A Hybrid Method to Improve Forecasting Accuracy

A Hybrid Method to Improve Forecasting Accuracy

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Abstract

Formerly, following method was proposed by us.

Focusing that the equation of exponential smoothing method was equivalent to (1,1) order ARMA model equation, the new method of estimation of smoothing constant in exponential smoothing method was proposed.

In this paper utilizing the above stated method, the revised forecasting method is proposed. In making forecast such as sales data, seasonal trend is removed by dividing the original time series by the Monthly Ratio which is calculated using moving average data. Utilizing the above stated method, forecasting is executed with this data in which seasonal trend is removed. The Monthly Ratio is multiplied after that, and forecasting data is obtained. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. These method are examined by the data of airline passengers. These results are compared with those of ARIMA model. Some favorable results are obtained.

Keywords: exponential smoothing method, ARMA model, ARIMA model, auto correlation function

1. Introduction

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM)[1]-[4]. Among these, ESM is recognized to be a practical simple method.

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For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend [5], utilizing Kalman Filter [6], Bayes Forecasting [7], adaptive ESM [8], exponentially weighted Moving Averages with irregular updating periods [9], making averages of forecasts using plural method [10] are presented. For example, Maeda [6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he couldn't grasp observation noise. It can be said that it doesn't pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii [11] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn't show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before [13]. Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing the above stated method, revised forecasting method is proposed. In making forecast such as sales data, seasonal trend is removed by dividing the original time series by the Monthly Ratio which is calculated using moving average data. Utilizing the above stated method, forecasting is executed with this data in which seasonal trend is removed. The Monthly Ratio is multiplied after that, and forecasting data is obtained. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. These method are examined by the data of airline passengers. These results are compared with those of ARIMA model.

The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. Seasonal Trend removing method is referred in section 3 and combined method of 2. and 3. are explained in section 4. ARIMA model is referred in section 5. Forecasting is executed in section 6, 7 and estimation accuracy is examined.

2. Description of ESM using ARMA model

2.1. ESM and ARMA model

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$\hat{x}_{t+1} = \hat{x}_t + \alpha(x_t - \hat{x}_t) \tag{1}$$

$$= \alpha x_t + (1-\alpha)\hat{x}_t \tag{2}$$

Here,

$\hat{x}_{t+1}$: **forecasting at** $t+1$

x_t : **realized value at** t

α : **smoothing constant** $(0 < \alpha < 1)$

(2) is re-stated as :

$$\hat{x}_{t+1} = \alpha(1-\alpha)^l x_{t-1} \tag{3}$$

By the way, we consider the following (1,1) order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} \tag{4}$$

Generally, (p, q) order ARMA model is stated as :

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{5}$$

Here,

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process $x(t)$ $t = 1,2,\cdots,N,\cdots$

$\{e_t\}$: Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (5) is supposed to satisfy convertibility condition.

Utilizing the relation that :

$$E[e_t | e_{t-1}, e_{t-2}, \cdots] = 0$$

we get the following equation from (4).

$$\hat{x} = x_{t-1} - \beta e_{t-1} \tag{6}$$

Operating this scheme on $t+1$, we finally get :

$$\begin{aligned} \hat{x}_{t+1} &= \hat{x}_t + (1-\beta)e_t \\ &= \hat{x}_t + (1-\beta)(x_t - \hat{x}_t) \end{aligned} \tag{7}$$

If we set $1-\beta=\alpha$, the above equation is the same with (1), i. e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is -1 [1][3].

