors in damping analysis: (a) a separation of mating surfaces (motion normal to interface); and (b) interface shear effects (relative motions of mating surfces in the plane of the interface). Of the two type of motion, the one which appears to offer the greater potential for dissipating energy is relative interface shear[2)3)]. When a variable external load is applied, the shear stress on the interface is augmented, and slip eventually occurs. This implies that the points on the opposite sides of the interface experience small relative displacements. The above behavior of the interface suggests two possible situations: slide and slip[4)]. The terminology "slide" is here defined as the phenomenon that gross and uniform shear motion occurs on a single interface and homogeneous shear

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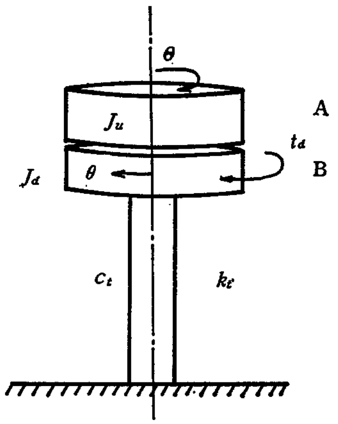
displacement occurs over the entire interface between rigid bodies. On the other hand, "slip" means that inhomogeneous local slide or shear occurs.

2. Dynamics of system with contact surface

Consider the dynamics of the system shown in Fig. 1. A stands for the upper part of the coupling rig and B the lower part. The dynamic behavior of the system is classifiable to the motion that A and B rotate as a solid elastic body or that a relative displacement between A and B exists. Once a relative displacement occurs between A and B, the torque transferred to B becomes the frictional torque generated by the relative angular velocity between A and B.



Fig, 1. Model of vibration.

The rigid body B with moment of inertia J_d is fixed to a spring with torsional spring constant k_t and a dashpot with viscous damping constant c_t. The rigid body A is in close contact with B, and excited at angular frequency ω. The solid friction torque t_d is assumed to act on the interface and the center axis of the model to be straight during twist.

The equation of motion for the system is

$$J_d\frac{d^2\theta}{dt^2}+c_t\frac{d\theta}{dt}+k_t\theta=t_d\,, \tag{1}$$

where θ is the angular displacement of B. The angular displacement of A is given by

$$\Theta=\bar{\Theta}_0e^{j\omega t}=\Theta_0e^{j(\omega t+\phi)}\,, \tag{2}$$

where Θ_0 stands for amplitude, ω angular frequency, ϕ phase angle between θ and Θ, and $\bar{\Theta}_0$, complex amplitude. When θ is not sinusoidal wave, ϕ is defined by the phase angle between Θ and the component of the exciting frequency in θ. The absolute value of solid friction torque t_{d0} is a constant independent of the magnitude of the relative velocity between two solid bodies and its direction is opposite to the relative velocity. The above yields the relation

$$t_d = -t_{d0} S\left[\frac{d\theta}{dt} - \frac{d\Theta}{dt}\right], \tag{3}$$

where S is sign function.

For the sake of convenience in analysis, equation (1) shall be transformed into the dimensionless form by the use of the following dimensionless variables.

Now let

$$n = \sqrt{\frac{k_t}{J_d}} \qquad h = \frac{c_t}{2\sqrt{J_d k_t}} \qquad x = \frac{k_t \theta}{t_{d0}} \qquad \mu = \frac{\omega}{n}$$
$$\tau = nt \qquad y = \frac{k_t \Theta}{t_{d0}} \qquad \varepsilon = \frac{\pi n}{\omega} \qquad \nu = \frac{\theta_0}{\Theta_0}$$
$$C = \frac{\Theta_0 k_t \pi}{t_{d0}\varepsilon} \qquad D = \frac{\theta_0 k_t \pi}{t_{d0}\varepsilon} \qquad \alpha = \frac{k_t \Theta_0}{t_{d0}} \qquad \kappa = \frac{J_u}{J_d}. \tag{4}$$

Then equation (1) becomes

$$\ddot{x} + 2h\dot{x} + x = -S, \tag{5}$$

and from equation (2)

$$\ddot{y} = jCe^{j\left(\frac{\pi}{\varepsilon}\tau + \phi\right)} \tag{6}$$

is obtained. Similarly S in equation (3) is reduced as

$$S\left[\frac{d\theta}{dt} - \frac{d\Theta}{dt}\right] = S\left[n\frac{t_{d0}}{k_t}\frac{dx}{d\tau} - n\frac{t_{d0}}{k_t}\frac{dy}{d\tau}\right] = S\left[\dot{x} - \dot{y}\right], \tag{7}$$

where the dot denotes differentiation with τ.

