Effect of Backmixing on Mass Transfer Characteristics in Multistage Vibrating Disk Gas Absorption Column

Effect of Backmixing on Mass Transfer Characteristics in Multistage Vibrating Disk Gas Absorption Column

Kakuji Tojo,* Kei Miyanami** and Takeo Yano**

A simple method is presented for determining the number of transfer units (N.T.U.) in a multistage vibrating disk contactor of gas-liguid co-current flow, in which backmixing occurs appreciably.

The back flow model is employed to describe the flow pattern of fluid in the contactor. The effects of backmixing coefficients on the N. T. U. are examined and the values may be decreased as much as 20 per cent by neglecting the effect of backmixing in the contactor.

1. Introduction

The methods for design and evaluation of the continuous mass transfer operations have been usually established on the assumption of plug flow or perfect mixing.[1,2,3] However, there are many cases in which neither of these two types of flow corresponds to the actual flow, and it is frequently said that backmixing occurs appreciably in cases of a gas bubble or a spray column.[4,5,6,7]

Tadaki et al.[8] estimated the performance of bubble columns using one dimensional diffusion model with taking account of longitudinal mixing. Hartland and Mecklenburgh[9] described the method for estimating the overall stage efficiency for stagewise processes with backmixing. J. A. Watson et al.[10] developed a simple method for estimating the effects of axial mixing on counter-current column performances in terms of column efficiency.

The prediction of the effects of backmixing on mass transfer in stagewise contactors, however, is not yet possible for many cases. The authors have already shown that the liquid phase mixing in the multistage vibrating disk contactor could be expressed by the back flow model which consists of the equal volume perfect mixers in series of twice the number of the actual stages.[12]

This work deals with an investigation of the effect of backmixing on mass transfer characteristics of the multistage vibrating disk column for gas-liquid contacting operations.

2. Mathematical model

Consider a stagewise gas absorber consisting of N perfectly mixed stages with backmixing between them. In order to simplify the mathematical treatment, the following assumptions are made.

1. Variations in holdup, backmixing coefficients and volumetric mass transfer coefficents from stage to stage in the absorber are negligible.

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2. The composition of the solute in the gas phase is constant throughout the column. This assumption will be satisfied in the case of absorption of pure gas.

Then, the material balance of the solute gas in the liquid phase is given from the following equation (see Fig. 1).
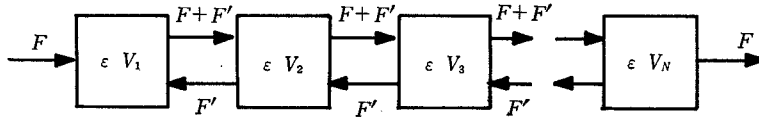

Fig. 1 Stagewise process with backmixing between stages

For the first stage:

$$\varepsilon V_1 \frac{dC_1}{dt} = -(F+F')C_1 + FC_0 + F'C_2 + K_L a \varepsilon V_1 (C^* - C_1) \tag{1}$$

for the i-th stage $(2 \leq i \leq N-1)$:

$$\varepsilon V_i \frac{dC_2}{dt} = -(F+2F')C_i + (F+F')C_{i-1} + F'C_{i+1} + K_L a \varepsilon V_i (C^* - C_i) \tag{2}$$

and for the last stage:

$$\varepsilon V_N \frac{dC_N}{dt} = -(F+F')C_N + (F+F')C_{N-1} + K_L a \varepsilon V_N (C^* - C_N) \tag{3}$$

where V_i = volume of the i-th stage,

ε = liquid holdup,

F = main flow rate of liquid,

F' = back flow rate of liquid,

C^* = solubility of the gas to be absorbed in the liquid.

As a steady state, Eqs. (1) to (3) are reduced

$$(1+q+K_1)X_1 - q \cdot X_2 = K_1 \tag{4}$$

$$-(1+q) \cdot X_{i-1} + (1+2q+K_i) \cdot X_i - q \cdot X_{i+1} = K_i \tag{5}$$

$$-(1+q) \cdot X_{N-1} + (1+q+K_N) \cdot X_N = K_N \tag{6}$$

respectively, where the dimensionless parameters and variables in these equations are defined as

backmixing coefficient: $q = F'/F$,

dimensionless concentration: $X_i = (C_i - C_0)/(C^* - C_0)$,

number of transfer units per stage: $K_i = K_L a \cdot \varepsilon \cdot V_i / F$.