2.2. Estimation of Smoothing Constant Utilizing System Identification of ARMA model

Comparing with (4) and (5), we obtain :

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases}$$

From (1), (7),

$$\alpha = 1-\beta .$$

Therefore, we get :

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases} \tag{8}$$

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below.
Let (5) be :

$$\tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} \tag{9}$$

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{10}$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (9), (10), we get the following non-linear equations which are well known [3].

$$\left.\begin{array}{l} \tilde{r}_k = \begin{cases} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} & (k \le q) \\ 0 & (k \ge q+1) \end{cases} \\ \tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2 \end{array}\right\} \tag{11}$$

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only b_1, so it can be solved in the following method.
From (4) (5) (8) (11), we get :

$$\left.\begin{aligned} & q=1 \\ & a_1=-1 \\ & b_1=-\beta=\alpha-1 \\ & \tilde{r}_0=\left(1+b_1^2\right)\sigma_e^2 \\ & \tilde{r}_1=b_1\sigma_e^2 \end{aligned}\right\} \qquad (12)$$

If we set :

$$\rho_k=\frac{\tilde{r}_k}{\tilde{r}_0}$$

the following equation is derived.

$$\rho_1=\frac{b_1}{1+b_1^2} \qquad (13)$$

We can get b_1 as follows.

$$b_1=\frac{1\pm\sqrt{1-4\rho_1^2}}{2\rho_1} \qquad (14)$$

In order to have real roots, ρ_1 must satisfy :

$$|\rho_1|\leq\frac{1}{2} \qquad (15)$$

From invertibility condition, b_1 must satisfy :

$$|b_1|<1 \qquad (16)$$

From (14), using the next relation,

$$(1-b_1)^2\geq 0$$

$$(1+b_1)^2\geq 0$$

(16) always holds.

As

$$\alpha = b_1 + 1$$

b_1 is within the range of :

$$-1 < b_1 < 0$$

Finally we get :

$$\left.\begin{aligned} b_1 &= \frac{1-\sqrt{1-4\rho_1^2}}{2\rho_1} \\ \alpha &= \frac{1+2\rho_1-\sqrt{1-4\rho_1^2}}{2\rho_1} \end{aligned}\right\} \tag{17}$$

which satisfy above condition. Thus we can get a theoretical solution by a simple way.

Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter.

It can be estimated only by calculating 0th and 1st order autocorrelation function.

3. Seasonal Trend Removing

Seasonal Trend removing method is used to exclude seasonal trend from time series and its procedure is described as follows.

(a) Firstly, take moving average of n terms from the original time series. If n is the even number, we can not obtain the corresponding item to the original time series. Therefore we have to make one more moving average.

(b) Next, calculate Moving average Ratio.
Moving average Ratio
= (realized value of the original time series)/(moving average of n terms to the corresponding month)

(c) Make average of (b) by each month.

(d) Calculate Monthly Ratio

Monthly Ratio
= (c)/(average of (c) throughout the year)
(e) Adjust the original time series by Monthly Ratio
Adjusted value of the original time series
= (realized value of the original time series)/(Monthly Ratio)

Seasonal trend is removed from the original time series by the above stated method. Then forecasting is executed by the proposed method which minimizes the variance of forecasting error is ESM. After multiplying Monthly Ratio, we obtain forecasting value. Forecasting accuracy would be improved by this newly proposed method.

4. Combined Seasonal Trend Removing Method and Exponential Smoothing Method

Seasonal trend is removed from the original time series by the above stated method. Then forecasting is executed by the proposed method which minimizes the variance of forecasting error in ESM. After multiplying Monthly Ratio, we obtain forecasting value. Forecasting accuracy would be improved by this newly proposed method.

In the Seasonal Trend Removing Part, we set $n = 4,6,8$ and obtain Monthly Ratio from 3(a)～(d).

In the case $n = 4$ (an even number), as is stated at 3(a), it follows

$$\left\{\frac{(n-2)\cdot month+(n-1)\cdot month+n\cdot month+(n+1)\cdot month}{4}\right.$$
$$\left.+\frac{(n-1)\cdot month+n\cdot month+(n+1)\cdot month+(n+2)\cdot month}{4}\right\}\div 2$$
$$=\frac{(n-2)\cdot month}{8}+\frac{(n-1)\cdot month}{4}+\frac{n\cdot month}{4}+\frac{(n+1)\cdot month}{4}+\frac{(n+2)\cdot month}{8}$$

Weight of the edge part of Moving Average is 1/2 to those of another parts.

