# Generation of Triangular Wave Using GCS Amplifier

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This paper presents a method of generating an ultra low frequency triangular signal with the aid of switched mode amplifier having the logarithmic characteristics between the input signal and the average value of output voltage. The exponential signal of low frequency is converted into the triangular signal of high power owing to the amplification characteristics of switched mode amplifier.

Since the switched mode amplifier, in general, shows its excellent ability in a low frequency region, the method presented in this paper is suitable for the generation of that frequency region.

In the first place of this paper, the principles of nonlinear waveform conversion with GCS amplifier are described.

And the frequency analysis of the output signal is studied together with the experimental results. The estimation of the output triangular signal is discussed.

## 1. Introduction

Several methods of generating a triangular signal have been reported. Most of them are based on the principles of integrating a rectangular waveform. The integrating method may use only the linear part of the exponential signal which is the integrated waveform of a rectangular waveform and its oscillating frequency comes up to about $10^{-3}$ (c/s) at the utmost. It comes to the difficult of the construction of the system to generate a triangular signal in a lower frequency region.

This paper deals with the system of waveform conversion which converts an exponential signal into a triangular signal by repeating leading edge modulation and trailing edge one of switched mode amplifier. The exponential waveform is employed as the specified bias voltage and the input signal of the switched mode amplifier.

This is an effective method generating a lower frequency triangular signal by making full use of the exponential signal to the extent of nonlinear part.

Since GCS is employed as the switching element, the generation of high power output signal can be performed.

## 2. Basic operation of GCS amplifier

The basic diagram of GCS amplifier is shown in Fig. 1, in which GCS and the load with a smoothing filter are connected in series with D.C. anode source voltage. In order to keep the normal operation of GCS amplifier, a diode is connected in paralell

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with the filter. On the other hand, the gate circuit is so designed that it may produce the gating signal to keep the switch "on" or "off" condition according to the positive or negative value of the resultant voltage of specified bias and input signal. GCS turns on at the instant that the resultant voltage of specified bias voltage having the integrated waveform of anode source voltage in each half-cycle and input signal is equal to zero.

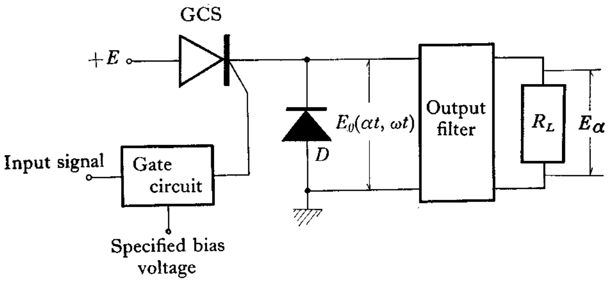


Fig. 1. Basic diagram of GCS amplifier.

If GCS conducts during the constant sampling period, the average output voltage is proportional to the input signal voltage. Suppose that the D.C. anode source voltage and the sampling period are indicated by $E$ and $T$, respectively. And provided that D.C. voltage of $E_i$ is applied as the input signal, the average output voltage $\bar{E}_{out}$ during the sampling period is given by the next equation;

$$\bar{E}_{out}=\frac{1}{T}\int_{\frac{T}{2}(1-E_i)}^{T} Edt=\frac{E}{2}(1+E_i) \qquad \cdots\cdots(1)$$

where $E_i$ is the normalized input signal voltage defined as $-1\leq E\leq 1$.

From Eq. (1), it is apparent that this amplifier is able to do the linear amplification of the input signal.

## 3. Principles of generating a triangular waveform

In general, thyristor amplifier may have a variety of characteristics by employing various waveforms of the specified bias voltage.