In matrix notation, Eqs. (4) to (6) can be written as

$$AX = K \tag{7}$$

where A is the $(N \times N)$ tridiagonal matrix,

$$A = \begin{pmatrix} b_1 & c_1 & 0 & \cdots\cdots & 0 \\ a_2 & b_2 & c_2 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & c_{N-1} \\ 0 & \cdots\cdots & 0 & a_N & b_N \end{pmatrix} = \begin{pmatrix} (1+q+K_1), & -q, & 0\cdots\cdots & 0 \\ -(1+q), & (1+2q+K_2), & -q & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -q \\ 0 \cdots\cdots & 0 & -(1+q), & (1+q+K_N) \end{pmatrix}$$

and X is the column vector of concentrations. Eq. (7) can be solved recursively by the following formulas (Thomas' method[11]);

$$w_1 = c_1/b_1, \quad w_i = c_i/(b_i - a_i w_{i-1}), \quad 2 \leq i \leq N$$

$$g_1 = K_1/b_1, \quad g_i = (K_i - a_i g_{i-1})/(b_i - a_i w_{i-1}), \quad 2 \leq i \leq N \tag{7'}$$

$$X_N = g_N, \quad X_i = g_i - w_i X_{i+1}, \quad 1 \leq i \leq N-1$$

For the two limiting cases of fluid mixing, the concentration in the liquid phase can be calculated from the following equations,

$$X_N = \frac{C_N - C_0}{C^* - C_0} = 1 - \exp(-K) \tag{8}$$

for plug flow ($q = 0$, $V_i = V/N$, $N \to \infty$), and

$$X_N = \frac{K}{1+K} \tag{9}$$

for perfectly mixed flow ($N = 1$, $q \to \infty$),

where $\qquad K = K_L a \cdot \varepsilon \cdot V/F$.

If all the K_i's in Eqs. (4) to (6) are the same and constant, Eq. (7) can be solved analytically and X_i is given by

$$X_i = 1 - (A\lambda_1{}^i + B\lambda_2{}^i) \qquad (i = 1, 2, \cdots, N) \tag{10}$$

where $\qquad \lambda_1 = (1 + 2q + K + \sqrt{(K+1)^2 + 4qK})/2q$

$$\lambda_2 = (1 + 2q + K - \sqrt{(K+1)^2 + 4qK})/2q$$

$$A = \left[\lambda_1(1+q+K-q\lambda_2) + \frac{\lambda_1{}^{N-1}}{\lambda_2{}^{N-2}}(1+q+K-q\lambda_2) \right]^{-1}$$

$$B = A \cdot \frac{\lambda_1}{\lambda_2} \left[\frac{1+q - (1+q+K)\lambda_1}{(1+q+K)\lambda_2 - (1+q)} \right]$$

It is seen from Eqs. (7), (8), (9) and (10), that the outlet concentration in the liquid phase X_N may be given by the expression in general;

$$X_N = f(N; q, K) \tag{11}$$

Because X_N is directly measurable quantity and q is obtained from the experimental results shown in Eq. (15), and also the number of transfer units K may be calculated by Eq. (11), as

$$K = g(N; q, X_N) \tag{12}$$

It is difficult, however, to solve Eq. (11) for K explicitly, so that the value of K can be better determined by adjusting it using trial and error method until the observed outlet concentration becomes equal to the concentration calculated by Eq. (7) or (10). Once the value of K is determined, H.T.U. or the volumetric mass transfer coefficient K_La can be calculated from Eq. (13) or (14), respectively.

$$\text{H.T.U.} = Z/(N \cdot K) \tag{13}$$

$$K_La = F \cdot K/(\varepsilon \cdot V) \tag{14}$$

where Z is the length of the contactor. These values thus obtained can be regarded as being corrected for the effect of backmixing.