By this method, we make forecasting and examine forecasting accuracy comparing with the original data.

Forecasting error is calculated as :

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i \tag{18}$$

where

$$\varepsilon_i = \hat{x}_i - x_i \tag{19}$$

and variance is calculated as :

$$\sigma_\varepsilon^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^2 \tag{20}$$

5. ARIMA Model by Using Delay Operator

p-th order AR model is stated as :

$$x_t + a_1 x_{t-1} + \cdots + a_p x_{t-p} = e_t \tag{21}$$

Using the delay operator z^{-1} which means :

$$z^{-1} x_t = x_{t-1} \tag{22}$$

define :

$$A\left(z^{-1}\right) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_p z^{-p} \tag{23}$$

Then AR model is stated as :

$$A\left(z^{-1}\right)x_t = e_t \tag{24}$$

q-th order MA model is also stated as :

$$x_t = e_t + b_1 e_{t-1} + \cdots + b_q e_{t-q} \tag{25}$$

And define :

$$B\left(z^{-1}\right) = 1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_q z^{-q} \tag{26}$$

then MA model is stated as :

$$x_t = B(z^{-1})e_t \tag{27}$$

ARIMA model

$$x_t + a_1 x_{t-1} + \cdots a_p x_{t-p} = e_t + b_1 e_{t-1} + \cdots + b_q e_{t-q} \tag{28}$$

is stated as :

$$A(Z^{-1})x_t = B(Z^{-1})e_t \tag{29}$$

(p,d,q) order ARIMA model of d times difference from the original data is stated as :

$$A(z^{-1})(1-z^{-1})^d x_t = B(z^{-1})e_t \tag{30}$$

6. Forecasting Passengers in Airline Companies－Forecasting By Newly Proposed Method

6.1. Forecasting

By using 3 airlines data for 3 years from Nov, 2000 to Dec, 2003 (Skymark Airlines, Jal Express, Hokkaido Airsystem), we set $n=4,6,8$ and obtain monthly Ratio from 3(a)～(d). By these data, we forecast number of passengers and examine forecasting accuracy comparing with the original data.

(1) Skymark Airlines

The Monthly Ratio of Skymark Airlines are exhibited in table 1, ρ_1 and α in Table 2 and Forecasting value in Table 3.

Table 1: The Monthly Ratio in Skymark Airlines

	n = 4	n = 6	n = 8
Jan	0.9810065	0.9801751	0.9527500
Feb	0.8221172	0.7921729	0.7949144
Mar	1.1701837	1.1408536	1.1297137
Apr	1.0017728	1.0121489	0.8656834
May	1.0341462	1.0162757	1.0434111
Jun	0.8390509	0.8411696	0.8804007
Jul	0.9648480	0.9918018	1.0082967
Aug	1.2624671	1.2953940	1.3401963
Sep	0.9645545	1.0165651	1.0446596
Oct	0.9637141	0.9776716	1.0372043
Nov	0.9756568	0.9587748	0.9513286
Dec	1.0204822	0.9769969	0.9514411

Table 2: ρ_1 and α in Skymark Airlines

	ρ_1	α
Exponential Smoothing Method	−0.32376	0.63252
n = 4	−0.05571	0.94411
n = 6	−0.16592	0.82924
n = 8	−0.23813	0.74658

Table 3: Forecasting in Skymark Airlines (number of passengers)

	The Original Data	Exponential Smoothing Method	Newly Proposed Method		
			n = 4	n = 6	n = 8
Jan	114,312	109,655	111,016	109,654	105,698
Feb	82,504	112,601	93,925	90,225	90,189
Mar	128,670	93,564	98,621	100,240	102,068
Apr	105,492	115,769	126,413	123,181	102,980
May	138,221	109,269	110,290	110,022	113,632
Jun	127,077	127,582	114,494	111,964	115,149
Jul	143,897	127,262	123,116	127,056	129,080
Aug	189,634	137,784	180,515	182,910	187,457
Sep	152,030	170,580	180,398	184,368	184,929
Oct	155,365	158,847	148,398	153,533	164,256
Nov	149,073	156,645	151,508	149,234	148,526
Dec	138,042	151,855	152,481	146,741	143,535

