A Theoretical Investigation on the Displacement Effect of a Cylindrical Pitot Tube

A Theoretical Investigation on the Displacement Effect of a Cylindrical Pitot Tube*

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This paper deals with a theoretical investigation on the displacement effect of a cylindrical pitot tube in a transversely non-uniform flow, especially in the peripheral flow of an impeller.

The solution is made on the assumption that the flow field around the pitot tube is ir-rotational, and is compared with one of the other treatments considering a linear shear flow. The result derived here may be one of the clues that should be necessary in the measurement of flow within a turbomachinery and its discussions.

1. Introduction

A cylindrical pitot tube is used willingly in the measurement of flow within a turbomachinery as well as an ordinary duct, for the convenience of installation. This instrument has usually two pressure-detect holes which are drilled at regular intervals on the circumference of the cylindrical tube. The direction and the intensity of the flow are then obtained by balancing the pressures detected on these holes (*i.e.* by turning the cylindrical pitot tube), on the supposition that the stagnation pitot of the flow locates at the mid point of the two holes. For this purpose, the cylindrical tube has to include two conduits of pressure in its body, and it is difficult to make up the cylindrical tube of too small dimensions. A cylindrical pitot tube with only one pressure-detect hole is adopted occasionally, for its simplified structure. If the static pressure angle of this pitot tube had been calibrated in a realistic flow, the pitot tube might be able to estimate the location of stagnation point and the magnitude of static- and dynamic- pressure of the flow from the measured pressure distribution around the cylindrical tube. However, the perfect calibration that should be performed under the established non-uniform flow is seldom carried out. Because the objective velocity profile, which is about to be treated, is in itself unknown beforehand, and moreover it is not easy to generate the suitable velocity profile[1], granting that it is predicted.

The pressure distribution of a cylindrical pitot tube of 3 *mm* radius with single pressure hole, which is obtained in a calibration channel (a paraller flow), is illustrated in Fig. 1, together with the typical variation of the pressure distribution revealed in the peripheral flow of an impeller of no discharge. As may readily be seen from this figure, the maximum point of the pressure distribution does not settle on the tangential direction

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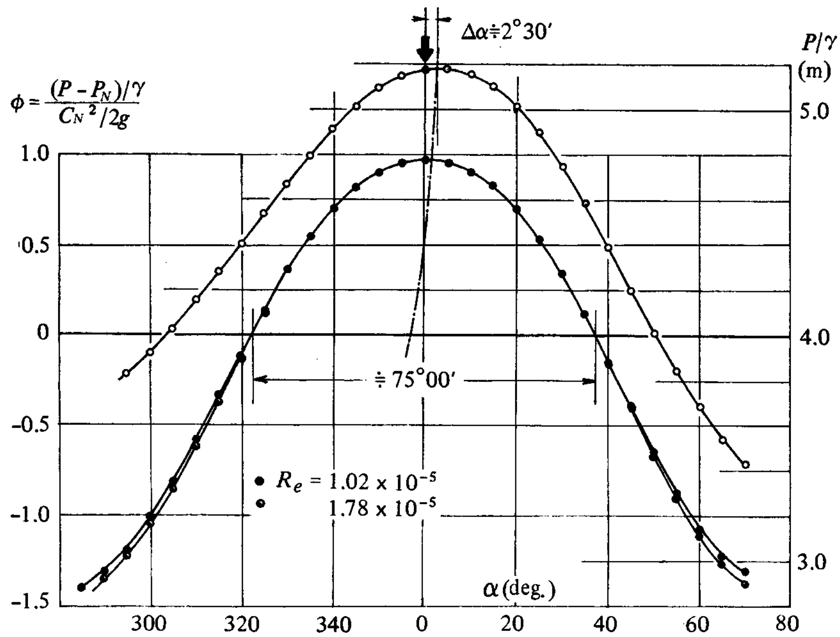

Fig. 1. Experimental results showing the pressure distributions of a cylindrical pitot tube.
The solid and semi-solid symbols illustrate the symmetrical pressure distributions calibrated in the uniform flows. (The ordinate is left-hand side.)
The open symbols represent the distortion of the pressure distribution obtained in the peripheral flow of an impeller. (The ordinate is right-hand side.)
The chain curve describes the center line of the deformed pressure distribution, and the arrow indicates the tangential direction of the impeller of no discharge.

