Theory of Arrester and Ground Wire (Two-Conductor System with Corona Loss)

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Masao KIDO*, Hiroaki KAWABATA*, Yoshio INAGAKI* and Yoshiaki SHIRAO*

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This paper dealt with the method of calculation for the attenuation of the wave traveling along the transmission lines, due to corona loss. In the following examples, the application of this method is restricted to the system having a ground wire, on which there is the incident wave on the lightning stroke.

1. Introduction

For computing the a-c loss of corona, semi-empirical formulas have been devised by many students. It is very interesting to know about the mechanism of corona loss in distorting and attenuating the wave moving along the transmission line. There may be two causes to make deformation of wave shape; one is slowing down of wave velocity and another is decreasing of crest value. Considering the ionization which occurs around conductors according to high surge voltage as illustrated in Fig. 1, this corona region increases the capacitance to ground but does not change the inductance since it is conducting radially but not axially.[1] Therefore the wave front travels at a velocity which is slower than the velocity of the wave toe, and thus traveling at slower speed will slip back flattening the front and decapitating the crest. Skilling and Dykes proposed a method of calculation for the slip back of the wave front and Foust Menger dealt with a new method of evaluation of the crest values.

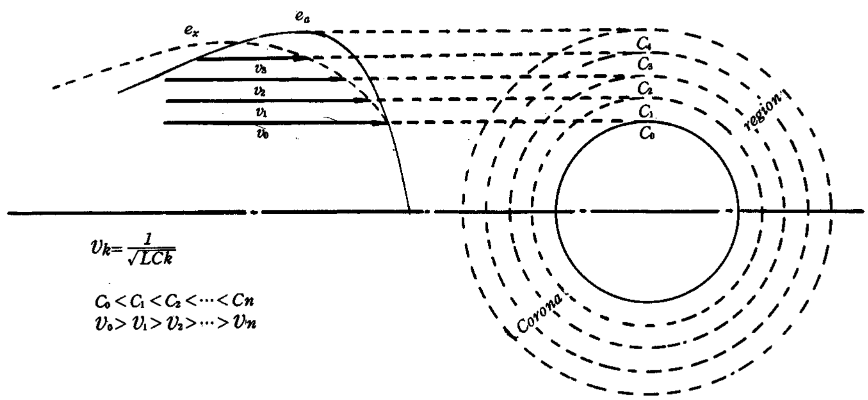

Fig. 1. Effect of corona on wave distortion.

* Department of Electrical Engineering, College of Engineering.

The conventional treatment of transmission line transients given in previous paper,[2] for the direct stroke near the station with an arrester and a ground wire, is based on consideration of two-conductor system and ignores the loss due to corona. However there are many times in the study of traveling waves when the corona loss can not be neglected. Sometimes their influence is so vital as to change the characteristics of the phenomena completely, and entirely erroneous results are obtained if they are not considered. Problems of this type are of interest in connection with the design of ground wires, and protective schemes.

The present paper is concerned only with certain theoretical aspects of ground wires and arresters. More general considerations and procedures, than the previous report, will be given to enable the engineer to design for adequate protection of his system. In this paper the calculation is restricted to two-wire circuit, since this simple multiconductor circuit adequately illustrates the method of analysis with a minimum amount of algebraic exercise. Increasing the number of conductors involved merely magnifies the amount of evaluation that must be done, without serving any other useful purpose.

2. Method of Calculation

Fig. 2 illustrates the power system and the several factors that determine the formation of traveling waves, originating from the direct stroke to the tower, on the ground wire and line conductor. The ground wire is there shown grounded through the tower footing resistance $1/G$. The many devices in a station, such as transformers, individually act as a simple capacitance to ground. This fortunate situation makes it possible to calculate the effect of traveling waves on station apparatus.

(1) Effect of Corona

It is well known that corona loss is expressed by square law in comparatively wide range and there is the following equation given by Peek

$$W = K(V - V_0)^2 \qquad (\mathrm{W/m}) \tag{1}$$

where

V=crest value of surge voltage

V_0 =critical corona voltage

$K=12(f+25)(r/2h)^{1/2} \times 10^{-11}$

r=*radius of conductor*

h=*height*

f=*frequency*.

