Design of Vibration Absorbers minimizing Human Discomfort

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This paper is concerned with the synthesis of vibration absorber minimizing the discomfort felt by a passenger on a vehicle under random excitation. Optimum control theory in frequency domain is applied to solve the minimizing problem. Theoretical implementation is accomplished for a single-degree-of-freedom system.

1. Introduction

A purpose of the suspension system for a passenger vehicle is to reduce the discomfort felt by a passenger. In our previous papers,[1,2] an optimum absorber minimizing the absolute acceleration of the vibratory mass was analytically studied. It is found, however, that such an absorber does not necessarily minimize the human discomfort. When a passenger vehicle is designed, the human discomfort should be minimized rather than the absolute acceleration of the vehicle. The human discomfort generally depends on the human response to vibration. A human discomfort criterion has been established for harmonic vibrations, but has not yet established for random vibrations[3]. A random vibration, however, can be considered to be a sum of an infinite number of harmonic vibrations of appropriate amplitude and phase. Thus, the criterion established for the harmonic vibration may be extended to a random vibration.

In this paper, a criterion is developed to design the absorbers minimizing the discomfort under random vibrations. Referring to the human vibration sensitivity curve, a filter or a discomfort criterion function is proposed to represent the transfer function from the vibration acceleration, to which a passenger is exposed, to the acceleration felt by the passenger i.e., the perceived acceleration. The optimum vibration absorber for a passenger vehicle is defined as an absorber minimizing the perceived acceleration. We apply Chang's optimum control theory[4] to the optimization. Theoretical implementation is proposed on a single-degree-of-freedom system. The performance of the optimum absorber thus obtained is compared with that of the absorber minimizing the absolute acceleration of the vibratory mass.

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2. Single-Degree-of-Freedom System

Let us design the vibration absorbers that will minimize the discomfort felt by a passenger on the vehicle which is primarily undergoing vertical vibrations. A schematic diagram of the simple dynamical model of the vehicle is shown in Fig. 1. The vibratory mass consists of the mass of the vehicle body and passengers. The absorber, whose configuration is not specified, is idealized as a massless element; providing forces between the body and the foundation. The foundation is subjected to the stationary random excitation. The equation of motion of the system is

$$m\ddot{x}_1(t)=f(t) \tag{2.1}$$

where $f(t)$ is the force generated by the absorber. The characteristic of the candidate absorbers to be synthesized is assumed to be linear. Then, the absorbing force $f(s)$ may be characterized by the relation:

$$f(s)=mF(s)\ddot{x}_0(s) \tag{2.2}$$

where $F(s)$ is an unspecified transfer function and s is the Laplace transform variable. Taking the Laplace transform of Eq. (2.1) and substituting it into Eq. (2.2), we obtain the following transfer functions:

$$\frac{\ddot{x}_1(s)}{\ddot{x}_0(s)}=F(s) \tag{2.3}$$

$$\frac{x_r(s)}{\ddot{x}_0(s)}=\frac{1}{s^2}(F(s)-1) \tag{2.4}$$

where $x_r(=x_1-x_0)$ is the relative displacement between the mass and the foundation. The perceived acceleration representing the discomfort felt by a passenger is defined by (see Appendix):

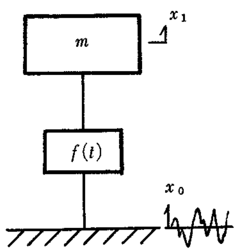

Fig. 1 Single-degree-of-freedom system

$$\ddot{y}(s) \quad G(s)\ddot{x}_1(s)=G(s)F(s)\ddot{x}_0(s) \tag{2.5}$$

where $\ddot{y}(s)$: perceived acceleration,

$G(s)$: transfer function from the vibration acceleration to the acceleration perceived by a passenger.

Let us define the problem "Design the linear vibration absorber so that the perceived acceleration may be minimized with a prescribed bound on the relative displacement." This is formulated as the optimum control problem:

"Under the constraint

$$\langle x_r^2 \rangle \leq M \tag{2.6}$$

synthesize the vibration absorber to minimize the performance index:

$$I=\langle \ddot{y}^2 \rangle + \lambda^2 \langle x_r^2 \rangle \tag{2.7}$$

where M : given constant,

λ^2 : Lagrange's undetermined multiplier,

$\langle \; \rangle$: time average."

