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Generation of Sinusoidal Wave Using SCR Amplifier

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This paper presents a method of generating the sinusoidal signal with the aid of switched mode amplifier having cosine characteristics between the input and the average output voltage. Since the switched mode amplifier shows an excellent ability in the ultra low frequency region owing to its switched mode operation, the method described in this paper is a suitable one for generating a high power sinusoidal signal of ultra low frequency.

In the first place, the principles of waveform conversion using the SCR amplifier with cosine amplification characteristics are introduced. The next place describes the frequency analysis of the output signal and the experimental results which are also satisfactory to the theoretical one. Finally, the distortion of the sinusoidal output signal is studied.

1. Introduction

Recently, an ultra low frequency signal generator has been used for not only a regulator of servo-mechanisms having long period but an instrument of various investigations in the ultra low frequency region such as the study of ferroelectric materials and the medical electronics.

Authors have developed a new type of method to generate the ultra low frequency signal by using the switched mode amplifier of simple circuit. This method is able to permit a high power and ultra low frequency signal generation owing to its switched mode operation.

In the first place, we deal with a method of generating a sinusoidal signal in the ultra low frequency signal generator employing the switched mode operation.

A sinusoidal signal may be sometimes converted from a triangular one by using the electronic devices or circuits of which the characteristics between the input and the output voltage are shown by a cosine curve. It is very useful for generating the sinusoidal signal having long period or high power because of the inherent feature that the large reactive elements are needless for forming the signal.

It is well known that the SCR amplifier, in which the average output voltage (or current) during the half-cycle of the SCR anode supply voltage is considered to be proportional to the input voltage at the conducting instant of SCR, is able to perform a linear power amplification of ultra low frequency signal. By modifying properly the waveform of specified bias voltage of the linear SCR amplifier, it shows the amplification characteristic that is not linear but accurate cosine curve. This paper clarifies the theory and utilization

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on converting a triangular signal into sinusoidal one by making use of the SCR amplifier with cosine amplification characteristics.

2. SCR Amplifier with Cosine Amplification Characteristics and Principles of Waveform Conversion

The basic circuit of the SCR amplifier with cosine amplification characteristics is shown in Fig. 1. Thyristors, SCR_1 and SCR_2, are connected in a full-wave rectifier type with respect to the A.C. power source. The input signal and the specified bias voltage which determines the amplification characteristics are applied to the input terminals of the gate circuit. The circuit of Fig. 1 has the same construction as that of ordinary SCR amplifier

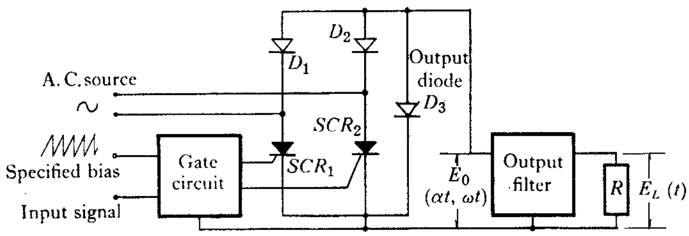


Fig. 1. Basic circuit of SCR amplifier with cosine amplification characteristic.

with linear amplification characteristics except that the specified bias is replaced by a saw-tooth waveform synchronized with the each half-cycle of anode source voltage, $E_m \sin \omega t$, in order to make the cosine amplification characteristics. In this case, the average output voltage during the each half-cycle of the anode source is given by

$$\bar{E}_0 = \frac{1}{\pi} \int_{\frac{\pi}{2}(1-E_s)}^{\pi} E_m \sin \omega t \cdot d\omega t = \frac{E_m}{\pi} \left\{ 1 + \cos \frac{\pi}{2} (1 - E_s) \right\} \quad \cdots\cdots\cdots (1)$$

where E_s is the normalized input signal voltage defined as $-1 \leq E_s \leq 1$. From Eq. (1), it is apparent that the static characteristic of the SCR amplifier employing the saw-tooth bias voltage shows the accurate cosine amplification characteristics.

Then, if the triangular input signal shown in Fig. 2 (b) is fed into the SCR amplifier with cosine amplification characteristics, a sinusoidal signal shown in Fig. 2 (c) is obtained. The relations illustrated in Fig. 2 are the process of the waveform conversion generating a sinusoidal signal from a triangular input signal.

3. Analysis of Output Signal

The method of waveform conversion using the SCR amplifier with cosine amplification characteristics takes an average value of the output signal voltage during each half-cycle of anode source as shown in Fig. 2. From this matter, the following conditions are indispensable for generating the actual sinusoidal wave.

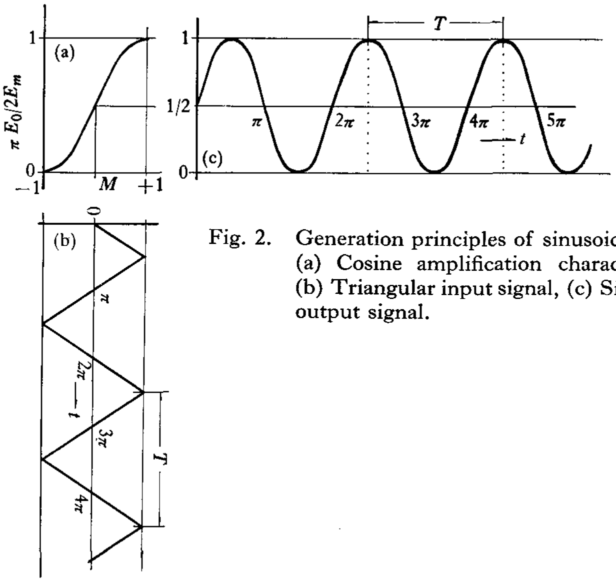


Fig. 2. Generation principles of sinusoidal wave. (a) Cosine amplification characteristics, (b) Triangular input signal, (c) Sinusoidal output signal.