of the revolving impeller, and the symmetry of the pressure distribution is deformed, in spite of the fact that the meridional component of aboslute flow leaving the impeller is considered to be equal to zero (since the structure of this pump is arranged so as not to arise the internal leakage of through flow). Consequently, the accurate value of the static pressure angle is uncertain, and if one try to deal with this crude result without an inquiry, some of errors may be included in the quantitative evaluation. The above-mentioned distortion of the pressure distribution is called a displacement effect of the cylindrical pitot tube. Therefore, the correct interpretation and evaluation of the displacement effect are important in the experimental study of boundary layers, wakes and the peripheral flows of impeller, whose velocity profile is transversely non-uniform.

The problem has already been investigated experimentally by Young & Maas[2], Livesey[3], Davies[4], and theoretically by Hall[5], Arie[6], Lighthill[7], and moreover with another treatment by Ikui & Inoue[8]. The theoretical analyses, however, have dealt with only the pitot tube in a shear flow whose velocity profile is assumed to be linear and vorticity to be constant.

Hereupon, the peripheral flow of an impeller canges periodically with the number of repetitions ($Z \times N$) by the influence of the impeller with finite number of blades (Z) and the revolution per second (N), but it is ordinary that the absolute flow leaving the impeller is regarded as being steady and uniform on the circumference of the impeller. On the other hand, the flow system around the impeller is considered to be a vorticity field by the influence of wall friction, *etc.*, thus the velocity- and the pressure- distribution of the flow must be non-uniform transversely. It is, however, difficult to perceive the actual velocity profile of the flow in advance. Accordingly, the conventional treatments stated above, *i.e.* in which the velocity profile is assumed as non-uniform but linear, may not be applied for directly. For this reason, it would be rather advantageous that the flow system around the pitot tube is considered to be ir-rotational (but it has constant vorticity in regard to the center of the impeller), and from this assumption the displacement effect of the pitot tube may be obtained more easily than the conventional method, and the tendencies of the displacement effect can be grasped as a function of the flow rate of the impeller and of the relative dimensions of the pitot tube against the impeller. (However, it should be noticed that the calculated results do not completely agree with the experimental results, because of the omission of viscosity or stretching vortex[5)]).

A part of the numerical results obtained by the present treatment is compared with the result checked by Arie's solution in paragraph 3.

2. Mathematical Formulation of the Problem

Consider a complex plane with its origin O at the center of a cylindrical pitot tube of radius a, and let point B on the real axis be a source of strength m, as shown in Fig. 2.

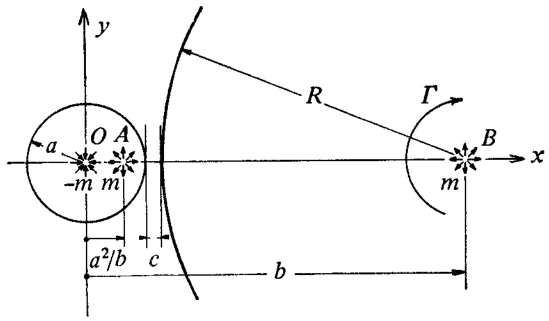

Fig 2. The scheme of sources, sink and circulation to generate the flow around a cylindrical pitot tube.

The point B, which is at the distance b from the origin, represents the center of an impeller with radius R. If both walls of a vortex chamber around the impeller are parallel with each other, the meridional component of absolute flow leaving the impeller is expressed as follows;

$$c_m = m/2\pi r \tag{1}$$

and

$$m = Q/W \tag{2},$$

where Q is the flow rate discharged from the impeller, W is the perpendicular width to the meridional flow and r is an arbitrary radius (but larger than R). Besides, assuming that the loss head wasting in the external rgeion of the impeller may be neglected, then the tangential component of absolute flow is derived as follows from the relation of angular momentum;

$$c_u = \Gamma/2\pi r \tag{3}$$

and denoting the input head of the impeller as H_{th}, the circulation Γ is

$$\Gamma = H_{\mathrm{th}} \cdot 2\pi g/\omega \tag{4}$$

From a simple combination of Eq. (1) and (3), it is evident that the path of the absolute flow becomes an equiangular spiral, in other words, it is a potential flow.