Suppose that the input signal $e_s(t)$ and the specified bias voltage $e_g(t)$, of which waveforms are exponential, are applied to GCS amplifier as shown in Fig. 2.

$$e_s(t)=-E_s\cdot\exp[-\alpha t/\alpha\tau_s] \qquad \cdots\cdots(2)$$

$$e_g(t)=E_g\cdot\exp[-\omega t/\omega\tau_g] \qquad \cdots\cdots(3)$$

where the maximum value and recurring frequency of the input signal and specified bias voltage are denoted as $E_s$, $\alpha$ and $E_g$, $\omega$, respectively. And the time constants of the integral circuits of the former and the latter are denoted as $\tau_s$ and $\tau_g$, respectively. The gate circuit is so designed that it may generate the sufficient pulse to turn "on" or "off" GCS at the instant that the resultant voltage of $e_s(t)$ and $e_g(t)$ becomes zero. That is, GCS is controlled at the instant $t_p$ that the next equation is valid in the gate circuit.

$$e_s(t_p)+e_g(t_p)=0 \qquad \cdots\cdots (4)$$

From Eqs. (2) (3) (4), the control phase angle of GCS, $\varphi_p$, is given as follows;

$$\begin{aligned}\varphi_p&=\omega t_p=(\omega\tau_g/\alpha\tau_s)\cdot\alpha t_p+\omega\tau_g\cdot\ln(E_g/E_s)\\ &=\varLambda\xi_1+\eta \qquad \cdots\cdots (5)\end{aligned}$$

where $\varLambda=\omega\tau_g/\alpha\tau_s\,,\quad \xi_1=\alpha t_p\,,\quad \eta=\omega\tau_g\cdot\ln(E_g/E_s)$

From Eq. (5), there is a linear relation between the phase angle to control GCS and that of the input signal.

GCS may be controlled in such way that it keeps "on" state during the leading portion of the control phase angle and "off" the rest portion in succeeding each cycle of the specified bias voltage and vice versa owing to the behavior of the gate circuit.

When the leading part is considered as object, the average output voltage of GCS amplifier is proportional to $\xi_1$. On the other hand, for the trailing portion it is proportional to $\pi-\xi_1$. The former and the latter are referred as trailing edge modulation and leading edge one, respectively.

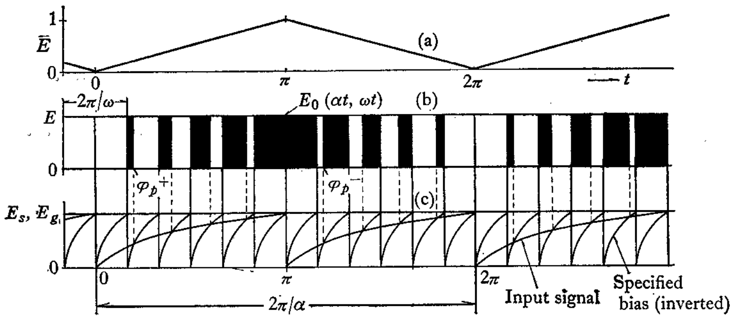


Fig. 2. Principles of triangular waveform generation (a) Triangular output signal (b) Segments of anode voltage (c) Waveforms of input signal and specified bias.

Thus, leading and trailing edge modulations are repeated in each half-period of the input signal in such manner that the former is performed for the positive going part of output triangular signal and the latter for the negative going part of it. Consequently, the average output voltage of GCS amplifier becomes a triangular waveform shown in Fig. 2 (a). Of course, a saw-tooth waveform may be obtained with only leading or trailing edge modulation.

## 4. Analysis of output signal

As mentioned above, the general aspect of the output voltage across GCS is a train of pulses of which the shapes are the segments of anode source voltage waveform as shown in Fig. 2 (b). The output signal is a function of the periodic variable of the input signal, $at(=\xi_1)$, and $\omega t(=\xi_2)$ of the specified bias voltage.

The output voltage $E_0(at, \omega t)$ which is expressed as the following double Fourier series may be given;

$$E_0(at, \omega t)=\sum_{\sigma_1,\sigma_2=-\infty}^{+\infty} C_{\sigma_1,\sigma_2}\cdot\exp\,[i(\sigma_1\xi_1+2\sigma_2\xi_2)] \qquad \cdots\cdots(6)$$

And

$$C_{\sigma_1,\sigma_2}=\frac{E}{2\pi^2}\left[\int_0^{\pi}\int_0^{\varphi_p^+}\exp\,[-i(\sigma_1\xi_1+2\sigma_2\xi_2)]\,d\xi_1 d\xi_2 +\int_{\pi}^{2\pi}\int_{\varphi_p^-}^{\pi}\exp\,[-i(\sigma_1\xi_1+2\sigma_2\xi_2)]\,d\xi_1 d\xi_2\right] \qquad \cdots\cdots(7)$$

where $E$ is the amplitude of D.C. anode source voltage.