(1) The operation of sampling for the input signal must be sufficient. That is, it is necessary that the period of the anode source frequency of SCR is short enough with respect to that of input signal.

(2) The output filter must be employed to take an average of output signal voltage of the SCR amplifier with cosine amplification characteristics.

This section deals with the analysis of the output signal waveform to investigate these conditions in detail.

In the circuit shown in Fig. 1, the switching elements of SCR_1 and SCR_2 turn on at the instant that the resultant voltage of the saw-tooth bias voltage and the triangular input signal is equal to zero and turn off at the end of each half-cycle of anode source voltage. The output signal voltage across a diode of D_3, in this case, is a train of pulses whose shapes are segments of anode supply voltage as shown in Fig. 3 (a). This output signal waveform can be analyzed by applying a double Fourier series expansion which is a function of periodic variables of triangular input signal and of anode source frequency, as mentioned in a previous paper[1].

Letting that the period of triangular input signal is $T=(2\pi/\alpha)$, and that φ_p is turn-on switching phase angle in each half-cycle of the anode source voltage, the output signal voltage $E_0(\alpha t, \omega t)$ of Fig. 3 (a) is given as

$$E_0(\alpha t, \omega t) = \sum_{\sigma_1=-\infty}^{\infty} \sum_{\sigma_2=-\infty}^{\infty} C_{\sigma_1,\sigma_2} \cdot \exp\left[j(\sigma_1 \alpha t + 2\sigma_2 \omega t)\right] \qquad \cdots\cdots\cdots\cdots (2)$$

And

$$C_{\sigma_1,\sigma_2} = \frac{1}{2\pi^2} \int_0^{2\pi} \int_{\varphi_p}^{\pi} E_0(\xi_1, \xi_2) \cdot \exp\left[-j(\sigma_1 \xi_1 + 2\sigma_2 \xi_2)\right] \cdot d\xi_1 \cdot d\xi_2 \qquad \cdots\cdots\cdots (3)$$

where σ_1, $\sigma_2 = 0, 1, 2, 3, \cdots$, $\xi_1 = \alpha t$, $\xi_2 = \omega t$,

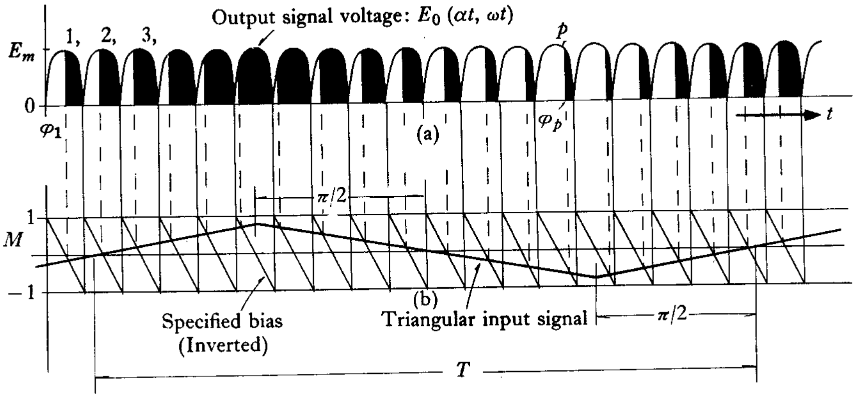


Fig. 3. Relation between input and output signal voltage. (a) Output signal voltage, (b) Input signal and turn-on phase angles.

The turn-on switching phase angle of SCR is determined graphically by the intersection of the triangular input signal and the inverted saw-tooth bias voltage as illustrated in Fig. 3 (b) and satisfies the following relations.

(a) In a part of positive gradient of the triangular input signal. $[0, \pi/2]$

$$\varphi_p = -M\xi_1 + \pi/2 \qquad \cdots\cdots\cdots\cdots (4)$$

(b) In a part of negative gradient. $[\pi/2, 3\pi/2]$

$$\varphi_p = M(\xi_1 - \pi) + \pi/2 \qquad \cdots\cdots\cdots\cdots (5)$$

(c) In a part of positive gradient. $[3\pi/2, 2\pi]$

$$\varphi_p = -M(\xi_1 - 2\pi) + \pi/2 \qquad \cdots\cdots\cdots\cdots (6)$$

where M is the ratio of the amplitude of specified bias voltage to that of triangular input signal and $0 \leq M \leq 1$.