If it be assumed that corona loss of traveling wave is given by equation (1), wave distortion may be determined by the following method.

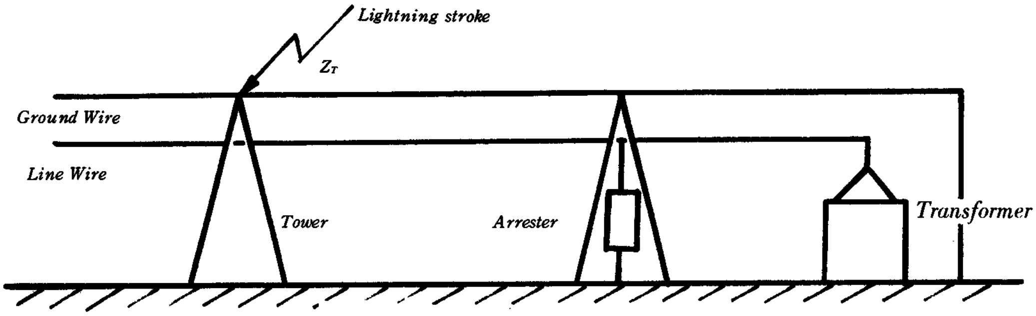

Fig. 2. Striken tower and typical station.

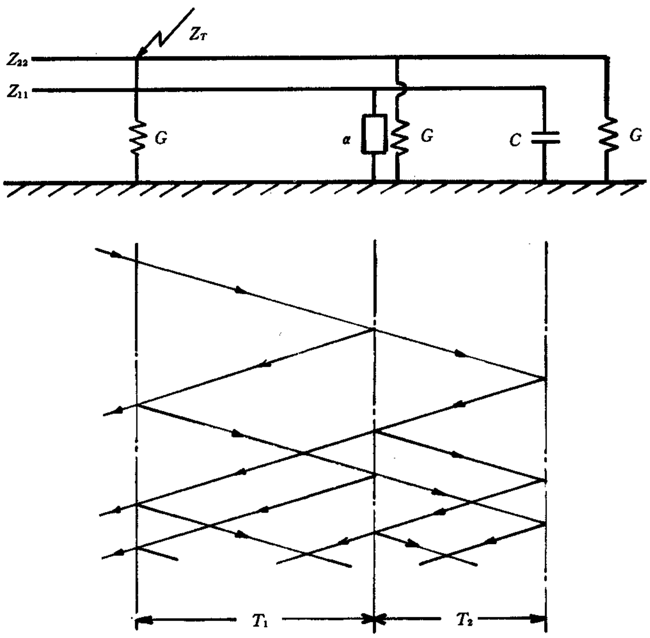

Fig. 3. Equivalent circuit and lattice diagram used to obtain transient voltages at station.

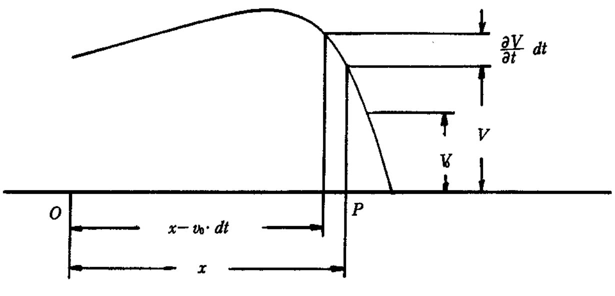

Fig. 4. Wave front.