In Eq. (7), $C_{\sigma_1,\sigma_2}$ is the Fourier coefficient which indicates the amplitude of each frequency component composing the output signal voltage. Where, $\sigma_1$ is the integer which gives the order of harmonics of fundamental frequency component of the input signal and $\sigma_2$ is that of the specified bias.

The first and second terms in Eq. (7) are corresponding to trailing and leading edge modulation, respectively. Suppose that $\varphi_p^+$ and $\varphi_p^-$ are the control phase angles of GCS. They may be derived from Eqs. (4) (5) as;

$$\varphi_p^+=A\xi_1+\eta \qquad \cdots\cdots(8)$$

$$\varphi_p^-=A(\xi_1-\pi)+\eta \qquad \cdots\cdots(9)$$

By inserting Eqs. (8) (9) into Eq. (7), the values of Fourier coefficients corresponding to the amplitudes of all components in the output signal are given as follows;

(1) D.C. Component ($\sigma_1=0$, $\sigma_2=0$)

$$C_{0,0}=E/2 \qquad \cdots\cdots(10)$$

(2) Fundamental Frequency Component of Input Signal and Its Harmonic Components ($\sigma_1 \neq 0$, $\sigma_2=0$)

$$C_{\sigma_1,0}=\frac{E}{2\pi^2}\cdot\frac{1}{i\sigma_1}\left\{\left(\frac{2A}{i\sigma_1}-\pi+A\pi+\eta\right)\cdot\left(1-e^{-i\sigma_1\pi}\right)\right\} \qquad \cdots\cdots(11)$$

(3) Even-multiple Frequency Components of Anode Source Frequency ($\sigma_1=0$, $\sigma_2\neq 0$)

$$C_{0,\sigma_2}=0 \qquad \cdots\cdots(12)$$

(4) Compound Frequency Components of Even-multiple of Anode Source Frequency and Fundamental Frequency Component of Input Singal or its Harmonic Frequencies ($\sigma_1\neq 0$, $\sigma_2\neq 0$)

(4–1) when $\sigma_1$ is odd

$$C_{\sigma_1, \sigma_2}=\frac{E}{2\sigma_2\pi^2}\cdot\left\{\frac{e^{-2i\sigma_2\eta}}{(\sigma_1+2\sigma_2\Lambda)}\cdot\left(e^{-2i\sigma_2\Lambda\pi}+1\right)-\frac{2}{\sigma_1}\right\} \quad \cdots\cdots(13)$$

(4–2) when $\sigma_1$ is even

$$C_{\sigma_1, \sigma_2}=0 \quad \cdots\cdots(14)$$

Referring to Eqs. (10)∼(14), it is to be noted that the notation of anode source frequency component is equivalent to the frequency component of the specified bias.

From the above theoretical results, it is apparent that the output signal across the diode is composed of D.C. component, the fundamental frequency component of input signal and its harmonics and the components which locate at the distances of integral multiples of fundamental frequency of the input signal from the position of even-multiples anode source frequency. But the components of anode source frequency and its harmonics do not appear.

Under the conditions as shown in Fig. 2, the following relations are satisfied.

$$E_s=E_g \quad \cdots\cdots(15)$$

$$\alpha\tau_s=\omega\tau_g \quad \cdots\cdots(16)$$

By inserting Eqs. (15) (16) into Eq. (5), the next relations are given;

$$\Lambda=1, \quad \eta=0 \quad \cdots\cdots(17)$$

Then the control phase angles of GCS become as follows;

for a period of trailing edge modulation

$$\varphi_p^{+}=\xi_1 \quad \cdots\cdots(18)$$

for a period of leading edge modulation

$$\varphi_p^{-}=\xi_1-\pi \quad \cdots\cdots(19)$$

By using these control phase angles, Fourier coefficients given by Eqs. (10)∼(14) are represented as follows.