Substituting the turn-on phase angles of Eqs. (4)~(6) into Eq. (3), the Fourier coefficients, C_{σ_1,σ_2}, which indicate the amplitudes of components composing the output voltage are derived as follows;

1) D.C. component. $(\sigma_1 = \sigma_2 = 0)$

$$C_{0,0} = E_m/\pi \qquad \cdots\cdots\cdots\cdots (7)$$

2) Fundamental frequency component of triangular input signal. $(\sigma_1 = 1, \sigma_2 = 0)$

$$C_{1,0} = j\frac{2M \cdot E_m}{\pi^2(M^2-1)} \cdot \cos\frac{M\pi}{2} \qquad \cdots\cdots\cdots\cdots (8)$$

3) Harmonic frequency components of triangular input signal. $(\sigma_1 > 1, \sigma_2 = 0)$

$$C_{\sigma_1,0} = \frac{M \cdot E_m}{\pi^2(M^2-\sigma_1^2)} e^{j\sigma_1(\pi/2)}\{1-(-1)^{\sigma_1}\} \cdot \cos\frac{M\pi}{2} \qquad \cdots\cdots\cdots\cdots (9)$$

4) Even-multiple frequency components of anode source frequency. $(\sigma_1=0,\ \sigma_2\geq 1)$

$$C_{0,\sigma_2}=\frac{E_m}{\pi(1-4\sigma_2^2)}+j\frac{(-1)^{\sigma_2}\cdot 2E_m}{\pi^2(1-4\sigma_2^2)^2\cdot M}\left\{4\sigma_2\cdot\cos\sigma_2 M\pi\cdot\sin\frac{M\pi}{2}\right.$$
$$\left.-(4\sigma_2^2+1)\cdot\sin\sigma_2 M\pi\cdot\cos\frac{M\pi}{2}\right\} \quad\cdots\cdots(10)$$

5) Compound frequency components of even-multiples of anode source frequency and integral multiples of input signal frequency. $(\sigma_1\neq 0,\ \sigma_2\geq 1)$

$$C_{\sigma_1,\sigma_2}=\frac{2M\cdot e^{-j\sigma_2\pi}\cdot E_m}{\pi^2\{M^2-(\sigma_1+2\sigma_2M)^2\}\{M^2-(\sigma_1-2\sigma_2M)^2\}}\left[4\sigma_2M^2\right.$$
$$\cdot\cos(\sigma_1+2\sigma_2M)\frac{\pi}{2}\cdot\sin\frac{M\pi}{2}-\{M^2(4\sigma_2^2+1)-\sigma_1^2\}$$
$$\left.\cdot\sin(\sigma_1+2\sigma_2M)\frac{\pi}{2}\cdot\cos\frac{M\pi}{2}\right]\cdot(\sin\sigma_1\pi+j\cos\sigma_1\pi) \quad\cdots\cdots(11)$$

From these coefficients, it can be seen that the output signal across the output diode is composed of the D.C. component, the frequency components composing the triangular input signal, the even-multiple components of anode source frequency and the components which locate at the distances of integral multiples of fundamental input signal from the position of even-multiple anode source frequency. And the amplitude of these components except the component of D.C. is a function of M.

If the amplitude of the specified bias voltage is equal to that of the triangular input signal voltage as illustrated in Fig. 2, the Fourier coefficients are represented by the following equations by inserting $M=1$ into Eqs. (7)~(11).

1) D.C. component.

$$C_{0,0}=E_m/\pi \quad\cdots\cdots(12)$$

2) Fundamental frequency component of input signal.

$$C_{1,0}=-jE_m/2\pi \quad\cdots\cdots(13)$$

3) Harmonic frequency components of input signal. $(\sigma_1\geq 2)$.

$$C_{\sigma_1,0}=0 \quad\cdots\cdots(14)$$

4) Even-multiple frequency components of anode source frequency.

$$C_{0,\sigma_2}=\frac{E_m}{\pi^2(1-4\sigma_2^2)}\left(\pi+j\frac{8\sigma_2}{1-4\sigma_2^2}\right) \quad\cdots\cdots(15)$$

5) Compound frequency components of even-multiples of anode source frequency and integral multiples of input signal frequency.

5-1) When $\sigma_1=\pm(2\sigma_2+1)$.

$$C_{\sigma_1,\sigma_2}=\mp j(-1)^{\sigma_2}\frac{E_m}{4\pi(1+2\sigma_2)} \quad\cdots\cdots(16)$$

5–2) When $\sigma_1=\pm(2\sigma_2-1)$.

$$C_{\sigma_1,\sigma_2}=\pm j(-1)^{\sigma_2}\frac{E_m}{4\pi(1-2\sigma_2)} \qquad \cdots\cdots(17)$$

5–3) When $\sigma_1\neq\pm(2\sigma_2\pm1)$ and σ_1 odd.

$$C_{\sigma_1,\sigma_2}=0 \qquad \cdots\cdots(18)$$

5–4) When σ_1 even.

$$C_{\sigma_1,\sigma_2}=j(-1)^{\sigma_1/2}\frac{8\sigma_2\cdot E_m}{\pi^2\cdot[(\sigma_1-2\sigma_2)^2-1][(\sigma_1+2\sigma_2)^2-1]} \qquad \cdots\cdots(19)$$

By making use of the above calculated results, the output signal voltage $E_0(\alpha t,\ \omega t)$ yielded when the triangular input signal is fed into the SCR amplifier with cosine amplification characteristics is given, as long as $M=1$ is satisfied, by