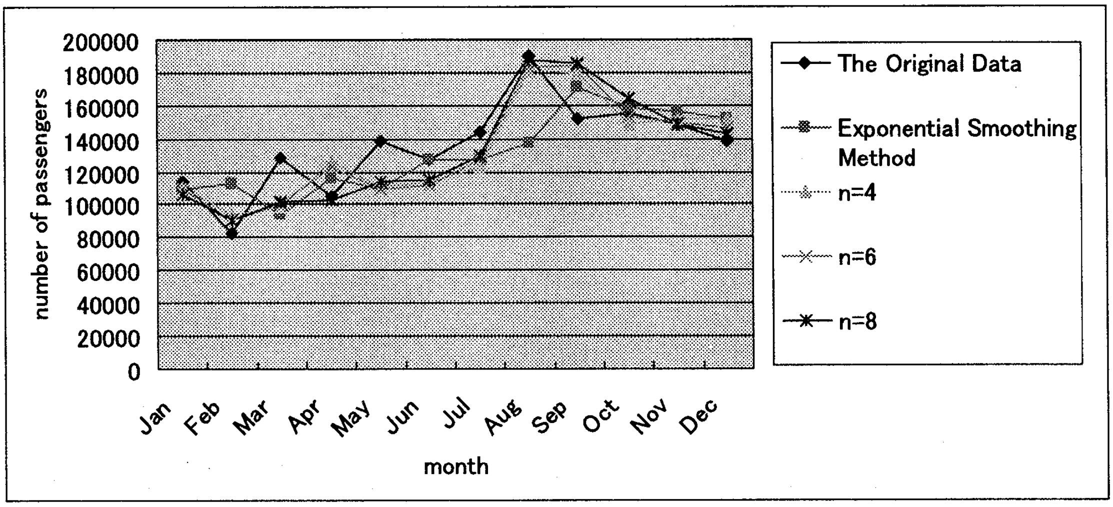

Fig. 1: Passengers of Skymark Airlines

Table 4: Variance of Forecasting Error by Skymark Airlines

Exponential Smoothing Method	Newly Proposed Method		
	n = 4	n = 6	n = 8
585,481,376.2	355,593,572.8	325,133,612.2	263,390,862.3

(2) Jal Express

The Monthly Ratio of Jal Express are exhibited in Table 5, ρ_1 and α in Table 6 and Forecasting value in Table 7.

Table 5: The Monthly Ratio of Jal Express

	n = 4	n = 6	n = 8
Jan	0.9137225	0.9086162	0.9040593
Feb	0.9678056	0.9286281	0.9213314
Mar	1.1082444	1.1556010	1.1438505
Apr	0.9330404	0.9476883	0.9025426
May	1.0759316	1.0796427	1.0877738
Jun	0.9718982	0.9402056	0.9495178
Jul	0.9282428	0.9261440	0.9190909
Aug	1.0082967	0.9876335	1.0232877
Sep	1.0306926	1.0617936	1.0376828
Oct	1.0602481	1.0896311	1.1180123
Nov	1.0805065	1.0825602	1.0861485
Dec	0.9213706	0.8918557	0.9067026

Table 6: ρ_1 and α of Jal Express

	ρ_1	α
Exponential Smoothing Method	−0.06176	0.93801
n = 4	−0.04067	0.95927
n = 6	−0.05353	0.94632
n = 8	−0.09755	0.90150

Table 7: Forecasting of Jal Express (number of passengers)