If $\dot{x}$ equals $\dot{y}$ and the slide follows immediately, $\ddot{x}$ is unequal to $\ddot{y}$. The condition for sustaining slide is

$$\ddot{x} > \ddot{y}$$

or

$$\ddot{x} < \ddot{y}. \tag{8}$$

Using

$$\ddot{x} = -2h\dot{x} - x - S$$

rewritten from equation (5), and the first condition in equation (8),

$$-2h\dot{x} - x - 1 > \ddot{y}$$

is obtained when S equals $+1$. From these equations, the necessary condition for the slide is given as

$$|\ \ddot{y} + 2h\dot{x} + x\ | > 1. \tag{9}$$

Since the conditions $\ddot{y} + 2h\dot{x} + x > 1$ and $\ddot{y} + 2h\dot{x} + x < -1$ are simultaneously satisfied, in what follows the former will mainly discussed. Equation (9) becomes

$$-\frac{\pi}{\varepsilon}Ce^{j\left(\frac{\pi}{\varepsilon}\tau + \phi\right)} + 2hjDe^{j\frac{\pi}{\varepsilon}\tau}\frac{\varepsilon}{\pi}De^{j\frac{\pi}{\varepsilon}\tau} > 1. \tag{10}$$

with the nomenclatures shown in equation (4), and also

$$C\sqrt{\left(\frac{\varepsilon}{\pi}-\frac{\pi}{\varepsilon}\cos\phi\right)^2+\left(2h\nu-\frac{\pi}{\varepsilon}\sin\phi\right)^2}\,e^{j\left(\Phi+\frac{\pi}{\varepsilon}\tau\right)}>1, \tag{11}$$

where

$$\Phi=\tan^{-1}\left\{(2h\nu-\frac{\pi}{\varepsilon}\sin\phi)\Big/(\frac{\varepsilon}{\pi}\nu-\frac{\pi}{\varepsilon}\cos\phi)\right\}.$$

The condition to satisfy equation (11) is

$$C\sqrt{\left(\frac{\varepsilon}{\pi}\nu-\frac{\pi}{\varepsilon}\cos\phi\right)^2+\left(2h\nu-\frac{\pi}{\varepsilon}\sin\phi\right)^2}>1. \tag{12}$$

Equation (12) can be rewritten as

$$a>\frac{1}{\sqrt{\nu^2-2\nu\mu^2\cos\phi+\mu^4+4h^2\nu^2\mu^2-4h\nu\mu^3\sin\phi}}. \tag{13}$$

The limit of the right hand side is

$$\lim_{\substack{\nu\to1\\ \phi\to1}}\frac{1}{\sqrt{\nu^2-2\nu\mu^2\cos\phi+\mu^4+4h^2\nu^2\mu^2-4h\nu\mu^3\sin\phi}}$$

$$=\frac{1}{\sqrt{(1-\mu^2)^2+(2h\mu)^2}}. \tag{14}$$

When $\ddot{x}$ equals $\ddot{y}$ from

$$\ddot{x}=-\left(\frac{\pi}{\varepsilon}\right)^2\frac{k_t\theta_0}{t_{d0}}e^{j\frac{\pi}{\varepsilon}\tau}$$

$$\ddot{y}=-\left(\frac{\pi}{\varepsilon}\right)^2\frac{k_t\Theta_0}{t_{d0}}e^{j\left(\frac{\pi}{\varepsilon}\tau+\phi\right)},$$

the relation

$$\theta_0=\Theta_0 \qquad \phi=0 \tag{15}$$

can be obtained. Since $\ddot{x}=\ddot{y}$ from equation (15), t_d may be given by