$$\begin{aligned}\frac{E_0(\alpha t,\ \omega t)}{E_m}&=\frac{1}{\pi}+\frac{1}{\pi}\sin\alpha t\\&+\sum_{\sigma_2=1}^{\infty}\frac{2}{\pi^2(1-4\sigma_2^2)}\sqrt{\left(\frac{8\sigma_2}{4\sigma_2^2-1}\right)^2+\pi^2}\cdot\sin(2\sigma_2\omega t+\theta)\\&+\sum_{\sigma_2=1}^{\infty}\frac{(-1)^{\sigma_2}}{2\pi\cdot(1-2\sigma_2)}[\sin\{2\sigma_2\omega-(2\sigma_2-1)\alpha\}t-\sin\{2\sigma_2\omega\\&+(2\sigma_2-1)\alpha\}t]+\sum_{\sigma_2=1}^{\infty}\frac{(-1)^{\sigma_2}}{2\pi(1+2\sigma_2)}[\sin\{2\sigma_2\omega+(2\sigma_2+1)\alpha\}t\\&-\sin\{2\sigma_2\omega-(2\sigma_2+1)\alpha\}t]+\sum_{\sigma_2=1}^{\infty}\sum_{\sigma_1/2=\pm1}^{\pm\infty}(-1)^{(\sigma_1+2)/2}\\&\cdot\frac{16\sigma_2}{\pi^2\cdot[(\sigma_1-2\sigma_2)^2-1][(\sigma_1+2\sigma_2)^2-1]}\cdot\sin(2\sigma_2\omega+\sigma_1\alpha)t \qquad \cdots\cdots(20)\end{aligned}$$

where

$$\theta=\tan^{-1}\cdot(4\sigma_2^2-1)\pi/8\sigma_2 \qquad \cdots\cdots(21)$$

That is, the harmonic components of the fundamental frequency of triangular input signal do not appear in the output signal provided that the cosine amplification characteristics SCR amplifier is driven by the triangular input signal under the condition of $M=1$. Comparing this matter with that for the case of $M\neq1$, there is a obvious improvement of the output sinusoidal waveforms.

By way of example, the output frequency spectrum for the case of $M=1$ calculated by Eq. (20) is shown in Fig. 4. These facts are also supported by the experimental results. In this figure, the maximum value of the anode source voltage is taken for unity and the ratio of anode source angular frequency to fundamental angular frequency of the triangular input signal is 60/7.5. The marks, $\times$, in Fig. 4 indicate the experimental results measured with a selective level meter.

Since the harmonics of the fundamental frequency of triangular input signal in this case are not in existence, the fundamental signal frequency component which locates

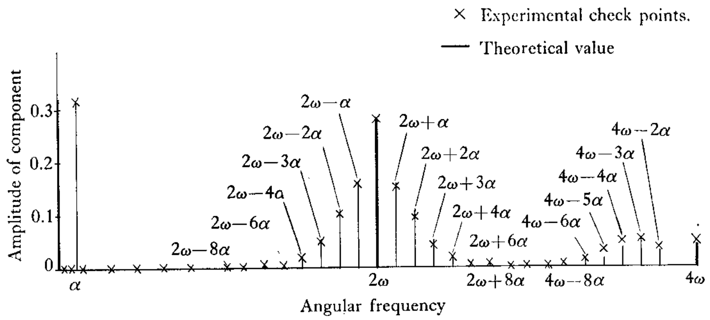


Fig. 4. Frequency spectrum of output signal voltage for $M=1$, $E_m=1$, $\omega/\alpha=60/7$.

in a low frequency region can be filtered with facility from the undesirable components (even-multiple harmonics of anode source frequency and the harmonics located at the distances of integral multiples of the fundamental frequency of the triangular input signal from the position of even-multiple anode source frequency). That is, the actual sinusoidal wave having an angular frequency corresponding to the recurring period of triangular input signal can be obtained by using a suitable low pass filter.

4. Frequency Region of Waveform Conversion and Output Filter

References of the theoretical results make it possible to examine a nature of each component composing the output voltage. From Eqs. (12)~(19), when σ_1 is odd number, only the components of $C_{\pm(2\sigma_2\pm1),\sigma_2}$ come into existence. These amplitudes and the values of C_{0,σ_2} decrease uniformly according to the increase of σ_2. When σ_1 is even number, $C_{\pm2\sigma_2,\sigma_2}$ for each σ_2 has the maximum value which does not exceed the value of $16\ \sigma_2/\pi^2(16\ \sigma_2^2-1)$. The frequency components having relatively high frequency among these undesirable components are much smaller than that corresponding to the fundamental frequency of triangular input signal and distribute in far distance from it. Therefore, it is considered that the undesirable components which locate in the vicinity of the second harmonic component of anode source frequency have principally influence on the fundamental signal frequency of α. As shown in Fig. 4, the amplitude of $C_{\sigma_1,1}$ decreases rapidly according to the increase of σ_1 and the value of $C_{6,1}$ is less than -50db compared with that of α component. This means that the undesirable components in the frequency region lower than $2\omega-6\alpha$ may be neglected in a practical use.

While, the variation of ω/α shifts only the relative position of each frequency component composing the output voltage and these amplitudes do not vary with it unless M is amended. If the ω/α is large, in other words, the recurring frequency of triangular input signal is relatively low as shown in Fig. 4, the undesirable frequency components concentrate in the vicinity of even-multiples of anode source frequency and can be eliminated from the output voltage by employing a suitable low pass filter. Consequently, the output signal of the filter is considered to be a pure sine wave. However, the undesirable components yielded when the triangular input signal of $\alpha > 2\omega/7$ is fed into the SCR amplifier with cosine amplification characteristics distribute in the region so far from each even-multiple component of anode source frequency and components of $2\omega-5\alpha$ and $2\omega-6\alpha$ may come close to the fundamental signal frequency or get into the frequency region lower than it. In such a case, it becomes difficult to eliminate all of the undesirable frequency components even if the output filter may be employed. And the output signal across the load is not acuurate sinusoidal but distorted one.