Let $\phi_{\ddot{y}}$, ϕ_{x_r} and $\phi_{\ddot{x}_0}$ be the power spectral densities of $\ddot{y}$, x_r and $\ddot{x}_0$, respectively. Using the well-known relationship between the power spectral densities of the input and the output, we have

$$\phi_{\ddot{y}}=GF\bar{G}\bar{F}\phi_{\ddot{x}_0} \tag{2.8}$$

$$\phi_{x_r}=\frac{1}{s^4}(F-1)(\bar{F}-1)\phi_{\ddot{x}_0} \tag{2.9}$$

where the shorthand notation F, G, $\bar{F}$ and $\bar{G}$ are used to represent $F(s)$, $G(s)$, $F(-s)$ and $G(-s)$, respectively. Thus, the mean square values can be calculated by the following relations:

$$\langle \ddot{y}^2 \rangle = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \phi_{\ddot{y}} ds \tag{2.10}$$

$$\langle x_r^2 \rangle = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \phi_{x_r} ds. \tag{2.11}$$

Substituting the above equations into Eq. (2.7), we get

$$I = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} (\phi_{\ddot{y}} + \lambda^2 \phi_{x_r}) ds. \tag{2.12}$$

The problem now is to find the unknown transfer function $F(s)$ so as to minimize the performance index Eq. (2.12). Applying the optimization theorem by S.S.L. Chang[4)], we get the condition for I to be minimum:

$$\frac{\partial \phi_{\ddot{y}}}{\partial \bar{F}} + \lambda^2 \frac{\partial \phi_{x_r}}{\partial \bar{F}} = R \tag{2.13}$$

where R is a function which does not possess any pole in the LHP (left-half-plane).

Substituting Eqs. (2.8) and (2.9) into Eq. (2.13) yields

$$\frac{\phi_{\ddot{x}_0}}{s^4}\{(G\bar{G}s^4+\lambda^2)F-\lambda^2\}=R \tag{2.14}$$

Solving formally Eq. (2.14), we have

$$F=\frac{Rs^4/\phi_{\ddot{x}_0}+\lambda^2}{G\bar{G}s^4+\lambda^2}. \tag{2.15}$$

If the functional form of $\phi_{\ddot{x}_0}$ is specified, the complete from of $F(s)$ can be determined by the procedure developed in our previous papers.[1,2]

Let us now take the simplest case of white noise excitation:

$$\phi_{\ddot{x}_0}=S_0=\text{const.} \tag{2.16}$$

Then, Eq. (2.15) is reduced to the following equation:

$$F=\frac{A_3s^3+A_2s^2+A_1s+A_0}{(s-r_1)(s-r_2)(s-r_3)(s-r_4)} \tag{2.17}$$

where A_0, A_1, A_2 and A_3 are undetermined coefficients and r_1, r_2, r_3 and r_4 are the roots of equation $G\bar{G}s^4+\lambda^2=0$, which are in the LHP. Substituting Eq. (2.17) into Eq. (2.14), we have

$$\frac{S_0}{s^2(-s)^2}\cdot\frac{T_2^2T_4^2}{(-T_2^2s^2+1)(-T_3^2s^2+1)}\{(s+r_1)(s+r_2)(s+r_3)(s+r_4)$$
$$\times(A_3s^3+A_2s^2+A_1s+A_0)-\lambda^2\frac{(-T_2^2s^2+1)(-T_3^2s^2+1)}{T_2^2T_4^2}\}=R \tag{2.18}$$

where

$$G(s)=\frac{T_2}{T_1}\cdot\frac{(T_1s+1)(T_4s+1)}{(T_2s+1)(T_3s+1)} \tag{2.19}$$

(see appendix).