(1) D.C. Component ($\sigma_1=0$, $\sigma_2=0$)

$$C_{0, 0}=E/2 \quad \cdots\cdots(20)$$

(2) Fundamental Frequency Component of Input Signal and Its Harmonics ($\sigma_1\neq0$, $\sigma_2=0$)

(2–1) when $\sigma_1$ is odd

$$C_{\sigma_1, 0}=-2E/\pi^2\sigma_1^2 \quad \cdots\cdots(21)$$

(2–2) when $\sigma_1$ is even

$$C_{\sigma_1, 0}=0 \quad \cdots\cdots(22)$$

(3) Even-multiple Frequency Components of Anode Source Frequency ($\sigma_1=0$, $\sigma_2\neq 0$)

$$C_{0,\sigma_2}=0 \qquad \cdots\cdots(23)$$

(4) Compound Frequency Components of Even-multiples of Anode Source Frequency and Fundamental Frequency Component of Input Signal or Its Harmonic Frequencies ($\sigma_1\neq 0$, $\sigma_2\neq 0$)

(4–1) when $\sigma_1$ is odd

$$C_{\sigma_1,\sigma_2}=-\frac{E}{\pi^2}\cdot\frac{2}{\sigma_1\cdot(\sigma_1+2\sigma_2)} \qquad \cdots\cdots(24)$$

(4–1) when $\sigma_1$ is even

$$C_{\sigma_1,\sigma_2}=0 \qquad \cdots\cdots(25)$$

From the above calculated results, the output signal is represented as follows;

$$E_0(\alpha t, \omega t)/E=\frac{1}{2}-\sum_{\sigma_1=1}^{\infty}\frac{4}{\pi^2\cdot(2\sigma_1-1)^2}\times\cos\{(2\sigma_1-1)\alpha t\}$$
$$+\sum_{\sigma_1=\pm 1}^{\pm\infty}\sum_{\sigma_2=1}^{\infty}\frac{4}{\pi^2}\cdot\frac{1}{(2\sigma_1-1)\{(2\sigma_1-1)+2\sigma_2\}}$$
$$\times\cos\{2\sigma_2\omega t+(2\sigma_1-1)\alpha t\} \qquad \cdots\cdots(26)$$

From Eq. (26), the output signal of GCS amplifier is composed of D.C. component, the fundamental frequency component of input signal and its odd-multiple harmonics and the components which locate at the distances of integral multiples of the input signal frequency from the position of even-multiples of anode source frequency. The frequency components of anode source and even-multiple harmonics of input signal frequency do not appear.

Fig. 3. shows an example of the output spectrum calculated with Eq. (26). In this figure, the maximum value of the anode source voltage is taken for unity and the ratio between the anode source angular frequency and the input signal has the relation of $\omega/\alpha=155/3.4$. The variation of $\omega/\alpha$ shifts only the relative positions of frequency components analyzed above and the amplitude of each component is kept constant unless the conditions given by Eqs. (15) (16) are disturbed. The marks, $\times$, in Fig. 3 indicate the experimental results measured with a selective level meter, and the results clarify that above mentioned theory.

While, the calculated results show that the amplitude of odd-multiple harmonic component of the input signal frequency decreases rapidly in inverse proportion to $\sigma_1^2$. Among the components which locate at the distances of integral multiples of the input frequency from the position of even-multiples anode source frequency, the component of $C_{-(2\sigma_2-1),\sigma_2}$ takes the maximum value if $\sigma_2$ is fixed. And the energy of undesirable frequency components in the region of $\sigma_1<-(2\sigma_2-1)$ decreases uniformly according to the increase of $\sigma_1$. Consequently, D.C. component and required components for com-