	The Original Data	Exponential Smoothing Method	Newly Proposed Method		
			n = 4	n = 6	n = 8
Jan	61,187	62,791	56,925	56,945	57,240
Feb	62,661	61,286	59,092	56,894	56,567
Mar	77,802	62,576	66,934	72,325	71,532
Apr	56,649	76,858	75,749	72,958	68,863
May	70,638	57,902	63,436	62,339	63,727
Jun	72,581	69,848	64,440	65,763	65,945
Jul	69,812	72,412	65,728	67,089	66,425
Aug	68,176	69,973	72,924	69,088	71,686
Sep	79,481	68,287	68,034	72,490	70,937
Oct	74,596	78,787	83,249	85,949	87,636
Nov	70,927	74,856	79,949	81,004	81,428
Dec	55,438	71,171	69,763	63,443	64,671

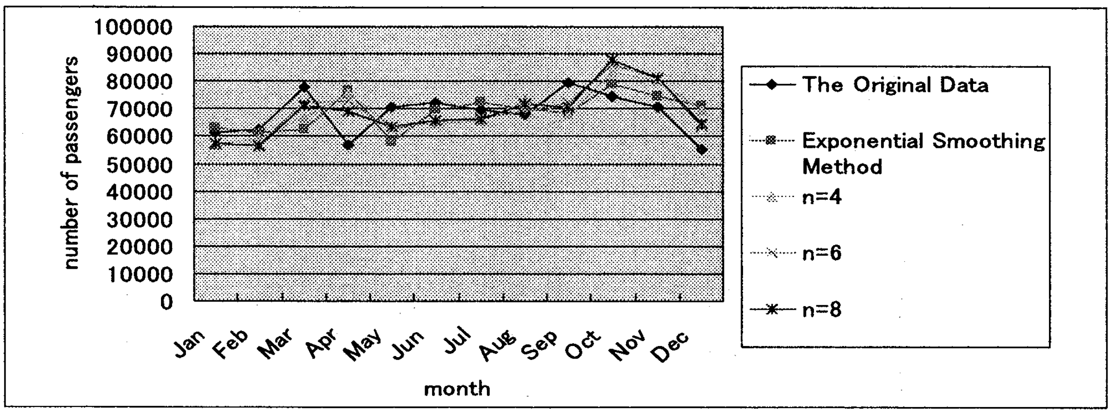

Fig. 2: Forecasting of Jal Express

Table 8: Variance of Forecasting Error by Skymark Airlines

Exponential Smoothing Method	Newly Proposed Method		
	n = 4	n = 6	n = 8
111,484,448.9	73,420,848.4	73,715,157.3	71,963,982.0

(3) Hokkaido Air system

The Monthly Ratio of Hokkaido Air system are exhibited in table 9, ρ_1 and α in Table 10 and Forecasting value in Table 11.

Table 9: The Monthly Ratio of Hokkaido Air system

Monthly Ratio	n = 4	n = 6	n = 8
Jan	1.0173781	1.0191080	0.9935807
Feb	0.9113113	0.8793517	0.8573289
Mar	1.1121404	1.1622387	1.1232543
Apr	0.8694269	0.8370323	0.8017868
May	1.0094122	1.0270025	1.0226790
Jun	1.0123213	0.9966569	1.0218539
Jul	1.0706485	1.1053245	1.1232377
Aug	0.9926509	1.0180237	1.0774171
Sep	1.0285635	1.0672029	1.0784930
Oct	1.0734351	1.0244139	1.0527615
Nov	1.0173459	1.0248803	0.9974890
Dec	0.8853657	0.8387647	0.8501181

Table 10: ρ_1 and α of Hokkaido Air system

	ρ_1	α
Exponential Smoothing Method	−0.22284	0.76483
n = 4	−0.10732	0.89142
n = 6	−0.23618	0.74893
n = 8	−0.24388	0.73958

Table 11: Forecasting of Hokkaido Air system (number of passengers)

