Tensile Rigidity of Flanged Strips Containing Holes

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(Received June 15, 1971)

The tensile rigidity of flanged strips containing holes was evaluated experimentally and theoretically, and it was confirmed that theoretical results nearly coincided with experimental results. From the results it was found that the tensile rigidity of flanged strips containing holes was mainly affected by the net cross sectional area and the pitch of holes, but hardly by the shape of the cross section.

1. Introduction

The authors previously carried out the investigation on tensile rigidity of perforated strips[1], and determined the effect of size and pitch of holes on the tensile rigidity of the perforated strips.

In the present study, the tensile rigidity of flanged strips containing holes is evaluated and the effect of flange on the tensile rigidity of flanged strips is determined.

2. Test specimen and experimental procedure

Specimens used in this study were steel H-beam and flat bar with one row of circular holes (diameter d, pitch p) as shown in Fig. 1. The flat bar specimens were made to cover the shortage of the previous results[1]. The results for flat bar specimens will be compared with the H-beam specimens in order to determine the effect of flange on the tensile rigidity of flanged strips containing holes.

Tension test of specimens were carried out by using Amsler type universal testing machine. For the purpose of evaluation of the tensile rigidity the measurements of displacement for gage length (100 mm or 150 mm) of the test specimens were performed by using optical extensometer.

3. Experimental results and discussions

We define the effective cross sectional area of flanged strip containing holes $\bar{A}$, using the mean strain per pitch of holes $\bar{\varepsilon}$ produced by the tensile load P_0, by

$$\bar{\varepsilon} = \frac{P_0}{E\bar{A}}$$

where E is Young's modulus. Then the value of $\bar{A}/A_0$, where A_0 is the gross cross sectional area, shows the effect of the holes on the tensile rigidity of flanged strip containing holes.

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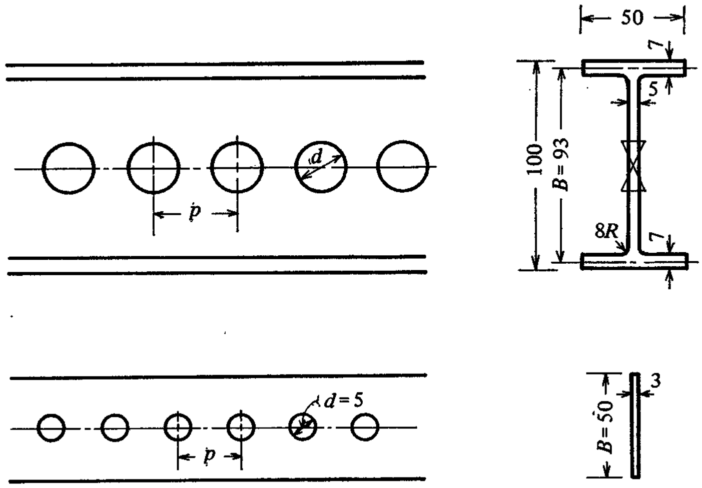

	d (mm)	d/B	A/A_0	p/d
H-beam	23.25	0.25	0.9	3.23 2.15 1.61 1.08
	46.5	0.5	0.8	3.23 1.61 1.08
	75	0.81	0.67	2.00 1.33 1.00
Flat bar	5	0.1	0.9	4.00 2.00 1.00

A_0 = gross cross sectional area of specimen
A = net cross sectional area of specimen

Fig. 1. Shapes and dimensions of specimens.

The experimental values of $\bar{A}/A_0$ is plotted against p/d in Fig. 2, where A/A_0 is taken as parameter (A is net cross sectional area). For the purpose of the comparison the experimental results for the perforated strips, which have been obtained both in the previous and present study, are also shown in Fig. 2. For given values of A/A_0 and p/d, the value of $\bar{A}/A_0$ for H-beam specimen seems to be about the same as the flat bar specimen. This means that the tensile rigidity of flanged stıip containing holes is mainly affected by the net cross sectional area and the pitch of holes, but hardly by the shape of the cross section.

The approximate formula of the evaluation of the tensile rigidity of flanged strips containing holes is given as follow (see **Appendix**):

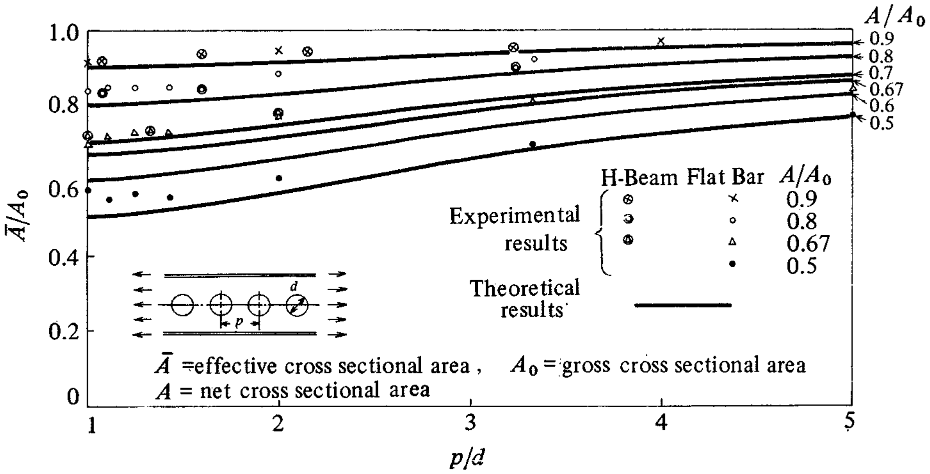

Fig. 2. Relation of $\bar{A}/A_0$ to p/d.