Fig. 2 shows the concentration profiles of the solute gas in the liquid phase along the column for the case of $N=14$, as calculated from Eq. (7). Although X_i is discontinuous between stages, all X_i's are plotted by smooth curves and it may be easily compared with the profile for the case of plug flow.
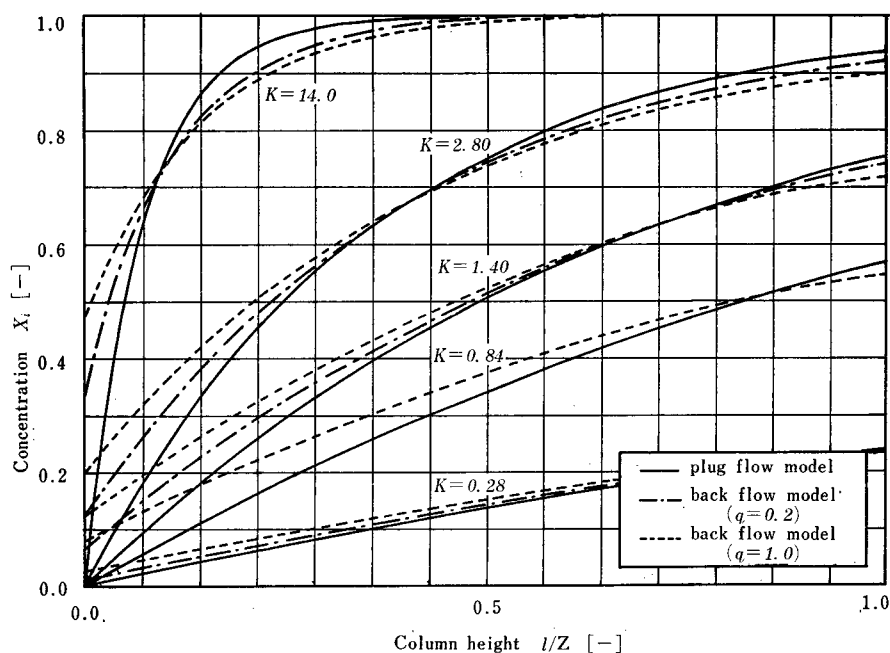

Fig. 2 Concentration profiles along the column

As shown in Fig. 2, there is a concentration jump or discontinuity at the inlet due to backmixing for the back flow model. On the other hand, no discontinuity occurs for the plug flow model.

Fig. 3 shows the effects of number of transfer units K on the concentration in the last stage for various degrees of mixing between plug flow and perfect mixing. If the concentrations in the inlet and the outlet liquid phase are known from experimental results, and the flow pattern in the eontactor can be represented by the backmixing coefficient q in the
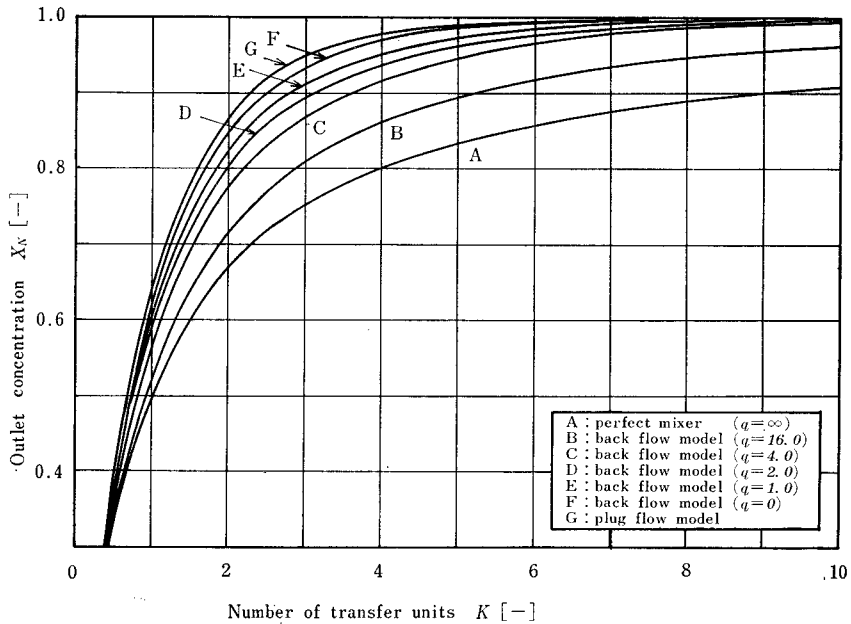
Fig. 3   Effect of number of transfer units K on X_N