	The Original Data	Exponential Smoothing Method	Newly Proposed Method		
			n = 4	n = 6	n = 8
Jan	14,401	13,440	13,371	13,735	13,412
Feb	14,234	14,175	12,999	12,460	12,145
Mar	16,466	14,220	15,834	16,524	15,969
Apr	11,802	15,938	14,106	13,310	12,732
May	13,780	12,775	12,398	13,178	13,156
Jun	15,202	13,544	13,785	13,496	13,837
Jul	16,822	14,812	16,092	16,342	16,590
Aug	15,096	16,349	16,505	16,605	17,548
Sep	15,341	15,391	15,698	16,436	16,616
Oct	15,586	15,353	16,458	15,731	16,168
Nov	11,225	15,531	15,828	15,915	15,488
Dec	10,669	12,238	10,355	10,321	10,495

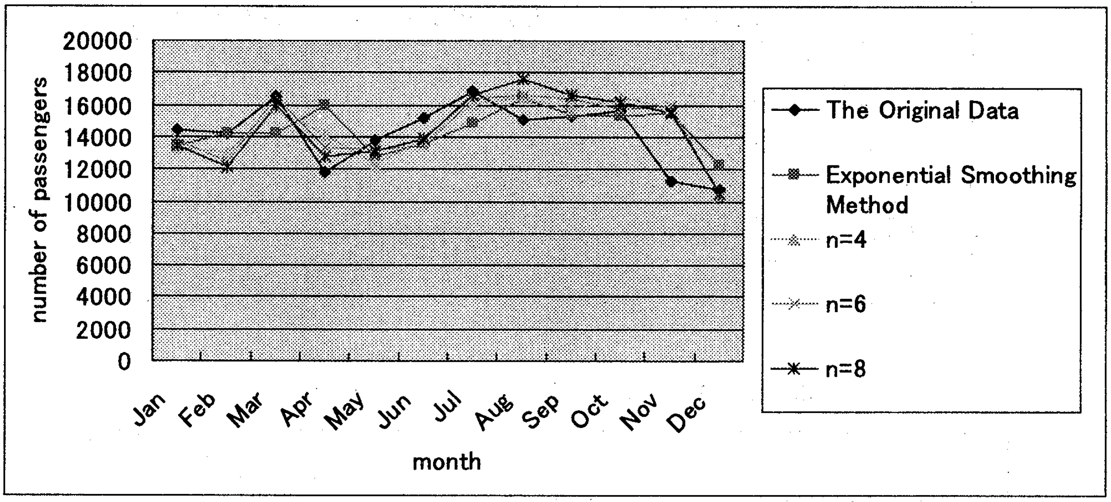

Fig. 3:Passengers of Skymark Airlines

Table 12:Variance of Forecasting Error by Hokkaido Air system

Exponential Smoothing Method	Newly Proposed Method		
	n = 4	n = 6	n = 8
4,789,781.0	3,295,508.3	3,091,041.1	3,081,759.8

6.2. Remarks

Variance of forecasting error of newly proposed method is smaller than that of ESM. Under newly proposed method, variance of forecasting error becomes smaller as n is set larger in Skymark Airlines and Hokkaido Air system. But in the case of Jal express, variance of forecasting error in $n=4$ is smaller than that in $n=6$. Therefore, decreasing rate of variance of forecasting error is examined at Table 13. Decreasing rate of Jal Express in the case ② in Table 13 is nearly equal 0. Considering these cases, variance of forecasting error generally decreases and accuracy of forecasting increases as n becomes large.

Table 13: Decreasing Rate of Variance of Forecasting Error by 3 companies

	①	②	③
Skymark Airlines	64.6%	9.4%	23.4%
Jal Express	51.8%	−0.4%	2.4%
Hokkaido Air system	45.3%	6.6%	0.3%

① (variance of forecasting error by the case n=4)/(variance of forecasting error by exponential smoothing method)

② (variance of forecasting error by the case n=6)/(variance of forecasting error by the case n=4)

③ (variance of forecasting error by the case n=8)/(variance of forecasting error by the case n=6)

7. Forecasting of Passengers in Airline Companies—Forecasting by ARIMA Model

7.1. Forecasting

We used TSP (Time Series Processor) in forecasting by ARIMA model.