$$\frac{\bar{A}}{A_0}=\frac{A/A_0}{1-\frac{8}{\pi^2}\left(1-\frac{d}{p}\right)\sum_{n=1}^{\infty}\frac{T_n}{(2n-1)^2}}$$

where

$$T_n=\frac{1+\frac{1}{\bar{w}_n}\tanh\bar{w}_n-\tanh^2\bar{w}_n}{\frac{1}{\bar{w}_n}\tanh\bar{w}_n-\tanh^2\bar{w}_n-1+\frac{2}{1-A/A_0}}$$

$$\bar{w}_n=\frac{(2n-1)\pi}{2\left(\frac{p}{d}-1\right)}$$

The numerical values which have been computed for n up to 6 are shown by full lines in Fig. 2. The theoretical results nearly coincide with the experimental results.

4. Conclusions

The tensile rigidity of flanged strips containing holes was evaluated experimentally and theoretically, and it was confirmed that theoretical results nearly coincided with experimental results. From the results it was found that the tensile rigidity of flanged strips containing holes was mainly affected by the net cross sectional area and the pitch of holes, but hardly by the shape ot the cross section.

Reference

1) Y. Fukumoto and Y. Okamura, Jour. Kansai Soc. Naval Architecture, 95 (1959).

Appendix

Approximate Calculation

We derive an approximate formula for tensile rigidity of flanged strip (such as H-beam) containing one row of square holes (sides d, pitch p) as shown in Fig. 3, instead of circular holes for the convenience of calculation.

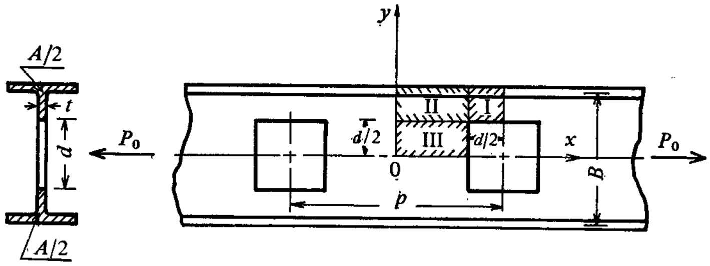

Fig. 3. Flanged strip containing one row of square holes.

Cartesian co-ordinates (x, y) has its origin mid-way between the holes and the x-axis lies on the center line of the holes.

Because of the symmetry of the stress system, the analysis will be developed only for the shaded region such as I, II and III (Fig. 3). We assume that the parts I and II are submitted to the action of normal stress σ_x, uniformly distributed at the cross section.

Denoting by $\frac{P_0}{2}$ and $\frac{P(x)}{2}$ the longitudinal forces in parts I and II respectively, extensions of parts I and II are given, respectively, by

$$\bar{u}_1 = \frac{P_0 d}{2EA} \tag{1}$$

$$\bar{u}_2 = \frac{1}{EA}\int_0^{(p-d)/2} P(x)dx \tag{2}$$

where E = Young's modulus

A = net cross sectional area of the flanged strip

If $P(x)$ can be found the mean strain of the flanged strip, denoted by $\bar{\varepsilon}$, is given by

$$\bar{\varepsilon} = \frac{2(\bar{u}_1 + \bar{u}_2)}{p} \tag{3}$$

and then the tensile rigidity can be evaluated.

We consider the state of stress in the part III to determine $P(x)$. The part III is treated as a case of plane stress, loaded only by shear stresses imposed on it by the part II. A stress function F is introduced into part III, represented by

$$F = \sum_{n=1}^{\infty} F_n = \sum_{n=1}^{\infty} f_n(y) \cos w_n x \qquad (4)^*$$

where

$$w_n = \frac{(2n-1)\pi}{p-d}$$

Here $f_n(y)$ is a function of y only, of such form as to satisfy the differential equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 F = 0 \qquad (5)$$

The y-function which meets the requirement is

$$f_n(y) = (A_n + C_n w_n y)\cosh w_n y + (B_n + D_n w_n y)\sinh w_n y \qquad (6)$$

The unknown constants A_n, B_n, C_n and D_n are determined from the boundary conditions.

The boundary conditions in this case are:

$$\left.\begin{aligned} &\text{for } y = 0\,, \\ &\tau_{xy} = 0\,, \qquad v = 0 \\ &\text{for } y = d/2\,, \\ &\qquad\qquad \sigma_y = 0 \end{aligned}\right\} \qquad (7)$$

where v is the displacement in y-direction.
To satisfy these conditions, we take

$$\left.\begin{aligned} B_n &= C_n = 0 \\ D_n &= -\frac{A_n}{\frac{w_n d}{2}\tanh\frac{w_n d}{2}} \end{aligned}\right\} \qquad (8)$$

Consequently the expression for the stress function contains only A_n as unknown constant.