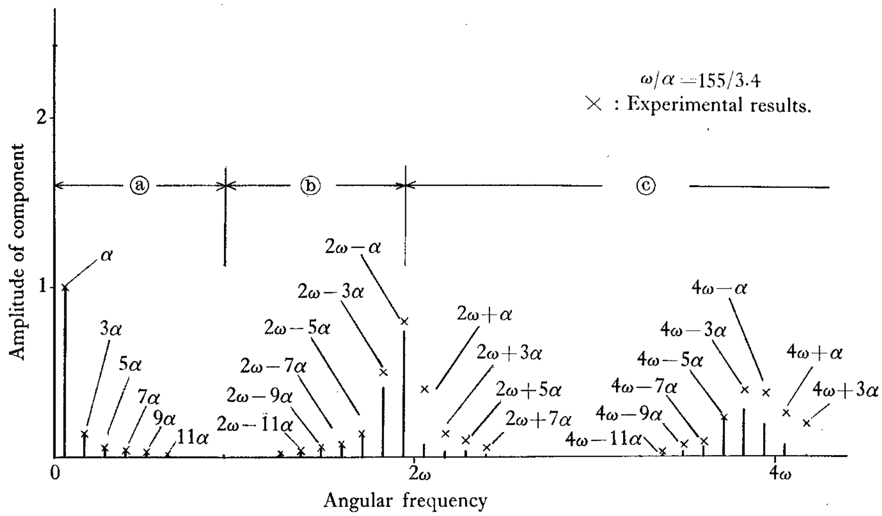


Fig. 3. Spectrum of output signal.

posing a triangular output signal may be easily separated from many other undesirable frequency components by using a proper low pass filter.

## 5. Practical circuit for generating a triangular waveform

The block diagram of the gate circuit is shown in Fig. 4. In this block diagram, a differential amplifier serves as a comparator. The differential amplifier produces a rectangular pulse whose leading edge is formed at the instant that the resultant voltage of the exponential input signal and the exponential specified bias voltage is equal to zero. As the result of applying the rectangular pulse to the differentiator, two differential wave-

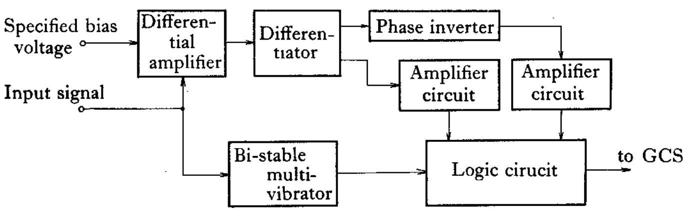


Fig. 4. Block diagram of gate circuit.

forms may be obtained, one of them is normal and the other is inversed. These signals are applied to the logic circuit which consists of four stages of AND circuit. On the other hand, the logic circuit is so designed that it may generate the controlling pulse with the suitable polarities for leading edge modulation or trailing edge one according to the state of the bi-stable multivibrator which is syncronized with the input signal.

This means that the operation of GCS amplifier is repeated leading edge modulation and trailing edge one in each half-cycle of the input signal.

## 6. Estimation of a triangular output signal and design basis of output filter

By referring to the calculated results, this section presents the theoretical estimation of the triangular output signal together with the consideration of the design criterion of the output filter.

In general, the linearity and ripple are considered as a major problem in estimating a triangular waveform. Therefore, these subjects will be considered in this paper.

This device has a low pass filter to average the output pulse train owing to the switching operation of GCS amplifier. It is considered that the higher order harmonics which are the required components to compose the triangular signal of the input signal frequency may be removed by the output filter in eliminating the undesirable components, and this gives a bad influence to the linearity of triangular output signal. From Eq. (26), the amplitude of triangular output signal is given by

$$E_{\infty}=-\frac{4E}{\pi^2}\cdot\sum_{\sigma_1=1}^{\infty}\frac{1}{(2\sigma_1-1)^2}\times\cos\{(2\sigma_1-1)\alpha t\} \qquad \cdots\cdots(27)$$

By inserting $\alpha t=0$, $\alpha t=\pi$ into Eq. (27), it is clarified that the phase angles of the components composing the triangular output signal are all in phase, and Eq. (27) is represented as

$$E_{\infty}=\mp\frac{4E}{\pi^2}\cdot\sum_{\sigma_1=1}^{\infty}\frac{1}{(2\sigma_1-1)^2} \qquad \cdots\cdots(28)$$

where it takes plus sign when $\alpha t=\pi$, and minus sign when $\alpha t=0$.
The peak value of the triangular output signal voltage is represented by $E_{\infty}$ in Eq. (28).