In the above equation (2.18), the coefficients A_0, A_1, A_2 and A_3 are determined so that all the poles in the LHP may be cancelled out. Therefore, we have

$$\begin{aligned}
A_0&=r_1r_2r_3r_4\triangleq\gamma_0\\
A_1&=-\{r_1r_2(r_3+r_4)+r_3r_4(r_1+r_2)\}\triangleq\gamma_1\\
A_2&=(T_2+T_3)\gamma_1-(T_2^2+T_2T_3+T_3^2)\gamma_0\\
A_3&=T_2T_3\gamma_1-T_2T_3(T_2+T_3)\gamma_0.
\end{aligned}$$

Now, we have completely determined the transfer function $F(s)$.
Substituting $F(s)$ thus obtained into Eq. (2.2), we get the absorbing force $f(s)$:

$$f(s)=m\frac{A_3s^3+A_2s^2+A_1s+A_0}{s^4+\gamma_3s^3+\gamma_2s^2+\gamma_1s+\gamma_0}\ddot{x}_0(s) \tag{2.20}$$

where

$$\begin{aligned}
\gamma_2&\triangleq r_1r_2+r_3r_4+(r_1+r_2)(r_3+r_4)\\
\gamma_3&\triangleq-(r_1+r_2+r_3+r_4).
\end{aligned}$$

The performance of this system will be discussed in the following section.

3. Comparison of the Performance between the Optimum System and Other System

In the previous paper[1)], we proposed the absorber configuration which minimized the absolute acceleration of the vibratory mass as shown in Fig. 2. In this paper, the absorber configuration is synthesized so as to minimize the perceived acceleration. Let us now compare the performance of the two systems from the view point of passenger comfort.

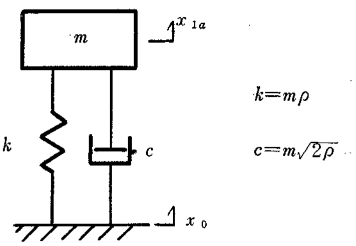

Fig. 2 Optimum suspension system minimizing the absolute acceleration

As shown in the previous paper[1)], the transfer function of the absorber minimizing the absolute acceleration is given by

$$F_a(s)=\frac{\sqrt{2\rho}s+\rho}{s^2+\sqrt{2\rho}s+\rho} \tag{3.1}$$

and the transfer functions of the system are

$$\frac{\ddot{x}_{1a}(s)}{\ddot{x}_0(s)}=\frac{\sqrt{2\rho}s+\rho}{s^2+\sqrt{2\rho}s+\rho} \tag{3.2}$$

$$\frac{x_{ra}(s)}{\ddot{x}_0(s)}=\frac{-1}{s^2+\sqrt{2\rho}s+\rho} \tag{3.3}$$

where ρ^2 is a Lagrange's undetermined multiplier and $x_{ra}(=x_{1a}-x_0)$ is the relative displacement. Thus, the perceived acceleration of the system can be calculated by the relation:

$$\ddot{y}_a \triangleq G\ddot{x}_{1a}=\frac{T_2}{T_1}\cdot\frac{(T_1s+1)(T_4s+1)}{(T_2s+1)(T_3s+1)}\cdot\frac{\sqrt{2\rho}s+\rho}{s^2+\sqrt{2\rho}s+\rho}\ddot{x}_0. \tag{3.4}$$

From Eq. (2.5), we have the perceived acceleration of the system synthesized in the preceding section:

$$\ddot{y}=\frac{T_2}{T_1}\cdot\frac{(T_1s+1)(T_4s+1)}{(T_2s+1)(T_3s+1)}\cdot\frac{A_3s^3+A_2s^2+A_1s+A_0}{s^4+r_3s^3+r_2s^2+r_1s+r_0}\ddot{x}_0. \tag{3.5}$$

For the fair judgement of the performance of the two systems, the values of $\ddot{y}$ and $\ddot{y}_a$ must be compared under the same condition.

In the following, comparison of the performance of the two systems is done when the mean square values of the relative displacement are equal. Thus, we set

$$<x_r^2>=<x_{ra}^2> \tag{3.6}$$

and determine the relation between the Lagrange's multipliers ρ and λ. Substituting the above relation into Eqs. (3.4) and (3.5), we get the perceived accelerations of the two systems. The values of the perceived accelerations and the relative displacements are calculated and plotted in Figs. 3 and 4, where the quantities are transformed to the nondimensional ones by the use of the constant S_0 and the frequency $\omega_2(=1/T_2)$. From Fig. 3, we see that the value of $\ddot{y}$ is about 92% of the value of $\ddot{y}_a$ when the relative displacements of the both systems are set to be equal. Therefore, the vehicle with the absorber minimizing the perceived acceleration is more comfortable than that with the absorber minimizing the absolute acceleration.

The final step to be done for the determination of the optimum configuration is to specify the value of the Lagrange's multiplier λ^2. It can be done with the aid of Fig. 4 once the constraint value M for the relative displacement is given.