From statics the condition of equilibrium of the normal stresses over any cross section is given by

$$P(x) + 2t\int_0^{d/2} \sigma_x \, dy = P_0 \qquad (9)$$

where t is thickness of part III.
Hence the equation for $P(x)$ becomes

$$\begin{aligned} P(x) &= P_0 - 2t\left[\frac{\partial F}{\partial y}\right]_0^{d/2} \\ &= P_0 - 2t\sum_{n=1}^{\infty} w_n \left\{\sinh\frac{w_n d}{2} - \frac{2}{w_n d}\cosh\frac{w_n d}{2}\right. \end{aligned}$$

* The conditions which can be satisfied at the end $x=(p-d)/2$ by this assumed form of stress function are:

$$\sigma_x = 0\,, \quad \sigma_y = 0 \text{ and } \tau_{xy} \neq 0\,.$$

Thus the stress function F cannot satisfy completely the condition of a plate with free edge.

$$-\cosh^2\frac{w_n d}{2}\operatorname{cosech}\frac{w_n d}{2}\Big\} A_n \cos w_n x \tag{10}$$

in which the unknown constant A_n is contained.

To determine the constant A_n we assume the condition that the longitudinal displacements of Parts II and III must be equal at their intersection. If u_2 is used to indicate the longitudinal displacement of the part II and u_3 of the part III, this condition is expressed as follow:

$$u_2 = [u_3]_{y=d/2} \tag{11}$$

From Eq. (10) the longitudinal strain in part II is derived and by integration the longitudinal displacement u_2 can be found as follow:

$$u_2 = \frac{P_0 x}{EA} - \frac{2t}{EA}\sum_{n=1}^{\infty}\left(\sinh\frac{w_n d}{2} - \frac{2}{w_n d}\cosh\frac{w_n d}{2} - \cosh^2\frac{w_n d}{2}\operatorname{cosech}\frac{w_n d}{2}\right) A_n \sin w_n x \tag{12}$$

On the other hand the longitudinal strain along the side $y=d/2$ in part III is given as follow:

$$\left(\frac{\partial u_3}{\partial x}\right)_{y=d/2} = \frac{1}{E}(\sigma_x - \nu\sigma_y)_{y=d/2} = \frac{1}{E}\left(\frac{\partial^2 F}{\partial y^2}\right)_{y=d/2}$$

By integration we obtain

$$[u_3]_{y=d/2} = -\frac{4}{Ed}\sum_{n=1}^{\infty}\cosh^2\frac{w_n d}{2}\operatorname{cosech}\frac{w_n d}{2}\cdot A_n \sin w_n x \tag{13}$$

The first term on the right hand of Eq. (12) can be expanded in the series

$$\frac{P_0 x}{EA} = \frac{P_0}{EA}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{4}{(p-d)w_n^2}\sin w_n x\,, \text{ for } -\frac{p-d}{2} \leq x \leq \frac{p-d}{2} \tag{14}$$

Substituting Eqs. (12), (13) and (14) into Eq. (11), we can determine the constant A_n

$$A_n = \frac{2P_0}{t(p-d)}\cdot\frac{(-1)^{n+1}}{w_n^2}\cdot\frac{\tanh\frac{w_n d}{2}\operatorname{sech}\frac{w_n d}{2}}{\tanh^2\frac{w_n d}{2} - \frac{2}{w_n d}\tanh\frac{w_n d}{2} - \left(\frac{2A_0}{td} - 1\right)} \tag{15}$$

From Eqs. (12) and (15), $\bar{u}_2$ is represented by the following expression:

$$\bar{u}_2 = \frac{P_0(p-d)}{2EA}\left[1 - \sum_{n=1}^{\infty}\frac{8}{(2n-1)^2\pi^2}\cdot T_n\right] \tag{16}$$

where

$$\left.\begin{aligned} T_n &= \frac{1 + \frac{1}{\bar{w}_n}\cdot\tanh\bar{w}_n - \tanh^2\bar{w}_n}{\frac{1}{\bar{w}_n}\cdot\tanh\bar{w}_n - \tanh^2\bar{w}_n - 1 + \frac{2}{1-\frac{A}{A_0}}} \\ \bar{w}_n &= w_n \times \frac{d}{2} = \frac{(2n-1)\pi}{2\left(\frac{p}{d} - 1\right)} \end{aligned}\right\} \tag{17}$$

From Eq. (3) we find

$$\bar{\varepsilon} = \frac{P_0}{EA}\left[1 - 8\left(1 - \frac{d}{p}\right)\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2\pi^2} T_n\right] \tag{18}$$

The effective cross sectional area of the flanged strip containing one row of holes under tension $\bar{A}$ is therefore given by

$$\bar{A} = \frac{A}{1 - \frac{8}{\pi^2}\left(1 - \frac{d}{p}\right)\sum_{n=1}^{\infty} \frac{T_n}{(2n-1)^2}} \tag{19}$$