From these facts, the period of anode source frequency must be more frequent than 7/2 times with respect to the period of triangular input signal. And, it is necessary that the upper limit of recurring frequency of the triangular input signal is $\omega/7\pi$ (c/s) to separate the α component from the undesirable component of $2\omega-6\alpha$. That is, the frequency region in which the waveform conversion can be performed without distortion is 0 (c/s)$\sim\omega/7\pi$ (c/s).

On the other hand, the output filter inserted between the SCR amplifier and the load must remove the undesirable components in the frequency region higher than $\omega/7\pi$ (c/s), as mentioned above. For the practical criterion, we consider such attenuation characteristics of the output filter that the each amplitude of the undesirable component is less than -50db compared with that of sinusoidal output signal. Then, the output filter may be defined by stipulating the following conditions.

(1) The cutoff frequency is about $\omega/7\pi$ (c/s).

(2) The attenuation at the point of 2ω is more than 47db, of which the slope is more than 20db per octave, and in the frequency region higher than 4ω, it approaches 30db$\sim$40db. The low pass filter having Wagner-Tchebycheff's attenuation characteristics is available.

(3) The values of elements composing the filter must be chosen so that the SCR amplifier may behave in normal manners, in other words, the output voltage appeared as a train of segments of the anode source voltage may not be disturbed.

Photos. 1 and 2 show the experimental results. It may be seen from these results that

(A)

(B)

(C)

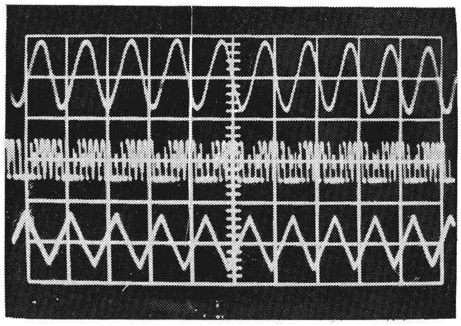


Photo 1. Oscillograms of waveform conversion. ($\alpha/2\pi=16$ (c/s)). (A) Sinusoidal output voltage (20 V/div), (B) Waveform of $E_0(\alpha t, \omega t)$ (140 V/div), (C) Triangular input signal (1 V/div).

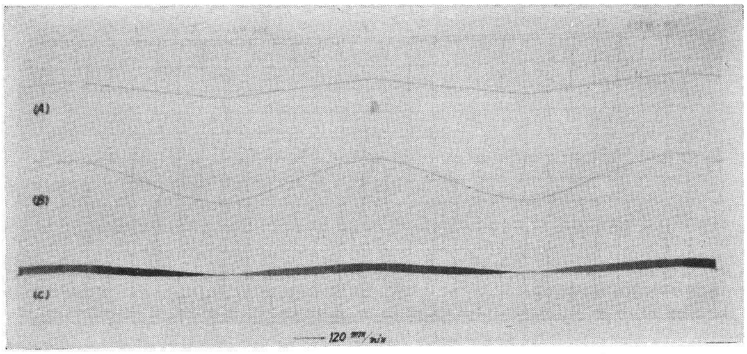


Photo. 2. Oscillograms of waveform conversion ($\alpha/2\pi \fallingdotseq 1.5\times 10^{-2}$ (c/s)).
(A) Triangular input signal, (B) Sinusoidal output signal,
(C) Waveform of $E_0(\alpha t, \omega t)$.

the above theory is appropriate. Furthermore, the method of waveform conversion presented in this paper is suitable for generating the sinusoidal signal of ultra low frequency. In the experiment, a derived-m low pass filter which satisfies the above conditions is used as the output filter.

5. Distortion of Sinusoidal Output Signal

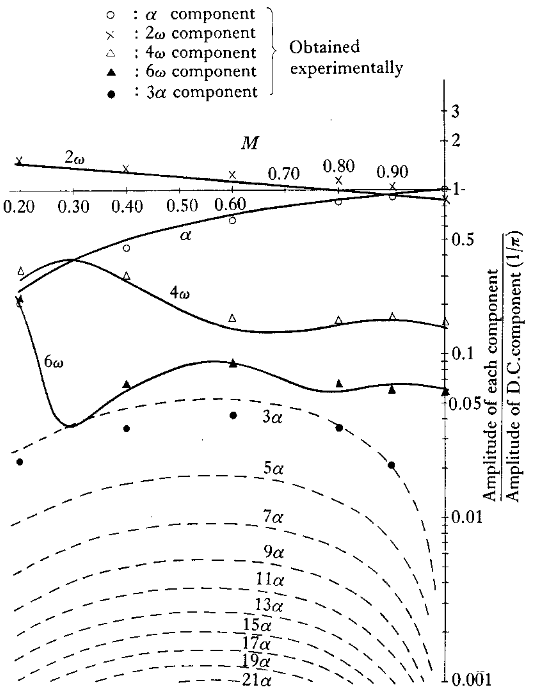
As long as $M=1$ is satisfied, it is considered that the output signal of the low pass filter which removes the undesirable components in the frequency region higher than $\omega/7\pi$ (c/s) presents the sine wave. However, for the case that the amplitude of specified bias voltage is not equal to that of triangular input signal, the odd-multiple harmonics of α given in Eq. (9) appear through the output filter. For these matters, the sinusoidal output signal is principally distorted owing to the amplitudes of odd-multiple harmonics of the fundamental angular frequency and the undesirable components ($C_{\sigma_1>7\sigma_2-1,\sigma_2}$) in the frequency region lower than the cutoff frequency of the output filter. And, the distortion factor $\varDelta$ which estimates the sinusoidal output signal is given by

$$\varDelta=\frac{\sqrt{\sum_{(\sigma_1-1)/2=1}^{r} C^2_{\sigma_1,0}+\sum_{\sigma_2=1}^{\infty}\sum_{\sigma_1=m}^{n} C^2_{\sigma_1,\sigma_2}}}{C_{1,0}}\times 100\ \% \qquad \cdots\cdots\cdots\cdots\cdots(22)$$