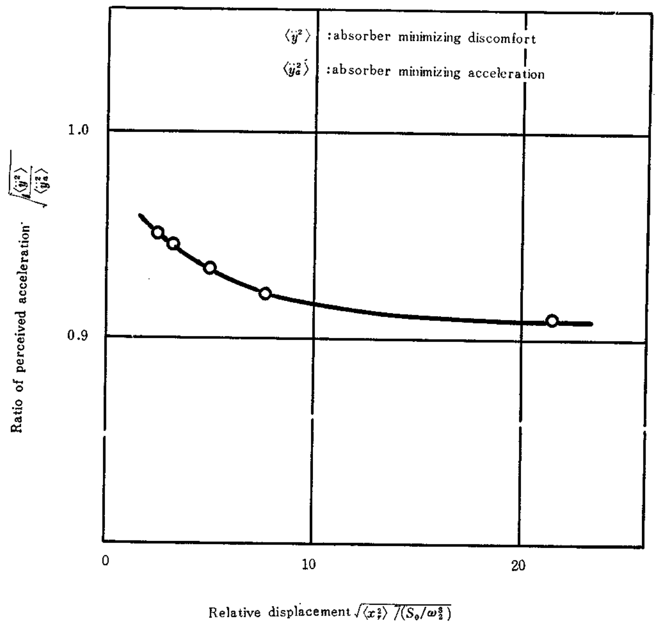

Fig. 3 Ratio of perceived acceleration of the absorbers

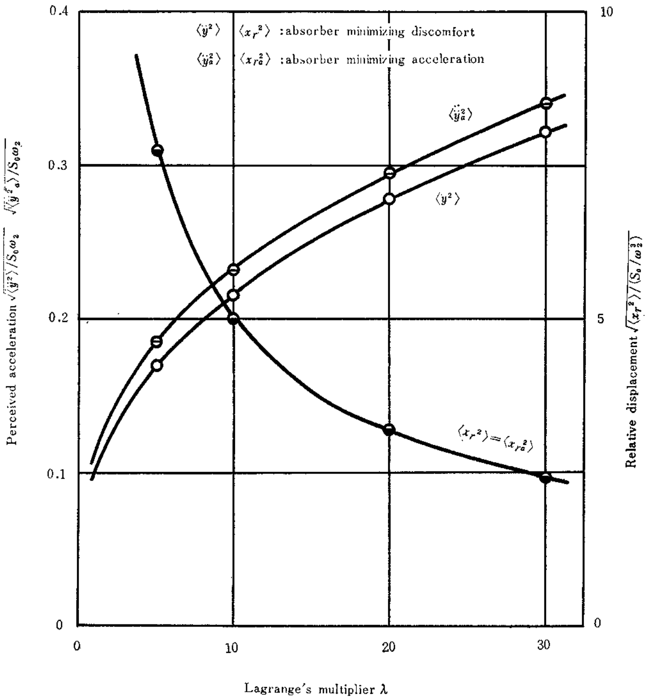

Fig. 4 Performance of the optimum absorbers

4. Conclusions

Linear optimum control theory enables us to obtain the optimum vibration absorber minimizing the discomfort felt by a passenger. The theoretical implementation has been accomplished for a single-degree-of-freedom system. As a design example, we have synthesized the optimum absorber for the system with white noise excitation from the foundation. It is found that the optimum absorber thus obtained can only be mechanized with active elements rather than the conventional spring-dashpot elements and that the human discomfort of the vehicle with the active elements' optimum absorber is about 8% smaller than that with the spring-dashpot absorber minimizing the absolute acceleration.

In this paper, no mention has been made of higher degree-of-freedom system. The described technique, however, can readily be extended to such systems. Such a study will be reported in the near future.

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Appendix

In designing the absorber to minimize the passenger discomfort, the criterion for discomfort must be first established. Generally speaking, the human discomfort is dependent on the amplitude and the frequency of the acceleration perceived by a passenger. The perceived acceleration is related to the human response to vibration or human vibration sensitivity. Three types of discomfort criterion have been proposed for representing the human response:

(1) A curve in which the lower limit of the acceleration perceived by a passenger is plotted against frequency,
(2) Frequency response curve of a part of the human body,
(3) A curve plotting an amplitude of acceleration at various frequencies in which a passenger perceives the same level of discomfort.