Supposing that the harmonics having higher order than $\sigma_1=l+1$ are removed by the output filter, the next equation indicates a maximum deviation of the output signal to an ideal triangular waveform.

$$E_M=\mp\frac{4E}{\pi^2}\cdot\sum_{\sigma_1=l+1}^{\infty}\frac{1}{(2\sigma_1-1)^2} \qquad \cdots\cdots(29)$$

Then, we define the next equation to express the ratio of the maximum deviation.

$$\frac{E_M}{E_{\infty}}\times 100=\frac{\sum_{\sigma_1=l+1}^{\infty}\frac{1}{(2\sigma_1-1)^2}}{\sum_{\sigma_1=1}^{\infty}\frac{1}{(2\sigma_1-1)^2}}\times 100=K(\%) \qquad \cdots\cdots(30)$$

The variation of $K$ according to $l$ is shown in Fig. 5. From this figure, it is apparent that the ratio of the maximum deviation to ideal triangular waveform is to be less than 3%, we must design to be in the pass band of the output filter up to 13th order harmonic component, and to be less than 1%, up to 41th order harmonic component.

In the next place, it may be considered that the influence of the undesirable components to the triangular output signal. These undesirable components, which locate at the distances of integral multiples of the input signal frequency from the position of even-multiples of anode source frequency may be attenuated by the output filter.

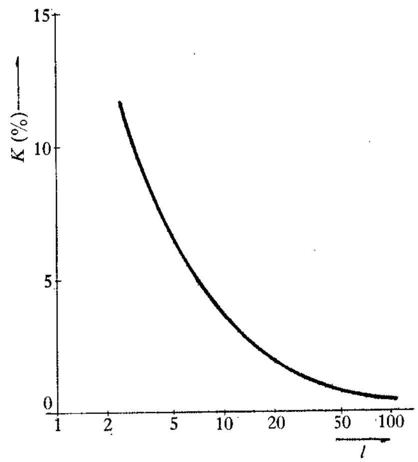


Fig. 5. Variation of $K$ for $l$.

For the practical criterion, each amplitude of these undesirable components is to be below -50dB as compared with the total value of the amplitudes of components composing the triangular output signal. This is derived from the point of view of the smoothness of a triangular waveform. In other words, a certain frequency component does not give a particular bad influence to the triangular output waveform even if there is a negligible ripple.

Now, we assort the distribution of the undesirable frequency components as shown in Fig. 3, ⓐ components in the pass band of the output filter ⓑ components in the frequency region of $\omega_c/2\pi \sim \omega/\pi$ $(c/s)$ ⓒ components higher than $\omega/\pi$ $(c/s)$. The notation of $\omega_c$ is a cutoff frequency of the output filter.

For the components of ⓐ, it is considered that these components give a bad influence to the triangular output signal without being attenuated by the output filter. Of course, many components of $C_{\sigma_1, 1}$, $C_{\sigma_2, 2}\cdots$ distributed in the pass band of the output filter, however, it is apparent from the calculated results that $C_{\sigma_1, 1}$ are most powerful of all. Therefore, only the components of $C_{\sigma_1, 1}$ are treated here.

By inserting $\sigma_2=1$ into the third term of Eq. (26), the amplitude of each component is given as

$$E_{\sigma_1, 1}=\frac{4E}{\pi^2}\cdot\frac{1}{(2\sigma_1-1)\cdot\{(2\sigma_1-1)+2\}} \qquad \cdots\cdots(31)$$

In order to keep each amplitude below $-50$dB compared with the total value of components composing the triangular output, the relation, $\sigma_1<-10$, must be satisfied. In other words, the frequency components higher than $2\omega-21\alpha$ must be removed by the output filter. The harmonics of the input signal frequency up to 13th order are to be in the pass band of the output filter if the maximum deviation is to be less than 3% as mentioned above.

Consequently, the cutoff frequency of the output filter is given by

$$f_c \doteqdot \omega/3\pi \qquad \cdots\cdots(32)$$

And the upper limit of the input signal frequency under these conditions is given by

$$f_{max} = \omega/42\pi \qquad \cdots\cdots(33)$$

Among the undesirable frequency components, the energy of $C_{-1,1}$ component shows the maximum value of $-1.6$dB compared with the total energy of each component composing the triangular output signal. Consequently, the output filter for this device must be composed under the following conditions.