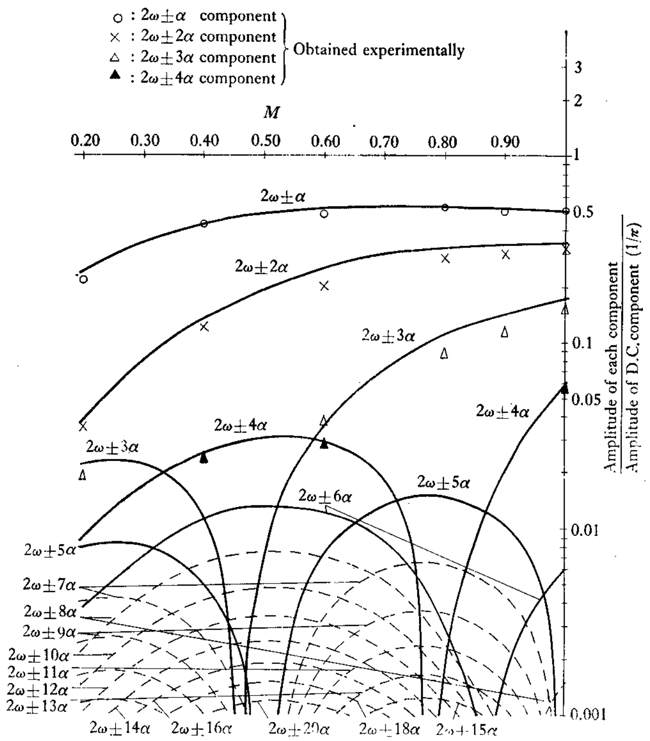
where r is the integer which gives the maximum order of odd-multiple harmonics of fundamental input frequency being distributed in the transmission band of the output filter. Similarly, n and m indicate the maximum and the minimum order of difference frequency components of even-multiple anode source frequency and integral multiple frequency of triangular input signal in the same transmission band.

(5-1) The case of $M<1$.

As described in the section **3**, the amplitudes of the components composing the input signal voltage $E_0(\alpha t, \omega t)$ of the output filter can be derived by making use of Eqs. (7)~(11).



(a) $\sigma_1\alpha$, $2\sigma_2\omega$ components.



(b) $2\omega \pm \sigma_1\alpha$ components.

Fig. 5. Amplitudes of major components of output signal as a function of M.

Figs. 5 (a) and (b) show the amplitudes of the major components calculated with a digital computer for the variation of M. In these figures, each value is normalized with respect to the amplitude of D.C. component and some experimental results measured with a selective level meter are plotted for the powerful components. The undesirable components indicated with solid lines can be eliminated by the output filter. And with dotted lines, the amplitudes of the components causing the distortion to the sinusoidal output signal are presented. The components of $4\omega\pm\sigma_1\alpha$, $6\omega\pm\sigma_1\alpha$, $\cdots$ etc. are not shown for making the figure simpler. The amplitudes which are seemed to cause the distortion among them are less than -50db compared with that of sinusoidal output signal. The dotted lines in Fig. 5 (b) show the distortion components for the case that the triangular input signal having the recurring frequency of $\omega/7\pi$ (c/s) is applied. For the general case that the recurring frequency of triangular input signal is lower than $\omega/7\pi$, only the components which satisfy the condition of $\sigma_1>2\omega(7\sigma_2-1)/7\alpha$ in $2\sigma_2\omega-\sigma_1\alpha$ components give rise to distortion. While, the dotted lines drawn in Fig. 5 (a) are the amplitudes of harmonic components of fundamental frequency α and show the distortion components for the case that the period of triangular input signal is nearly equal to zero. Since the higher order harmonics of triangular input signal are also eliminated by the output filter, only the odd-multiple harmonics satisfying the condition of $1<\sigma_1<2\omega/7\alpha$ become the actual components of the distortion. Thus, the waveform of the output signal across the load is varied considerably depending on the M and the period of triangular input signal.

From Figs. 5 (a) and (b), $2\sigma_2\omega-\sigma_1\alpha$ components distributing in the transmission band of the output filter are practically of small amplitude and may be kept negligible by the condition of $\alpha<2\omega/7$. Therefore, the output signal of the output filter is composed of the D.C. component and the odd-multiple harmonics of fundamental frequency α. And the output voltage $E_L(t)$ across the load is given by

$$E_L(t) \doteqdot \frac{E_m}{\pi}+\frac{4E_m}{\pi^2}M\cdot\cos\frac{M\pi}{2}\cdot(-1)^{\sigma_1+1}\cdot\sum_{(\sigma_1-1)/2=0}^{r}\frac{1}{(M^2-\sigma_1^2)}\sin\sigma_1\alpha t \quad \cdots\cdots(23)$$

Consequently, the distortion factor given in Eq. (22) may be estimated by the equation;

$$\Delta)_{M<1} \doteqdot (1-M^2)\sqrt{\sum_{(\sigma_1-1)/2=1}^{r}\left(\frac{1}{\sigma_1^2-M^2}\right)^2}\times 100\ \% \quad \cdots\cdots\cdots\cdots\cdots\cdots(24)$$

where r is the same notation as that of Eq. (22).