In this paper, the third curve is used for evaluating the human discomfort. Refering to the standard curve of ISO[5] for the third criterion, we propose the human response curve which is plotted in Fig. 5. As seen from the figure, ISO curve has been defined in the bandwidth: from 0.71 *Hz* to 90 *Hz* and has two corner frequencies—one at $f=2.8$ *Hz* and the other at $f=11.2$ *Hz*. The slope of the curve from 0.71 *Hz* to 2.8 *Hz* is -10 *dB* per decade of frequency and that from 11.2 *Hz* to 90 *Hz* is 20 *dB*. On the other hand, the proposed curve has four corner frequencies at the frequencies of 0.71 *Hz*, 2.8 *Hz*, 11.2 *Hz* and 90 *Hz*. The slope of the

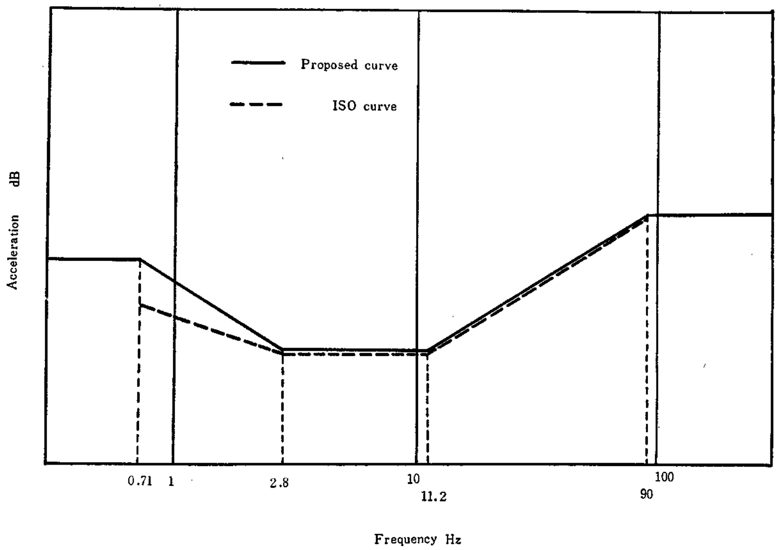

Fig. 5 Human response curve

proposed curve from 0.71 *Hz* to 2.8 *Hz* is different from that of ISO curve. The transfer function from vibration absorber to the perceived acceleration therefore is expressed as follows:

$$G(s)=\frac{T_2}{T_1}\cdot\frac{(T_1s+1)(T_4s+1)}{(T_2s+1)(T_3s+1)}$$

where

$$T_1=1/(2\pi\times0.71)$$
$$T_2=1/(2\pi\times2.8)$$
$$T_3=1/(2\pi\times11.2)$$
$$T_4=1/(2\pi\times90).$$

Fig. 6 is the plot of the transfer function against frequency.

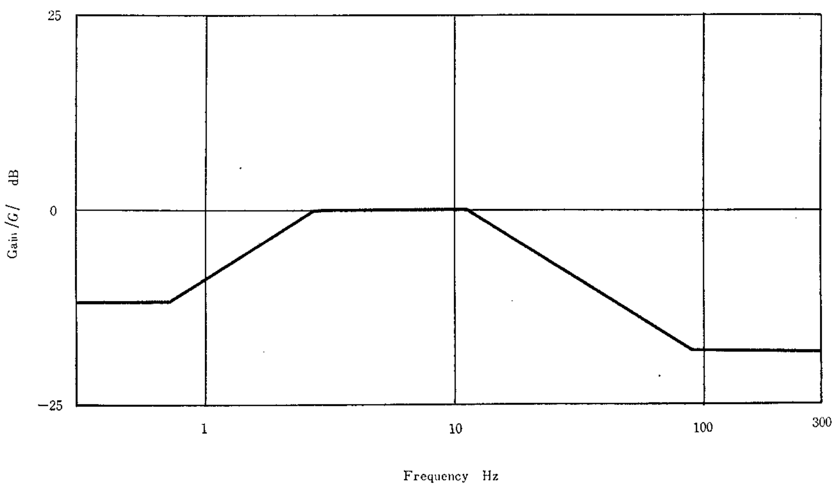

Fig. 6 Human discomfort criterion function