In Fig. 4, if the point P is taken at a fixed distance behind the origine and the wave is moving at a constant velocity v_0, then the voltage becomes, after dt time,

$$V + v_0 \frac{\partial V}{\partial x} dt$$

Thus the change of electro-magnetic energy stored in the space is

$$W_1 = c(V + v_0 \frac{\partial V}{\partial x} dt)^2 - c(V + \frac{\partial V}{\partial t} dt)^2 \tag{2}$$

in which c is the capacitance to ground per meter of line. This must be equal to the rate of energy change due to corona;

$$W_1 = K(V + \frac{\partial V}{\partial t} dt - V_0)^2 - K(V - V_0)^2 \tag{3}$$

Equating (2) and (3), and neglecting the higher orders of $(\frac{\partial V}{\partial t})$, there results

$$\frac{\partial V}{\partial t} = \frac{v_0}{1 + \frac{K}{c}(1 + \frac{V_0}{V})} \frac{\partial V}{\partial x} \tag{4}$$

It is evident that equation (4) shows that for the voltage more than V_0, its propagation velocity v_e is

$$v_e = \frac{v_0}{1 + \frac{K}{c}(1 - \frac{V_0}{V})} \tag{5}$$

In other words, the voltage on the front of the wave travels at a velocity which is slower than the velocity v_0 for the case ignoring the corona loss. Now, voltage ordinate travels at the propagation time T in considering no corona, and lags its original position in time τ. Hence

$$v_e(T + \tau) = v_0 T \tag{6}$$

$$\tau = \frac{K}{c} \cdot T(1 - \frac{V_0}{V}) \tag{7}$$

For our purpose the attenuation, which waves moving along the lines suffer, will be assumed to follow the Foust and Menger empirical formula;[3)]

$$\rho_{(t)} \cdot E = \frac{E}{1 + k v_0 T E} \tag{8}$$

where $\rho_{(t)}$ and k denote the attenuation factor and constant (approximately from 0.02×10^{-6} to 0.14×10^{-6}), respectively. Thus the higher the voltage, the greater the attenuation.

(2) Ideal Lines

Fig. 3 illustrates the part played by the attenuation and successive reflections in operation of the arrester. For the sake of simplicity the attenuation has been ignored. Without attenuation the cycle of oscillations repeats indefinitely, but when line losses are present the oscillations gradually diminish untill the line eventually reaches a steady state.

Refering to Fig. 3 the voltage of the apparatus to be protected at various instants of time is

$$e_c=\frac{2e}{\underline{s}+\theta}\Big[\Big\{(\alpha_1+\beta_1)\theta+(\gamma_1+\delta_1)\varphi\Big\}\varepsilon^{-s(T_1+T_2)}+\Big\{(\eta_1+\eta_2)\theta+(\eta_3+\eta_4)\varphi\Big\}\varepsilon^{-s(3T_1+T_2)}$$

$$+\frac{1}{\underline{s}+\theta}\Big\{\overline{(\eta_5'+\eta_6'\cdot\theta+\overline{\eta_7'+\eta_8'}\cdot\varphi)}\cdot s+(\overline{\xi_5+\xi_6}\cdot\theta+\overline{\xi_7+\xi_8}\cdot\varphi)\Big\}\varepsilon^{-s(T_1+3T_2)}+\cdots\cdots\Big] \quad (9)$$

where

$$\eta_5'=2\alpha_1\beta_1 p-\alpha_1\alpha_1'+\beta_1\gamma_1 k' \qquad \alpha_1=\frac{2}{\varDelta}(2+GZ_{22})$$

$$\eta_6'=2\beta_1^2 p-\alpha_1'\beta_1+\beta_1\delta_1 k'$$

$$\eta_7'=2\alpha_1\delta_1' p-\alpha_1\delta_1+\gamma_1\delta_1' k' \qquad \beta_1=-\frac{2}{\varDelta}GZ_{12}$$

$$\eta_8'=2\beta_1\delta_1' p-\beta_1\gamma_1+\delta_1\delta_1' k'$$

$$\xi_5=\alpha_1\alpha_1'\theta+2\alpha_1'\gamma_1\varphi+\beta_1\gamma_1 k'\phi' \qquad \gamma_1=-\frac{2}{\varDelta}\alpha Z_{12}$$

$$\xi_6=\alpha_1'\beta_1\theta+2\alpha_1'\delta_1\varphi+\beta_1\delta_1 k'\phi'$$