$$t_d=t_{d0}e^{j(\omega t+\phi_0)} \tag{16}$$

where ϕ_0 is phase angle between θ and t_d. Substitution of equation (16) into equation (1) gives

$$\ddot{x}+2h\dot{x}+x=e^{j\left(\frac{\pi}{\varepsilon}\tau+\phi_0\right)}, \tag{17}$$

or

$$a=\frac{1}{\sqrt{(1-\mu^2)^2+(2h\mu)^2}}. \tag{18}$$

This equation shows the critical condition at which the dynamic behavior of the model changes from a single body motion to two bodies motion after the relative velocity has occurred. Moreover equation (18) yields the frequency response curve which describes the amplitude ratio of the exciting torque to the response (angular displacement), if the amplitude of the frictional torque at the contact surface t_{do} equals the maximum static frictional torque.

It being difficult to observe t_{do} experimentally, however, we try to obtain the relationship between the amplitude of external torque t_{uo} subject to A and the one of response at critical condition for the rectangular wave response.

The equation of the single body motion is

$$(J_d+J_u)\frac{d^2\theta}{dt^2}+c_t\frac{d\theta}{dt}+k_t\theta=t_u, \tag{19}$$

where t_u is equivalent to $t_{u0}e^{j(\omega t+\delta)}$ and δ is phase angle between θ and t_u. From equation (4) and equation (19) we have the form

$$(1+\kappa)\ddot{x}+2h\dot{x}+x=\frac{t_{u0}}{t_{d0}}e^{j(\mu\tau+\delta)}, \tag{20}$$

whence

$$\frac{k_t\theta_0}{t_{u0}}=a\frac{t_{d0}}{t_{u0}}=\frac{1}{\sqrt{\{1-(1+\kappa)\mu^2\}^2+(2h\mu)^2}}. \tag{21}$$

Substituting equation (18) into equation (21) and rearranging the form, we have

$$\frac{t_{d0}}{t_{u0}}=\sqrt{\frac{(1-\mu^2)^2+(2h\mu)^2}{\{1-(1+\kappa)\mu^2\}^2+(2h\mu)^2}}. \tag{22}$$

The curve shown in Fig. 5 and Fig. 6 represents equation (21). Equation (22) describes the relationship between the torque transferred to B and the external torque subject to A.

3. Experimental apparatus and method

3.1. Apparatus

The experimental apparatus consists of a torsional vibration system with a conical contact surface (i. e. a coupling model) and a equipment which detects the torsional vibration. The side excited directly in the coupling model is input side and the side excited through interface is output side. The behavior of the coupling model is shown in Fig. 2, and these waves are input and output wave (time history). (a) shows the appearance of input wave and (b) output wave.

In the rectangular wave region, the coupling model behaves like two rigid bodies, whereas in the sinusoidal region, it behaves like two elastic bodies or a single body. Therefore, the system can be treated as a simple single degree of freedom in a single body motion, and the Coulomb slide may be considered in two bodies motion and in rigid bodies slide. For a single body motion, regarding the frictional torque of the critical condition for the slide as T_c, it is written as

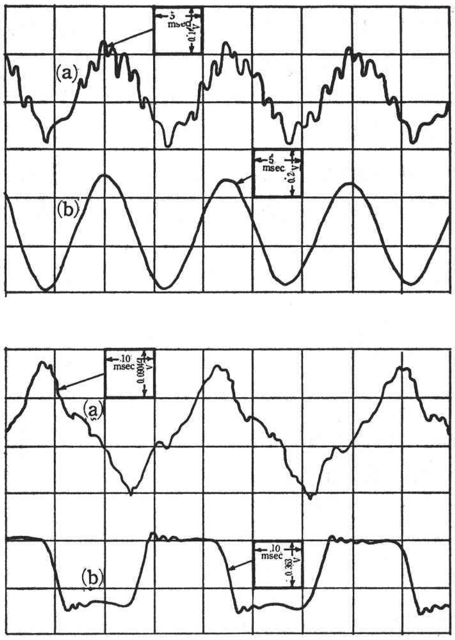


Fig. 2. Response waves in sinusodal and rectangular wave region.

$$T_c = \frac{f_s r_a}{\sin\beta + f_s \cos\beta} P = F r_a, \tag{23}$$

where f_s is the maximum static friction coefficient, P the axial load, β the half angle of the cone and F the frictional force in circular direction, r_a the mean radius of contact surface.