Now, let A be an inverse point of B with respect to the circle of the pitot tube, and imagine it to be equal to source m, furthermore let the origin O be a sink of equal but negative strength m. Here the position of A is determined so as to satisfy the relation of $\overline{OA}\cdot\overline{OB}=a^2$, that is $Z=a^2/b$. Thereupon, if the following complex flow potential is introduced, it can be proved easily that the surface of the pitot tube coincides with a stream line. (Refer to Appendix.)

$$F(z) = \frac{1}{2\pi}\{(m+i\Gamma)\cdot\log(Z-b)+(m-i\Gamma)\cdot\log(Z-a^2/b)-(m-i\Gamma)\cdot\log Z\} \tag{5}$$

Hence, the velocity on the surface of the pitot tube is

$$q = \left|\frac{dF(z)}{dZ}\right| = \frac{1}{2\pi}\cdot\left|(m+i\Gamma)\frac{1}{Z-b}+(m-i\Gamma)\frac{1}{Z-a^2/b}-(m-i\Gamma)\frac{1}{Z}\right| \tag{6}$$

and $\quad Z = a\cdot e^{i\alpha} = a(\cos\alpha+i\sin\alpha)$ (7).

By substituting Eq. (7) into Eq. (6), and rearranging the expression;

$$\frac{q}{\Gamma/\pi b} = \frac{|(m/\Gamma)\sin\alpha+\cos\alpha-(a/b)|}{1-2(a/b)\cos\alpha+(a/b)^2} \tag{8}$$

the denominator in this formula is rewritten as follows;

$$(1-a/b)^2+2(a/b)(1-\cos\alpha),$$

which is finite and positive. Then, the stagnation point of the flow is obtained by solving the following equation;

$$(m/\Gamma)\sin\alpha_s+\cos\alpha_s-(a/b) = 0 \tag{9}$$

that is $$\alpha_s = \cos^{-1}\left\{\frac{(m/\Gamma)\sqrt{1+(m/\Gamma)^2-(a/b)^2}+(a/b)}{1+(m/\Gamma)^2}\right\} \tag{10}$$

In order to define the coefficient of the pitot tube, it may be reasonable that the following absolute velocity and corresponding static pressure at the impeller tip are consiered;

$$c_2 = \sqrt{c_{m2}+c_{u2}} = \frac{\Gamma}{2\pi R}\sqrt{1+(m/\Gamma)^2} \tag{11}$$

Then, by applying Bernoulli's equation

$$\begin{aligned} \Phi &= \frac{(p-p_2)/\gamma}{c_2^2/2g} = 1-(q/c_2)^2 \\ &= 1-\frac{(2R/b)^2}{(m/\Gamma)^2+1}\left\{\frac{(m/\Gamma)\sin\alpha+\cos\alpha-(a/b)}{1-2(a/b)\cos\alpha+(a/b)}\right\}^2 \end{aligned} \tag{12}$$

The static pressure angle of the cylindrical pitot tube is obtained from the condition of $\Phi=0$, namely

$$\alpha_0 = \cos^{-1}\left\{\frac{MN \mp L\sqrt{M^2-N^2+L^2}}{M^2+L^2}\right\} \tag{13}$$

where

$$\left.\begin{aligned} L &= \mp(m/\Gamma)(R/b) \\ M &= (a/b)\sqrt{1+(m/\Gamma)^2}\pm(R/b) \\ N &= \frac{1+(a/b)^2}{2}\sqrt{1+(m/\Gamma^2)}\pm(a/b)(R/b) \end{aligned}\right\} \tag{14}$$

In these formulae, the plus-minus or minus-plus signs must be chosen coincidentally, *viz.* the upper signs refer to the inner static pressure angle bordering on the impeller with respect to the stagnation point, *v.v.* the lower signs refer to the outer static pressure angle receding from the impeller.