$$\xi_7=\alpha_1\gamma_1\theta+2\gamma_1^2\varphi+\gamma_1\delta_1' k'\phi' \qquad \delta_1=\frac{2}{\varDelta}(2+\alpha Z_{11})$$

$$\xi_8=\beta_1\gamma_1\theta+2\gamma_1\delta_1\varphi+\delta_1\delta_1' k'\phi' \qquad \varDelta=(2+\alpha Z_{11})(2+GZ_{22})-\alpha GZ_{12}^2$$

$$\theta=\frac{1+GZ_{22}}{C\{Z_{11}+G(Z_{11}Z_{22}-Z_{12}^2)\}} \qquad \varphi=\frac{-GZ_{12}}{C\{Z_{11}+G(Z_{11}Z_{22}-Z_{12}^2)\}}$$

$$p=\frac{-Z_{12}}{Z_{11}+G(Z_{11}Z_{22}-Z_{12}^2)} \qquad k''=\frac{Z_{11}}{Z_{11}+G(Z_{11}Z_{22}-Z_{12}^2)}$$

$$\phi=\frac{1}{CZ_{11}}\ ,\quad k'=2k''-1\ ,\qquad \phi'=\frac{2k\phi-\theta}{2k-1}$$

in which

Z_{11} = self surge impedance of the transmission line

Z_{22} = self surge impedance of the ground wire

Z_{12} = mutual surge impedance between the ground wire and the line wire

$1/G$ = tower footing resistance

C = equivalent capacitance of the station apparatus

α = instantaneous resistance of the arrester.

The negative traveling wave caused by the protective performance of the arrester is

$$E_c'=2(\theta+\varphi\frac{Z_{12}}{Z_{11}})\int_0^t E_f\,(t-\tau)\cdot\varepsilon^{-\theta\tau}\cdot d\tau \quad (10)$$

where $E_{f(t)}$ means the fictious voltage wave nullified or canceled by arrester discharging operation.

(3) Lines with Corona Loss

When the surge of voltage e moves along the line of distance l and arrives at the station, a reflection occurs. Thus, the first thing that is necessary to (shearing back of the wave front is to retard each voltage ordinate by the in (1), and the crest voltage at the station becomes

$$e_2=b'e=\frac{1+be}{1+kle}$$

where b' denotes a reflection index.

Wave arriving back at striken tower from the station are reflected therefrom as

$$e_1' = b'' e$$

and the same thing happens at all the reflection points. By means of lattices the potential at any point at any time may be readily calculated.

3. Numerical Examples

To estimate the surge voltage due to lightning stroke it is not permissible to assume arbitrary values for any of the paramerers in the above equations. As a numerical example, for two conductors let

Z_{11}=500 (Ω) Z_{12}=125 (Ω)

Z_{22}=500 (Ω) Z_T =400 (Ω)

G =1/30 (℧)

Taking a simple exponential incident wave having 1.5μs front,

$$E = 100(\varepsilon^{-0.041t} - \varepsilon^{-1.15t}) \quad (kV)$$

in which t is in μs.

The ideal arrester would not allow the surge voltage to rise appreciably above normal operating voltage. The 70 kV arrester, which is used in modern power system, possesses the characteristic α =0.000125 mhos. Thus the potential at the point to be protected has been plotted in Fig. 5,6 and 7 for the direct stroke near the station, and in these figures there is a comparison between results obtained with and without corona.

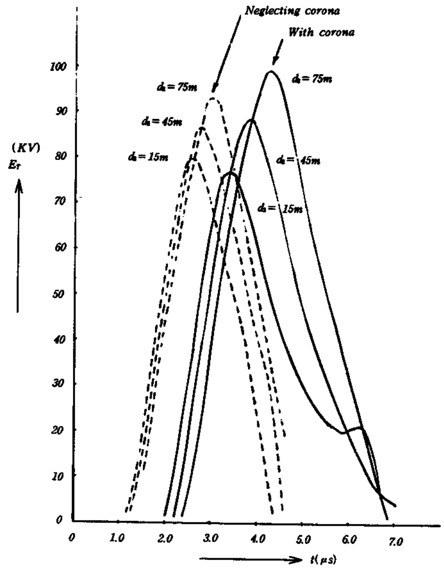

Fig. 5. Potentials at the protected apparatus as functions of time, where d_2' s are the distance from arrester to transformer. C = 1000μF.