back flow model and also the value of K can be obtained from Fig. 3. For example, if the values of X_N and q obtained experimentally are 0.834 and 2.0 respectively, the value of K is given as 2.31. On the other hand, in the cases of assumptions of plug flow or perfect mixing, K's are shown in the following equations

$$K = 1.79 \text{ (plug flow)}$$

$$K = 2.02 \text{ (CSTM with 7 stages)}$$

and so the value of K may be underestimated by as much as 22 or 13 per cent.

3. Experimental

The schematic diagram of the experimental apparatus is shown in Fig. 4. The contactor is a multistage vibrating disk contactor and its details are described elsewhere.[12]

The experiments were carried out with the system of pure carbon dioxide-water in co-current operation at 30°C ± 0.5°C. The experimental conditions are listed in Table 1.

After the concentration of the outlet liquid reached in steady state, liquid samples were taken from each stage.

The carbon dioxide in the liquid sample was analyzed by a titration method; 0.1 N standard sodium hydroxide solution and barium chloride were added to the samples, then the excess sodium hydroxide was back-titrated with 0.1 N standard hydrochloric acid using phenolphthalein as an indicator.
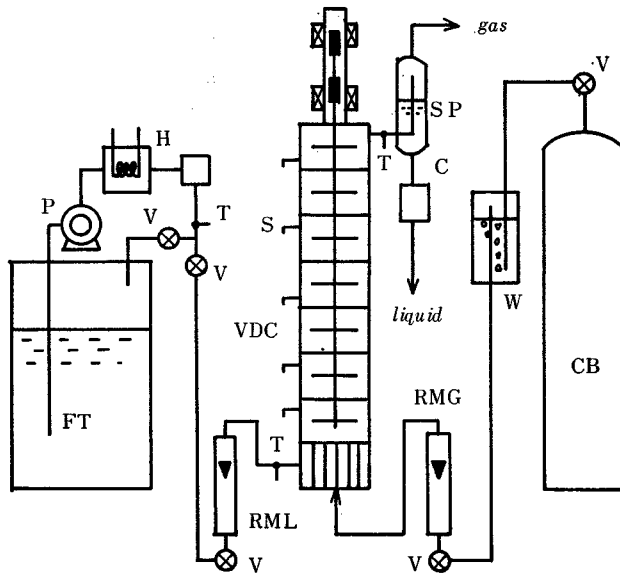
FT—Liquid feed tank, P—Pump, H—Heater, T—Thermometer, V—Valve, RML—Liquid flow meter, RMG—Gas flow meter, VDC—Contactor, S—Sampling cocks, SP—Separator, C—Electric conductivity cell, W—Water vapor saturator, CB—CO₂ cylinder.

Fig. 4 Schematic diagram of experimental apparatus

Table 1 Experimental conditions

number of stage $N/2$	7	[—]
vib. disk diameter d_d	30, 40	[mm]
hole diameter d_h	20, 40	[mm]
frequency ν	0, 50, 100, 200	[cpm]
vib. amplitude $2a'$	6—10	[mm]
liquid superficial velocity u_l	0. 16—1. 62	[cm/s]
gas superficial velocity u_g	1. 32—7. 02	[cm/s]
temperature	30±0. 5	[°C]

4. Results

The number of transfer units K (N.T.U.) obtained by the measurement of inlet and outlet liquid phase concentrations is correct only when the actual concentration profile coincides with the one calculated from Eq. (7). To see whether this is the case or not, concentration profiles are measured and plotted. Examples of them are shown in Fig. 5. The solid line in Fig. 5 represents the profiles predicted from Eq. (7). The backmixing coefficient q and liquid holdup ε are estimated from Eqs. (15) and (16), respectvely, which have been obtained by the another experiments.[12]