Finally, in order to compare the results derived here with those corresponding to an uniform flow, substituting the conditions that $(R/b)\doteqdot 1$, $(a/b)\rightarrow 0$ and $(m/\Gamma)\rightarrow 0$ into Eq. (10) and Eq. (12), respectively,

$$\cos\alpha_s = 0 \tag{10'}$$

$$\Phi = 1-4\cos^2\alpha \tag{12'}$$

Accordingly, the perfect agreements between these results and the ordinary results, which are well-known such as $\sin\alpha_s=0$ and $\Phi=1-4\sin^2\alpha$, can be confirmed by the consideration that there is the phase difference of $\pi/2$ in the way of taking the amplitude.

3. Numerical Results and Considerations

Fig. 3 shows an example of the calculated results drawn with a solid curve, which deals with the condition that $R=100$ *mm*, $a=3$ *mm*, $c=1$ *mm* and $m/\Gamma=0.00$, together with the ordinary curve corresponding to an uniform flow. These curves should be refered to the experimental result described in paragraph 1 (Fig. 1). It is evident from this figure that the pressure distribution of the pitot tube within the peripheral flow of an impeller is unsymmetric, and the stagnation point of the flow (the maximum point of the pressure distribution) shifts towards the impeller side with the degree of $\Delta\alpha(=1°40')$. The tendency of the calculated result almost coincides with the experimental result,

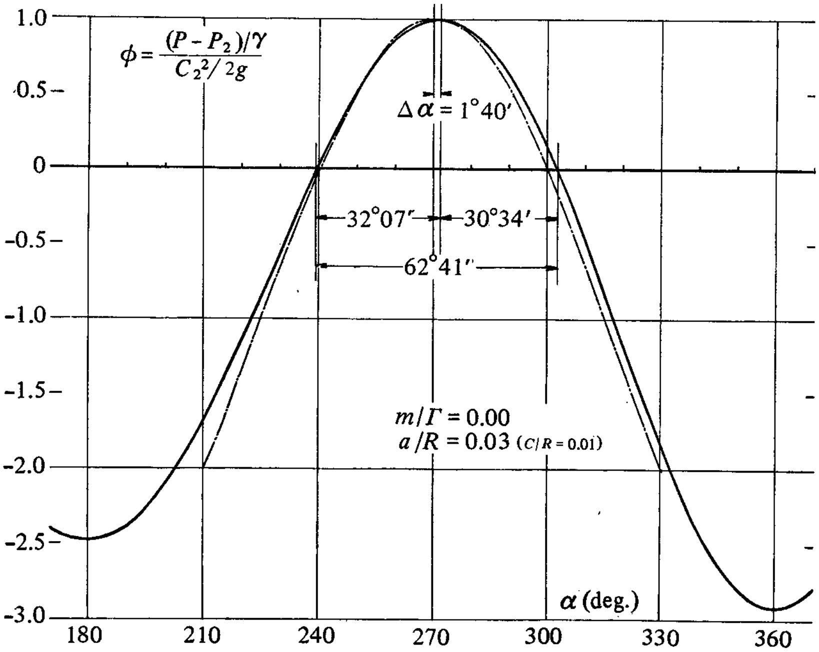

Fig. 3. The calculated results of pitot tube coefficient corresponding to the results shown in Fig. 1.

but the value of $\Delta\alpha$ is rather small comparing with the experimental result. This reason may be that the effect of viscosity is neglected throughout. Furthermore, the static pressure angle of the pitot tube becomes larger than the ordinary value that is known as $\pi/3$. Accordingly, care must be taken to the following two points.

1) The measurement of the peripheral flow of an impeller is liable to obtain the incorrect value of the meridional component of absolute velocity leaving the impeller. That is, the rate of flow of the impeller may be somewhat larger than the actual value.

2) If the magnitude of the absolute velocity is derived by supposing that the static pressure angle is unchangeable, (*i.e.* derived by the method of basing the makeshift static pressure which would be interpreted by using the static pressure angle calibrated within an uniform flow,) some of errors may be included in the obtained value. Namely, the absolute velocity leaving the impeller is apt to be evaluated rather smaller than the accurate value.