The coupling model which contacts each other in the conical surface is shown in Photo. 1. The specification of this vibration system is as given in Table 1. The material of the coupling model is mild steel, its contact surface is finished by

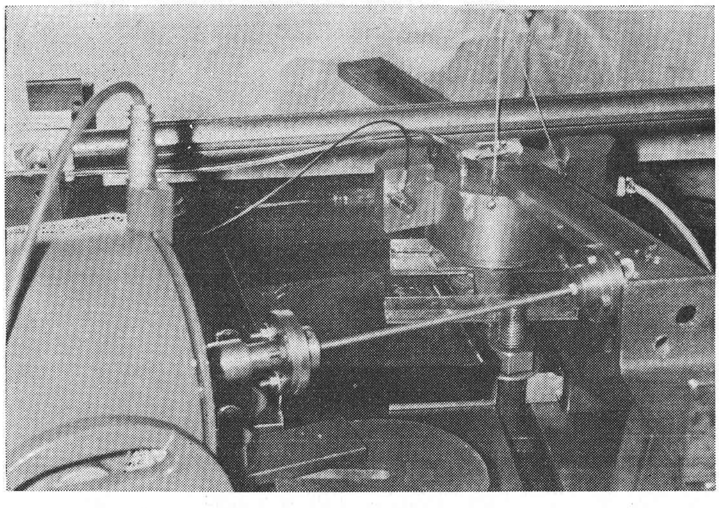


Photo. 1.

Table 1. Specifications for vibration system.

	Calculated	Experimental
Half angle of cone, degree	10	
Length of thin cylinder(from fixed end to neck), mm	80	
Spring constant of thin cylinder, mm kg/rad	4.89×10^6	4.94×10^6
Moment of inertia of concave part of coupling, mm kg sec^2/rad	1.08	
Moment of-inertia of convex part of coupling, mm kg sec^2/rad	0.416	
Moment of inertia of exciting lever, mm kg sec^2/rad	13.10	
Torsional natural frequency of convex part, c/s	544	500
Torsional natural frequency of total vibration system, c/s	92.3	88.3

lapping (#600) and roughness is almost 6μ H_{max}. An exciting lever is fixed to the upper coupling by six bolts (M 10).

The purchased electrodynamic exciter is used for the exciting force generator and the exciting torque is generated by means of the exciting lever. The Piezo accelerometer fixed to the upper coupling is utilized to pick up vibration and to detect Θ in input. The torsional vibration θ in output is observed by the rosette type wire strain gauge, and the exciting frequency is counted by a time counter.

3.2. Method

The test condition of the lubrication on the interface remains constant and the lubricating oil is used at the room temperature. The lubricating oil is poured onto

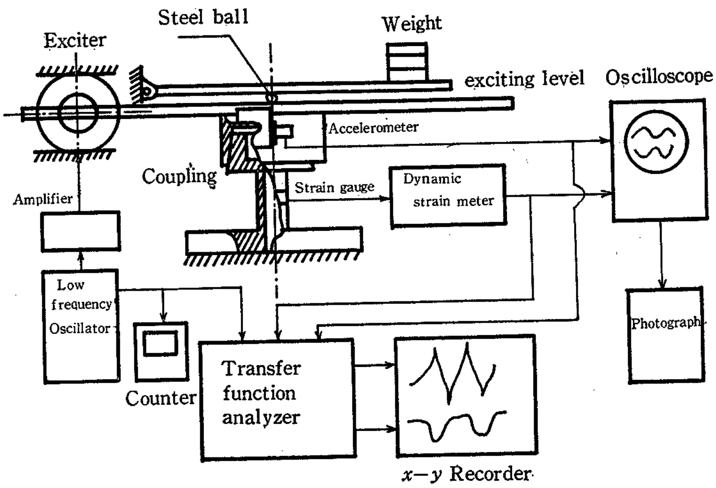


Fig. 3. Picture of measurement system.

the interface at constant pressure and the response of the vibration is observed by means of the torsional vibration forced to the coupling model by the exciter. Fig. 3 sketches the measurement system. The axial load is generated by dead weight and the bar which is supported by steel ball (1/2″) to prevent distortion. The weight of the bar and the upper coupling is counterbalanced by the deadweight loaded on two soft springs. The normal force may be calculated from equation (23).