Key	u_l [cm /s]	u_g [cm /s]	$\overline{q}$ [−]	K [−]
O	.0.48	2.52	2.0	2.30
⊙	0.16	1.32	0.96	1.40
▽	0.80	2.50	0.20	0.84
△	0.80	1.32	0.20	0.70

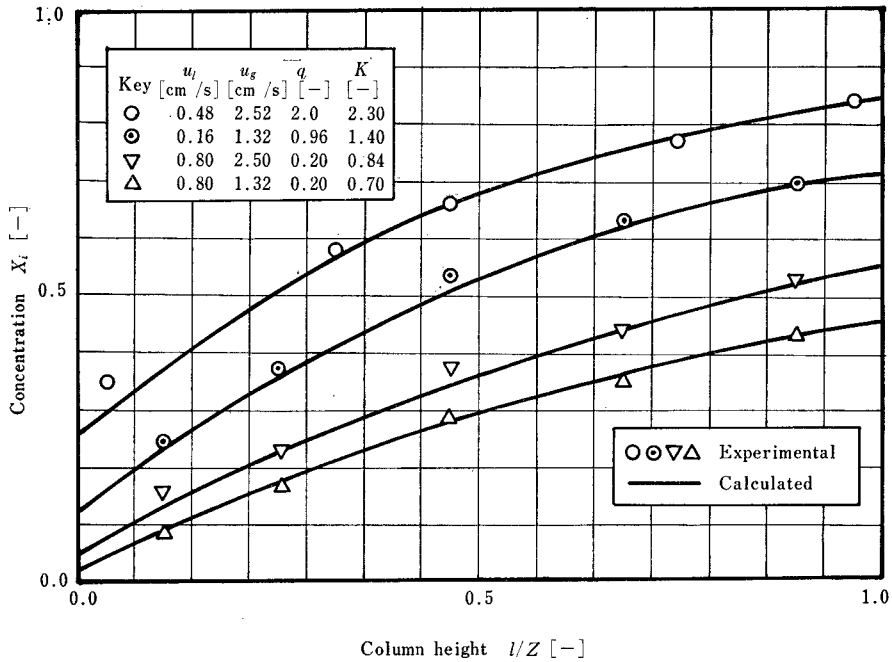OO▽△   Experimental
———   Calculated

Column height l/Z [−]

Fig. 5   Concentration profiles along the column

$$q = \left\{ -\frac{1}{2} + 2\nu\beta + \frac{2a'\nu}{u_t}\left(\frac{d_h}{d_i}\right)^2 \right\}\left(\frac{d_d}{d_i}\right)^2 \qquad (15)$$

$$\varepsilon = 1 - 0.08\, u_g \qquad (16)$$

where

ν = vibrating frequency

β = factor depend on frequency

a' = amplitude of vibration

u_t = superficial liquid velocity

d_h = diameter of hole in partition plate

d_d = diameter of vibrating disk

d_i = inner diameter of column.

The experimental results of concentration profiles are well agreed with the calculated ones. Hence, the present method can be expected to give corrected values of H.T.U. or N.T.U. for evaluating performances of the multistage vibrating disk contactor even if the backmixing is appreciable.

5. Conclusions

The present method for experimental determination of the corrected values of H.T.U. or N.T.U. is simple and easy to use for estimating the performance of a multistage vibrating disk contactor with co-current gas-liquid flow when backmixing occurs appreciably. The effects of backmixing on N.T.U. or H.T.U. of the contactor were reasonably estimated by the back flow model. The effects of the model parameter, the backmixing coefficient, on

these values were quantitatively investigated. It was also shown that the neglect of back-mixing in the contactor might lead to an underestimation of N.T.U. as much as 20 per cent.

The validity of this method was verified by measurements of concentration profiles along the contactor when used as an absorber of carbon dioxide with water.

The effects of the operating conditions and the design variables of the multistage vibrating disk contactor on N.T.U. or H.T.U. will be shortly reported.

Acknowledgement

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