These contradicting tendencies never offset each other, because the former (the displacement effect of the stagnation point) is more essential than the latter. Then, the obtained performance of the impeller may represent somewhat lower volumetric- and vane-[9] efficiencies than the actual values. Fig. 4 shows the variation of $\Delta\alpha$*) against

*) This value includes the deflection of amplitude $\Delta\alpha'$ due to the finite radius of the point tube, *i.e.*

$$\Delta\alpha = \Delta\alpha' + \Delta\alpha_s$$

where $\Delta\alpha_s$ is, so to speak, a net displacement quantity, and $\Delta\alpha'$ is formulated by Eq. (19) mentioned below.

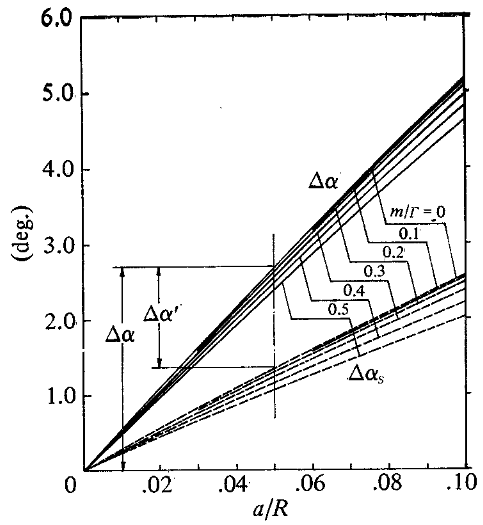

Fig. 4. The displacement of stagnation point *vs.* the dimensionless radius of pitot tube.

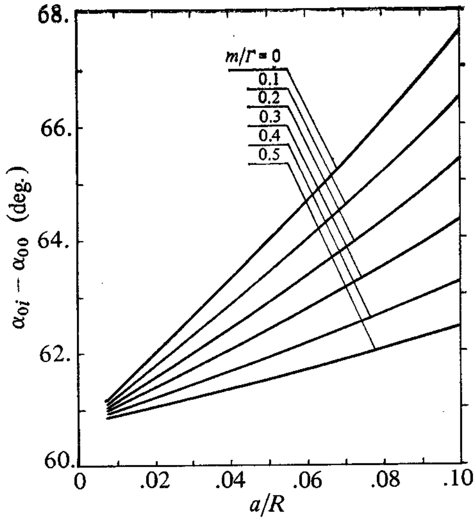

Fig. 5. The extent of static pressure angle *vs.* the dimensionless radius of pitot tube.

the various values of a/R, and Fig. 5 describes the relation of $(\alpha_{oi}-\alpha_{oo})$ *vs.* a/R with the parameter of m/Γ. As will be seen from these figures, the displacement effect increases with the increase of a/R and with the decrease of m/Γ. Furthermore, Fig. 6 represents the relation between m/Γ and the correction factor of the pitot tube coefficient that is defined as the ratio of the correct value Φ $(\alpha_{oi}-\alpha_{oo})$ and the makeshift value Φ $(\pi/3)$. The dimensionless radii of the pitot tube are written alongside of the curves. In this figure, it is noteworthy that the correction factor of a hypothetic pitot tube, of which radius is zero, does not equal to 1.0. The reason why this fact occurs is

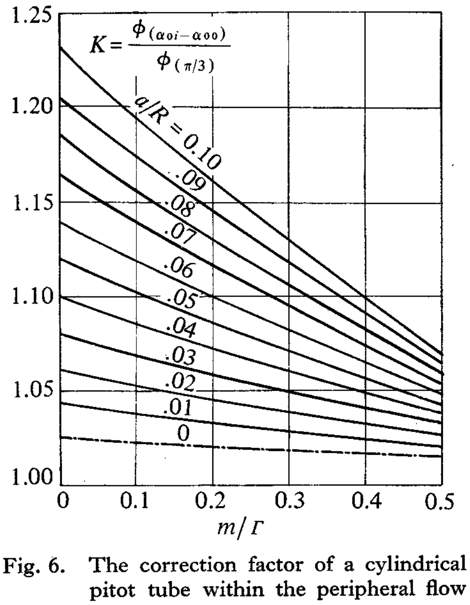

Fig. 6. The correction factor of a cylindrical pitot tube within the peripheral flow of an impeller.

that the situation of the flow varies along the streamline in the present treatment, saying it differently, the term of (R/b) in Formula (14) and then in Eq. (13) does not equal to 1.0.