Log L (Log of Likelihood Function) from the original data is calculated for each order in Table 14, 15, 16 and then AIC is obtained. AIC is calculated as follows.

$$AIC = n * Log\ L + 2 * (p + d + q)$$

(1) ARIMA model fitting

A. Skymark Airlines

Table 14 shows AIC value of Skymark Airlines in ARIMA model.

Table 14: AIC value in ARIMA model of Skymark Airlines

model	Log L	AIC	order	model	Log L	AIC	order
ARIMA (1,1,0)	−395.767	−14243.612	1	ARIMA (1,1,1)	−391.757	−14097.252	5
ARIMA (2,1,0)	−394.733	−14204.388	2	ARIMA (1,1,2)	−390.918	−14065.048	8
ARIMA (0,1,1)	−393.353	−14156.708	3	ARIMA (2,1,1)	−391.72	−14093.92	6
ARIMA (0,1,2)	−392.104	−14109.744	4	ARIMA (2,1,2)	−391.717	−14091.812	7

B. Jal Express

Table 15 shows AIC value of Jal Express in ARIMA model.

Table 15: AIC value of Jal Express in ARIMA model.

model	Log L	AIC	order	model	Log L	AIC	order
ARIMA (1,1,0)	−367.647	−13231.292	1	ARIMA (1,1,1)	−361.653	−13013.508	8
ARIMA (2,1,0)	−367.319	−13217.484	2	ARIMA (1,1,2)	−365.831	−13161.916	5
ARIMA (0,1,1)	−362.569	−13048.484	6	ARIMA (2,1,1)	−366.484	−13185.424	3
ARIMA (0,1,2)	−361.714	−13015.704	7	ARIMA (2,1,2)	−365.795	−13158.620	4

C. Hokkaido Airsystem

Table 16 shows AIC value of Hokkaido Airsystem in ARIMA model.

Table 16: AIC value of Hokkaido Airsystem in ARIMA model

model	Log L	AIC	order	model	Log L	AIC	order
ARIMA (1,1,0)	−312.835	−11258.06	3	ARIMA (1,1,1)	−311.284	−11200.224	5
ARIMA (2,1,0)	−311.636	−11212.896	4	ARIMA (1,1,2)	−311.02	−11188.720	6
ARIMA (0,1,1)	−314.953	−11334.308	1	ARIMA (2,1,1)	−310.874	−11183.464	7
ARIMA (0,1,2)	−313.246	−11270.856	2	ARIMA (2,1,2)	−309.089	−11117.204	8

AIC value is minimum by ARIMA (1,1,0) model at Skymark Airlines and Jal Express. This is the best fitting model. While Hokkaido Airsystem has chosen ARIMA (0,0,1) model as best fitting model. Next, we make forecast from January 2003 to December 2003 by each method, i.e., newly proposed method (n=8) and ARIMA model which is minimum in AIC. Then both methods are compared concerning forecasting accuracy.

(2) Forecasting results

A. Skymark Airlines

Forecasting results by newly proposed method and ARIMA model are exhibited in Table 17, 18.

Table 17: Forecasting results of Skymark Airlines (number of passengers)

	The Original Data	Newly Proposed Method (n=8)	ARIMA (1,1,0)
Jan	11,4312	105,698	112,807
Feb	82,504	90,189	117,159
Mar	128,670	102,068	119,184
Apr	105,492	102,980	122,223
May	138,221	113,632	124,820
Jun	127,077	115,149	127,609
Jul	143,897	129,080	130,315
Aug	189,634	187,457	133,057
Sep	152,030	184,929	135,783
Oct	155,365	164,258	138,516
Nov	149,073	148,526	141,247
Dec	138,042	143,535	143,978

Table 18: Variance of Forecasting Error of Skymark Airlines

Newly Proposed Method (n=8)	ARIMA (1,1,0)
263,390,862.3	480,064,041.5

B. Jal Express

Forecasting results by newly proposed method and ARIMA model are exhibited in Table 19, 20.