In order to determine the critical condition for the rectangular wave response, we vary the exciting amplitude or acceleration under the constant frequency. The observation of the critical condition gives the limit of the exciting amplitude or acceleration in the sinusoidal wave region.

4. Experimental retults and disscusion

4.1. Relationship between maximum static friction torque and axial load

The results in Fig. 4 which are obtained under the static torque disclose the following linear relationship between maximum static frictional torque T_c and axial load P.

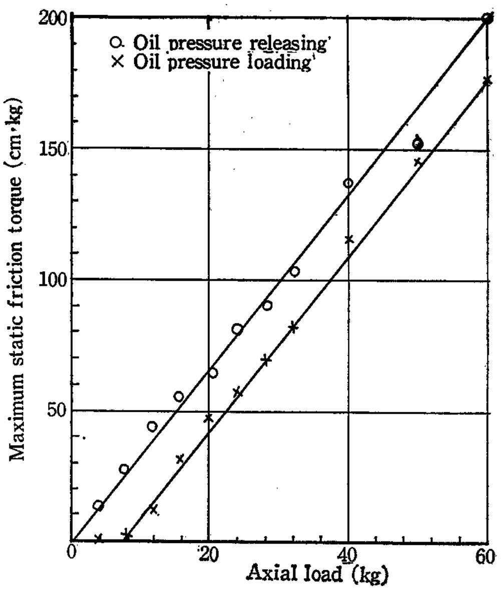


Fig. 4. Relationship between maximum static frictional torque and axial load.

$$T_c = f_{eq} P,$$

where f_{eq} is a constant and the equivalent coefficient of friction such as

$$f_{eq} = (f_s r_a) / (\sin\beta + f_s \cos\beta)$$

derived from equation (23). Suppose the pressure distribution on the conical surface is constant, then the value of f_s may be obtained about 0.3 from this equation. The neglect of generating line friction on the conical surface brings a value of 0.1.

4.2. Critical condition for rectangular wave response

For small axial load and large exciting amplitude, the system generates the rectangular wave response. The observation of the critical condition is plotted in Fig. 5 and Fig. 6 after normalized by equation (21).

According to the experimental results, the transmission of vibration from the upper to the lower coupling depends much upon the friction on the interface. As the axial load becomes smaller and the exciting amplitude becomes larger, the restoring force is larger than the frictional force and the rectangular wave region appears. For small axial load and low frequency, the plotted points scatter about the curve which represents equation (21), because it is very difficult to observe

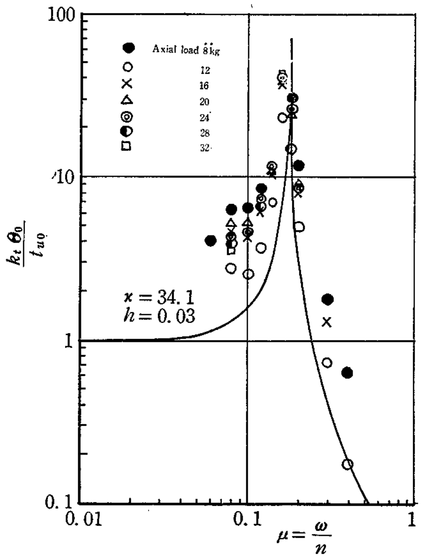


Fig. 5. Critical condition for rectangular wave response under oil pressure releasing.