Next, in order to compare the present solution with the other treatment[5)-7)], in which the velocity profile is considered to be only linear and parallel, the identical condition of flow is verified by using Arie's equation. Hereupon, this equation includes the terms of velocity gradient k as defined as follows;

$$c = c_o\left(1 + k\frac{y}{2a}\right) \tag{15}$$

where c_o is the typical velocity of the oncoming flow on the axis of cylindrical pitot tube, and y is a rectangular co-ordinate perpendicular to the axis as quoted in Fig. 7. Now, differentiating Eq. (15) with respect to y

$$\frac{dc}{dy} = k\frac{c_o}{2a} \tag{16}$$

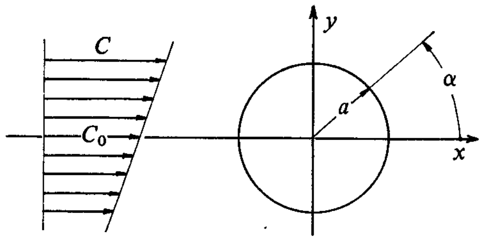

Fig. 7. Velocity profile assumed as a linear and parallel flow.

On the other hand, since the present flow system is assumed as equiangular spiral as shown in Fig. 8, the velocity gradient on the transverse direction is

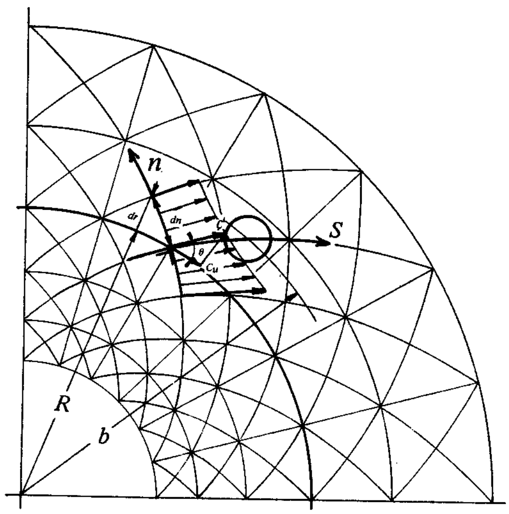

Fig. 8. Velocity profile within a peripheral flow of an impeller.

$$\frac{dc}{dn} = \frac{dc_u}{dr}\sec\theta\frac{dr}{dn} = -\frac{c_u}{r} \qquad (17).$$

Let *Eq.* (17) be equal to Eq. (16) under the condition of $r=R$, then

$$k = -2\frac{a}{R}\frac{c_{u2}}{c_2} = -2\frac{a}{R}\frac{1}{\sqrt{1+(m/\Gamma)^2}} \qquad (18).$$

Besides, it is necessary to take account of the effect of the curvature of flow path. Although the strict consideration of this subject is somewhat complicated, the following approximation may be approved. Let D denote an intersection of the circle of the pitot tube and an equiangular spiral passing the center of the pitot tube (Origin O), as shown in Fig. 9, the straight line connecting O and D may be able to be regarded as a quasi-axis of the pitot tube defined in Arie's treatment. Furthermore, let E denote an intersection of the cirlce of the pitot tube and a circle passing the origin, $\angle DOE$ may be approximately equal to $\tan^{-1}(m/\Gamma)$ with the restriction of small-sized (a/R). Since ΔBOE is an equilateral triangle, $\angle y'OE$ is expressed as follows;

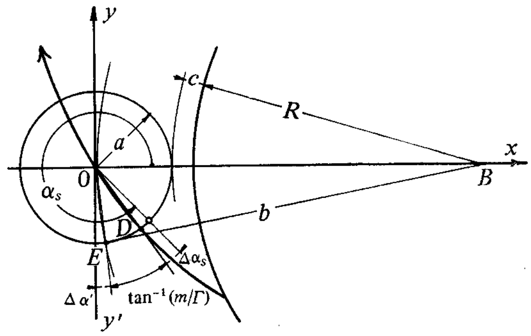

Fig. 9. Illustration of the amplitudes around the center of pitot tube.