Referring to Eq. (24), the distortion factor of the sinusoidal output signal may be kept under 3% for $0.88\leq M\leq 1.00$. If M is larger than 0.96, the sinusoidal output signal whose distortion factor is less than 1% is converted from the triangular input signal by using the SCR amplifier with cosine amplification characteristics.

(5-2) The case of $M>1$.

When the amplitude of the triangular input signal exceeds that of the specified bias voltage, the turn-on phase angle of SCR is fixed on 0 (rad), during the amplitude of the input signal exceeds the maximum value of the specified bias voltage. And for the smaller than the minimum value, the turn-on phase angle is fixed on π (rad). For this reason, the amplitudes of the components composing the output signal voltage of the SCR amplifier are different from the previous case.

Substituting the above conditions into Eq. (3), the Fourier coefficients for this case are given as

1) D.C. component

$$C_{0,0} = E_m/\pi \qquad \cdots\cdots(25)$$

2) Fundamental frequency component and its harmonics of triangular input signal.

2–1) When $\sigma_1 \neq M$

$$C_{\sigma_1,0} = j\frac{M^2 \cdot E_m}{2\pi^2 \cdot \sigma_1(M^2-\sigma_1^2)}(1-e^{-j\sigma_1\pi}) \cdot e^{-j(\sigma_1\pi/2M)} \cdot (e^{-j\sigma\left(\frac{M-1}{M}\right)\pi}-1) \qquad \cdots\cdots(26)$$

2–2) When $\sigma_1 = M$

$$C_{\sigma_1,0} = j(\cos M\pi - 1)E_m/4\pi M \qquad \cdots\cdots(27)$$

3) Even-multiple frequency components of anode source frequency

$$C_{0,\sigma_2} = \frac{E_m}{\pi^2(1-4\sigma_2^2)}\left\{\pi + j\frac{8\sigma_2}{M(1-4\sigma_2^2)}\right\} \qquad \cdots\cdots(28)$$

4) Compound frequency components of even-multiples of anode source frequency and harmonics of triangular input signal

$$C_{\sigma_1,\sigma_2} = j\frac{E_m}{2\pi^2(1-4\sigma_2^2) \cdot \sigma_1} e^{-j(\sigma_1\pi/2M)}\left[(1-e^{-j\sigma_1\pi}) \cdot (e^{-j\sigma_1\left(\frac{M-1}{M}\right)\pi}-1)\right.$$
$$\left. + \sigma_1 \cdot (1+e^{j(\sigma_1\pi/M)}) \cdot \left\{e^{-j\sigma_1\pi}\frac{\sigma_1+4\sigma_2 M}{M^2-(\sigma_1+2\sigma_2 M)^2} - \frac{\sigma_1-4\sigma_2 M}{M^2-(\sigma_1-2\sigma_2 M)^2}\right\}\right] \qquad \cdots\cdots(29)$$

In the same manner, the amplitudes of the components for $M>1$ given by Eqs. (25)~(29) are shown in Fig. 6. Fig. 6 (a) shows the amplitudes of $\sigma_1\alpha$ and $2\sigma_2\omega$ components whose ratio to the amplitude of D.C. component is larger than 0.001. Fig. 6 (b) shows the amplitudes of $2\omega \pm \sigma_1\alpha$ components whose ratio to the D.C. component is larger than 0.001. The marks point out the measured values and the amplitudes of the components causing the distortion to the sinusoidal output signal are drawn with the dotted lines as well as the case of $M<1$. The components of $4\omega \pm \sigma_1\alpha$, $6\omega \pm \sigma_1\alpha$, $\cdots$ etc. are not shown for making the figure simpler. The amplitudes which are seemed to cause the distortion among them are very small and less than -50db compared with that of sinusoidal output signal. From Figs. 6 (a) and (b), it may be seen that the output signal voltage $E_0(\alpha t, \omega t)$ is composed of the D.C. component, the fundamental frequency component of triangular input signal and its harmonics, in addition to the many other undesirable components of $2\sigma_2\omega \pm \sigma_1\alpha$.