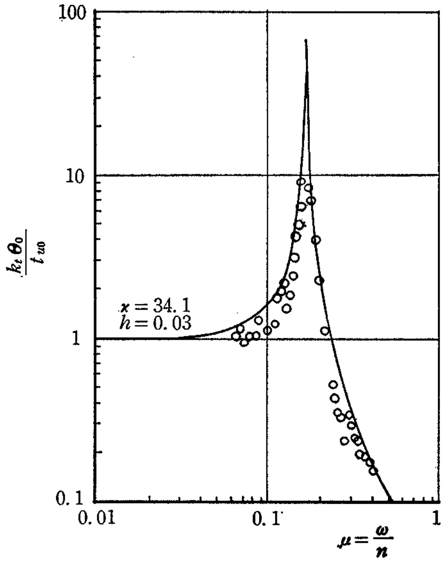


Fig. 6. Critical condition for rectangular wave response under oil pressure loading.

the difference between the sinusoidal wave and the rectangular wave. In the case of the rectangular wave response, the driving point mechanical impedance at the exciting lever is shown Fig. 7. The transformation of the value of this impedance at each frequency gives the relationship between $k_t\Theta_o/t_{uo}$ and μ as in Fig. 8. It is found from Fig. 9 that the relative slide on the interface does not occur in the sinusoidal wave region. The critical condition for the rectangular wave response differentiates the slide phenomenon from the nonslide.

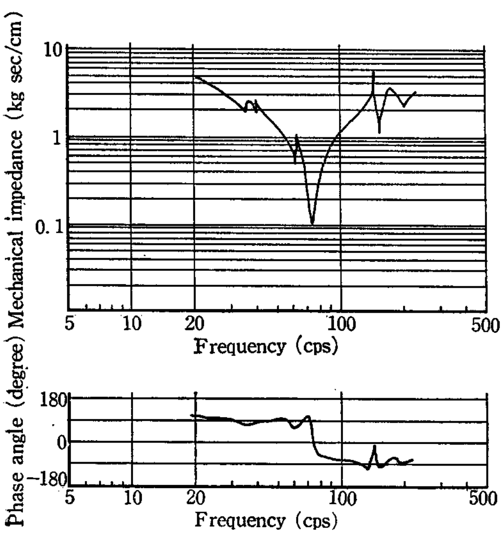


Fig. 7. Driving point impedance at exciting point on exciting lever.

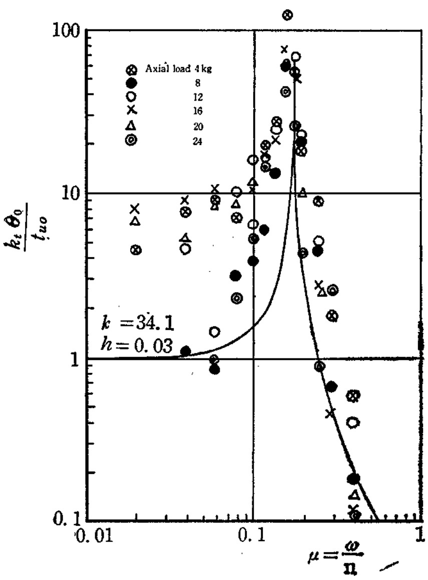


Fig. 8. Comparison of critical condition for slide and behavior in sinusoidal wave region (axial load 20 kg, oil psessure releasing).

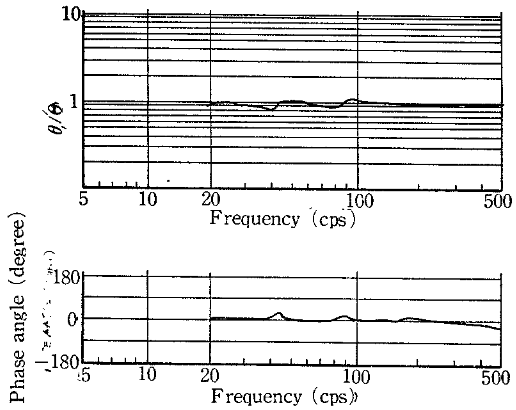


Fig. 9. Transfer frequency response diagram.

5. Conclusion

Whether the output wave is rectangular or sinusoidal depends on the exciting frequency, the amplitude of the upper coupling, the torsional spring constant, the maximum static friction torque, the moment of inertia of the upper and lower coupling, and the torsional damping constant.

With our model the behavior of the coupling at μ ranging from 0.04 to 0.2 reprensents with ease the rectangular wave response, as the axial load becomes smaller and the exciting amplitude larger.

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