$$\Delta\alpha' = \sin^{-1}\left(\frac{a}{2b}\right) \qquad (19).$$

Namely, this value is the geometrically deflection of the amplitude around the origin due to the finite raidus of the pitot tube. Fig. 10 shows a comparison of the result derived from the present solution with the result calculated by using Arie's equation, in the case of $a/R=0.10$ and $m/\Gamma=0.30$. In this figure, the broken line describes the direction of the quasi-axis of the pitot tube, and the open symbol on the solid curve indicates the stagnation 'point obtained from the present solution. The angle between the stagnation point and the direction of the negative y-axis is devided into three substances as annotated in Fig. 10. On the other hand, the solid symbol on the chain curve describes the stagnation point derived from Arie's equation. (The subscript -A refers to the results obtained from Arie's solution.) As may clearly be seen from this figure, the displacement effect explained by the present treatment is more emphatic than the result obtained from Arie's solution. This contrast is mainly caused by the divergent effect of the absolute

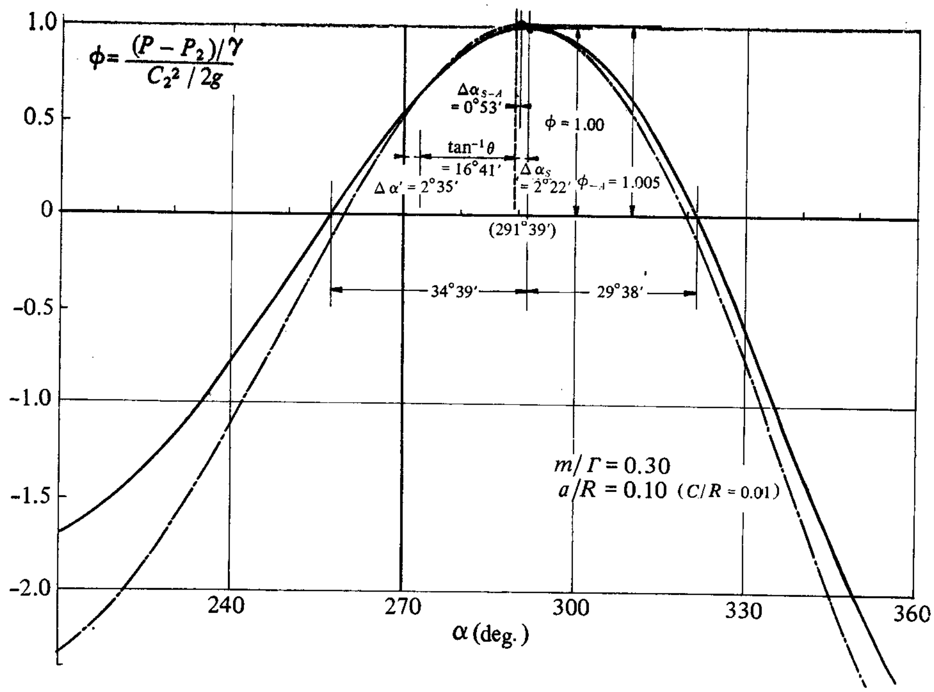

Fig. 10. A comparison of the result derived from the present solution with the result calculated by using Arie's equation[6].

flow. (The absolute velocity decreases along the flow path.) It may be therefore suggested that the only consideration of linearly non-uniform velocity profile is insufficient in connection with the displacement effect of the cylindrical pitit tube within the peripheral flow of an impeller.

According to Arie's solution, the value of the pitot tube coefficient at the stagnation point is slightly larger than 1.0. This may be caused by the way of taking the typical velocity of the oncoming flow.

4. Conclusions

The displacement effect of a cylindrical pitot tube within the peripheral flow of an impeller is investigated theoretically on the assumption that the flow field around the pitot tube is ir-rotational. In view of the calculated results, the following conclusions can be drawn.

1) The stagnation point of the peripheral flow of an impeller shifts towards the impeller side.

2) The static pressure angle within the peripheral flow becomes larger than the value of the non-uniform but parallel flow.

3) These effects increase with the increase of the radius of the pitot tube.

In sum, the obtained characteristics of the impeller by using a cylindrical pitot tube may represent somewhat lower volumetric- and vane- efficiencies than the correct values.

In order to avoid these defects, it is necessary that the cylindrical pitot tube with the radius as small as possible is used, even through the displacement effect within the peripheral flow of an impeller may not be perfectly eliminated.