Table 19: Forecasting Result of Jal Express (number of passengers)

	The Original Data	Newly Proposed Method (n=8)	ARIMA (1,1,0)
Jan	61,187	57,240	67,521
Feb	62,661	56,567	65,299
Mar	77,802	71,532	66,158
Apr	56,649	68,863	65,806
May	70,638	63,727	65,930
Jun	72,581	65,945	65,867
Jul	69,812	66,425	65,877
Aug	68,176	71,686	65,859
Sep	79,481	70,937	65,852
Oct	74,596	87,636	65,840
Nov	70,927	81,428	65,831
Dec	55,438	64,671	65,820

Table 20: Variance of Forecasting Error of Jal Express

Newly Proposed Method (n=8)	ARIMA (1,1,0)
71,963,982.0	62,190,494.5

C. Hokkaido Airsystem

Forecasting results by newly proposed method and ARIMA model are exhibited in Table 21, 22.

Table 21: Forecasting of Hokkaido Airsystem (number of passengers)

	The Original Data	Newly Proposed Method (n=8)	ARIMA (1,1,0)
Jan	14,401	13,412	14,176
Feb	14,234	12,145	14,228
Mar	16,466	15,969	14,280
Apr	11,802	12,732	14,333
May	13,780	13,156	14,385
Jun	15,202	13,837	14,437
Jul	16,822	16,590	14,489
Aug	15,096	17,548	14,541
Sep	15,341	16,616	14,593
Oct	15,586	16,168	14,646
Nov	11,225	15,488	14,698
Dec	10,669	10,495	14,750

Table 22: Variance of Forecasting Error of Hokkaido Airsystem

Newly Proposed Method(n=8)	ARIMA(1,1,0)
3,081,759.8	4,306,649.1

7.2. Remarks

Newly proposed method shows apparently good result in Table 8 and Table 22. While variance of forecasting error is slightly smaller by ARIMA model than those by newly proposed method in JAL Express (Table 20). As a whole, newly proposed method shows good result. Variance of the normalized original data is 480,064,041.5 for Skymark Airlines, 62,190,494.5 for Jal Express, 43,606,649.1 for Hokkaido Airsystem respectively. Variance of Jal Express is minimum. Most separate data of Monthly Ratio in $n=8$ from 1.0 is 1.34 for Skymark Airlines, 1.14 for Jal Express, 0.80 for Hokkaido Airsystem respectively and the peak fluctuation of time series is minimum at Jal Express. Considering these facts, newly proposed method is effective especially for the case that fluctuation of time series is big.

Table 23: Decreasing Rate of Variance of Forecasting Error by 3 companies

	(ARIMA)/(variance of forecasting error by the case of n=8)
Skymark Airlines	82.3%
Jal Express	－15.7%
Hokkaido Airsystem	39.7%

8. Conclusion

Formerly, following method was proposed by us. Focusing that the equation of exponential smoothing method was equivalent to (1,1) order ARMA model equation, new method of estimation of smoothing constant in exponential smoothing method was proposed.

In this paper, utilizing the above stated method, the revised forecasting method was proposed. In making forecast such as sales data, seasonal trend was removed by dividing the original time series by the Monthly Ratio which was calculated using moving average data. Utilizing the above stated method, forecasting was executed with this data in which seasonal trend was removed.

The Monthly Ratio was multiplied after that, and forecasting data was obtained. This was a revised forecasting method. These methods were examined by the data of airline passengers. Variance of forecasting error of this newly proposed method proved to be less than those of previously proposed method. These results were compared with those of ARIMA model. Newly proposed method showed good results.

The effectiveness of this method should be examined in various cases.

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