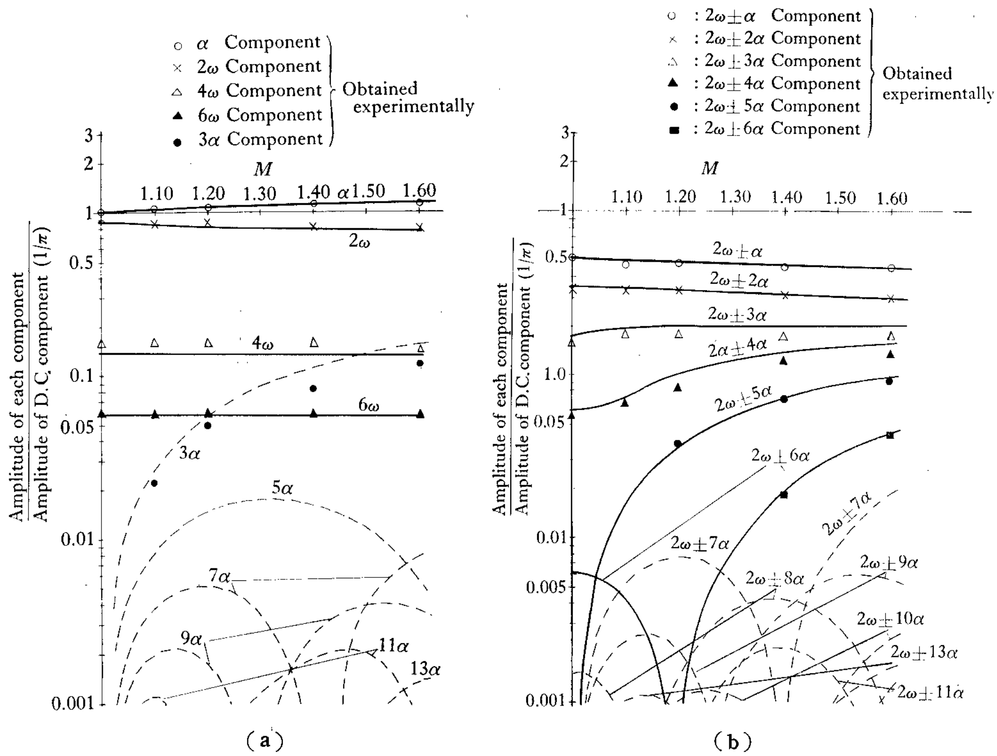


Fig. 6. Amplitudes of major components of output signal as a function of M. (a) $\sigma_1\alpha$, $2\sigma_2\omega$ components, (b) $2\omega\pm\sigma_1\alpha$ components.

Provided that the recurring frequency of the triangular input signal is $\alpha<2\omega/7$, the odd-multiple harmonics of the fundamental frequency α have principally influence on the sinusoidal output signal. Therefore, the distortion factor in this case is estimated as

$$\Delta)_{M>1} \doteqdot \frac{M^2-1}{\cos(\pi/2M)}\sqrt{\sum_{(\sigma_1-1)/2=1}^{r}\left\{\frac{\cos(\sigma_1\pi/2M)}{\sigma_1(M^2-\sigma_1^2)}\right\}^2}\times 100\ \% \quad \cdots\cdots\cdots\cdots(30)$$

where r is the same as in Eq. (24).

From Eq. (30), when M is regulated less than 1.10, the distortion factor of the sinusoidal output signal may be kept under 3%. And for $M<1.04$, it can be kept under 1%.

(5-3) The case of $M=1$.

Both the case of $M<1$ and $M>1$, the distortion of the sinusoidal output signal becomes smaller when the amplitude of the specified bias voltage is close to that of the triangular input signal, as mentioned above.

In the case $M=1$, $C_{\sigma_1,1}$ being located at the nearest position of the cutoff frequency of the output filter becomes a main component of the distortion. Accordingly, the upper limit of the theoretical distortion is given by

$$\Delta)_{M=1} \doteqdot \frac{16}{\pi\cdot[\sigma_1^2(\sigma_1^2-10)+9]}\times 100\ \% \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(31)$$

where σ_1 is the first even integral value that is larger than $2\omega(7\sigma_2-1)/7\alpha$.

From Eq. (31), the distortion factor of the case of $M=1$ can be kept under 0.16% and decreases uniformly according to the decrease of its oscillating frequency α.

In the study of distortion described above, an influence of undesirable components being distributed in the attenuation band of the output filter is ignored. The power energy of these undesirable components does not exceed the value of $(\pi^2-4)E_m^2/4\pi^2$. This is derived from the facts that the Schwartz inequality for the Fourier coefficients is given as

$$\sum_{\sigma_1,\sigma_2=-\infty}^{\infty} |C_{\sigma_1,\sigma_2}|^2 \leq \frac{E_m^2}{2\pi^2}\left[\int_0^{2\pi}\int_{\varphi_p}^{\pi} \sin^2\xi_2\cdot d\xi_1\cdot d\xi_2\right]$$

$$= \frac{E_m^2}{4\pi^2}\left[\pi^2+\frac{2}{M}\sin\frac{M\pi}{2}\right] \qquad \cdots\cdots(32)$$

Therefore when the attenuation in the attenuation band of the output filter is δ, the maximum distortion factor Δ_m due to these undesirable components is given as

$$\Delta_m < (\delta\sqrt{(\pi^2-4)/2})\times 100\ \% \qquad \cdots\cdots(33)$$

6. Conclusion

The method of waveform conversion presented in this paper is a kind of non-linear waveform conversion. However, this is a new system because of the inherent feature that the cosine amplification characteristic between the input and the average output voltage is employed. The distortion of sinusoidal output signal may be kept under 3% for the case of $0.88 \leq M \leq 1.10$. Especially, it is derived that the waveform conversion using the SCR amplifier with cosine amplification characteristics may theoretically yield the powerful sinusoidal signal of 0 (c/s)$\sim\omega/7\pi$ (c/s) with total harmonics distortion reducible to below 0.16% when the amplitude of the specified bias voltage is equal to that of triangular input signal.

The method of waveform conversion described in this paper can be realized by using other switched mode amplifiers employing newly developed switching elements such as GCS and SSS, in addition to SCR or switching transistors. This system has a good ability of stability and accuracy as well as high power in the operation of ultra low frequency region. Therefore, the application to the ultra low frequency signal generator can be conceivable.

Reference

1) S. Kaku, S. Minamoto and K. Miyakoshi, *Bull. Univ. Osaka Pref.*, A **15**, No. 1, 119 (1966).