Appendix

Z is defined anywhere on the complex plane. Then, let C be an arbitrary point on the circle of the pitot tube with radius a as shown in Fig. 11. Point C is expressed with respect to the origin O as follows;

$$Z = a \cdot e^{i\alpha} \qquad \text{(A.1)},$$

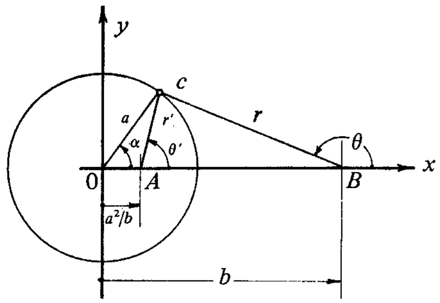

Fig. 11

and is also rewritten in regard to point B and A. That is

$$Z-b = re^{i\theta} \qquad \text{(A.2)}$$

$$Z-a^2/b = r'e^{i\theta'} \qquad \text{(A.3)},$$

where r, r' are absolute values, and θ, θ' are amplitudes connected with point B and A, respectivelly.

Substituting these relations into the complex flow potential (5) described in paragraph 2;

$$F(z) = \frac{1}{2\pi}\{(m+i\Gamma)(\log r+i\theta)+(m-i\Gamma)(\log r'+i\theta')-(m-i\Gamma)(\log a+i\alpha)\} \qquad \text{(A.4)}.$$

Hence, the stream function can be obtained form the imaginary part of this farmual;

$$\Psi = \frac{1}{2\pi}\left\{ m(\theta+\theta'-\alpha)+\Gamma\cdot\log\left(\frac{ar}{r'}\right)\right\} \qquad \text{(A.5)}.$$

Now, consider the triangles OBC and OCA in Fig. 11, the following relations may be found;

$$\angle BOC = \angle COA = \angle\alpha$$

and

$$\frac{\overline{OA}}{\overline{OC}} = \frac{a^2/b}{a} = \frac{a}{b} = \frac{\overline{OC}}{\overline{OB}}.$$

Therefor, the triangles OBC and OCA are similar to each other, and it is evident that

$$r/r' = b/a \tag{A.6}$$

$$\theta = \pi - \angle OBC = \pi - \angle COA = \pi - (\theta' - \alpha) \tag{A.7}$$

By using these results the stream function (A.5) can be reduced as follows;

$$\Psi = \frac{1}{2\pi}(m \cdot \pi + \Gamma \cdot \log b) = \text{const.} \tag{A.8}$$

Accrdingly, it is proved that the circle of the cylindrical pitot tube with radues a coincides with one of the stream lines.

Nomenclature

R	:	radius of impeller (m)
a	:	radius of cylindrical pitot tube (m)
b	:	center distance between the impeller and the pitot tube (m)
m	:	source strength of impeller (squ. m/sec)
Γ	:	circulation of impeller (squ. m/sec)
Q	:	discharge of impeller (cub. m/sec)
W	:	perpendicular width to the meridional flow (m)
H_{th}	:	input head of impeller (m)
g	:	gravitational accerelation (=9.8 m/squ. sec)
ω	:	angluar velocity (1/sec)
γ	:	specific weight of fiuid (kg/cub. m)
Φ	:	pitot tube coefficient
K	:	correction factor of the pitot tube coefficient
α	:	amplitude around the center of cylindrical pitot tube
$\Delta\alpha$	:	displacement of stagnation point
$\Delta\alpha'$	:	deflection of amplitude
c	:	absolute velocity of flow (m/sec)
c_u	:	tangential component of absolute velocity (m/sec)
c_m	:	meridional component of absolute velocity (m/sec)
q	:	local velocity of flow (m/sec)
p	:	static pressure of flow (kg/squ. m)
$F(z)$	:	complex flow potential
Ψ	:	stream function

and the following subscripts are introduced;

N	:	for the typical flow
2	:	for the flow at the impeller tip
s	:	for the stagnation pitot
o	:	for the static pressure angle
$.i$	:	for the inner side bordering on the impeller (with respect to the stagnation point)
$.o$	:	for the outer side receding from the impeller

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