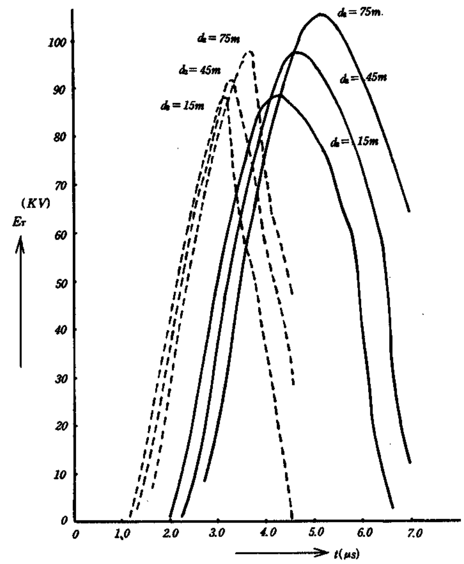

Fig. 6. Potentials at the protected apparatus as functions of time, where d_2' s are the distance from arrester to transformer. C = 2500 μF.

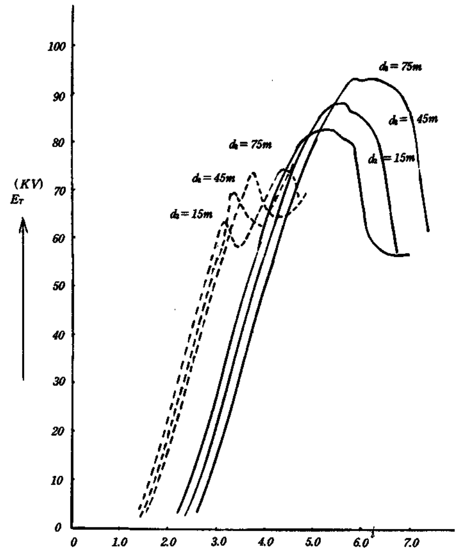

Fig. 7. Potentials at the protected apparatus as functions of time, where d_2' s are the distance from arrester to transformer. C = 4000 μF.

4. Conclusions

As a result of numerical calculations and theory, the following conclusions have been established concerning the station protection:

(1) Effect of distance from arrester to apparatus to be protected

In order that the arrester may be most sufficient, it should be placed as close as possible to the apparatus it is to protect. Above figures show the effect of installing the arrester at same distance from the point to be protected for a rate of rise (kV/μ sec) and complete reflection.

(2) Effect of equivalent capacitance

Generally speaking, the capacitances are very effective in reducing the surge voltages, but the entire station in the aggregate and the reflections is nothing more complicated than a very simple situation of a capacitance.

(3) Effect of Corona

The results obtained by Menger and Skilling's formula do not agree perfectly with those ignoring the corona loss. This difference between these two pairs is great and is of practical importance. The Foust and Menger formula is the simplest for estimating the attenuation, but the Skilling's formula is easier to operate upon mathematically.

Although these formulas have been adopted in this analysis, it should be noticed that all of them are based of the assumption that the line length at the station is less than 100 meters; in other words, Foust and Menger empirical formula may be ignored. Thus all that is necessary to estimate the voltage at protective apparatus is to retard each voltage ordinate on the front of the wave by amount that given in equation (7). Moreover, the experimental constants in equation (8) have to be determined from tests on the transmission line in question, and under the actual conditions that are to prevail.

References

1) L.V.Bewley, Traveling Waves on Transmission Systems, p.35, Dover Publication, Inc., New York (1951).

2) M.Kido, Y.Inagaki and Y.Shirao. Bull. Univ. Osaka Pref., A **15**,1 (1966).

3) L.V.Bewley, Traveling Waves on Transmission Systems, p.389, Dover Publication Inc., New York (1951).