(1) The cutoff frequency satisfies the condition of Eq. (32).

(2) The attenuation at the point of $2\omega$ is more than 48dB, of which the slope is more than 40dB per octave, and that in the higher than $4\omega$ approaches 40dB.

In the case that the output filter stipulating the above conditions is employed to this device, the followings deal with the influence of the undesirable components to the triangular output signal.

From the calculated results, it is apparent that each amplitude of these components decreases according to the increase of its harmonic order, $\sigma_1$. These components will be treated by the method of involving to the second power. This means that the power energy is considered of these components, in other words, the estimation is derived from the factor of power energy of these components for that of components composing the triangular output signal.

Suppose that the sum of each square of the components which distribute in the pass band of the output filter is given as $W_b$;

$$W_b = \frac{16E^2}{\pi^4} \cdot \sum_{\sigma_1=n}^{\infty} \frac{1}{(2\sigma_1-1)^2(2\sigma_1-3)^2} \qquad \cdots\cdots(34)$$

If the condition $n \gg 1$ is satisfied, Eq. (34) is reduced to the next equation.

$$\begin{aligned} W_b &\doteqdot \frac{16E^2}{\pi^4} \cdot \sum_{\sigma_1=n}^{\infty} \frac{1}{(2\sigma_1-2)^4} \\ &\doteqdot \frac{E^2}{\pi^4} \cdot \left\{ \frac{1}{n+1} - \frac{1}{2(n+1)(n+2)} \right\}^2 \\ &= \frac{E^2}{\pi^4} \cdot \left\{ \frac{2n+3}{2(n+1)(n+2)} \right\}^2 \qquad \cdots\cdots(35) \end{aligned}$$

For the upper limit of the input signal frequency, by inserting $n=11$ into Eq. (35), the next equation is given;

$$W_{b,u} = E^2/15400 \qquad \cdots\cdots(36)$$

Suppose that the sum of each square of the components composing the triangular output signal $W_t$, which indicates the power energy of the triangular output signal.

$$W_t=\frac{16E^2}{\pi^4}\cdot\sum_{\sigma_1=1}^{\infty}\frac{1}{(2\sigma_1-1)^4}=\frac{E^2}{6} \qquad \cdots\cdots(37)$$

From Eqs. (36) (37), the ratio of the power energy of components in the frequency region of ⓐ to that of triangular output signal is calculated as Eq. (38).

$$W_{b,u}/W_t\times 100=0.04\ (\%) \qquad \cdots\cdots(38)$$

On the other hand, next Schwartz's inequality is realized in the relation of each Fourier coefficient.

$$\sum_{\sigma_1,\sigma_2=-\infty}^{\infty}\left|C_{\sigma_1,\sigma_2}\right|^2\leq 1/2\pi^2\cdot\left[\int_0^{\pi}\int_0^{\varphi_p^+}d\xi_1 d\xi_2+\int_{\pi}^{2\pi}\int_{\varphi_p^-}^{\pi}d\xi_1 d\xi_2\right]$$
$$=1/2 \qquad \cdots\cdots(39)$$

From Eq. (39), the power energy of all the undesirable components except the D.C. component and harmonics of the input signal frequency is given as;

$$\sum_{\sigma_1=\pm1}^{\pm\infty}\sum_{\sigma_2=\pm1}^{+\infty}\left|C_{\sigma_1,\sigma_2}\right|^2\leq\frac{1}{2}-\frac{1}{4}-2\left(\frac{2}{\pi^2}\right)^2\cdot\sum_{\sigma_1=1}^{\infty}\frac{1}{(2\sigma_1-1)^4}$$
$$=\frac{1}{6} \qquad \cdots\cdots(40)$$

The power energy of ⓒ does not exceed the value of 1/6. Applying the output filter of which the attenuation is 40dB in the higher than $2\omega$, the ratio of this power energy to that of the triangular output signal is about 0.04%.

For the components of ⓑ, it will be reasonable to consider that the influence of this power energy is contained with in the treatment of ⓒ components.

The values of the estimation calculated above are derived from the results that every amplitude of the undesirable components is to be below $-50$dB compared with the total value of that of components composing the triangular output signal.

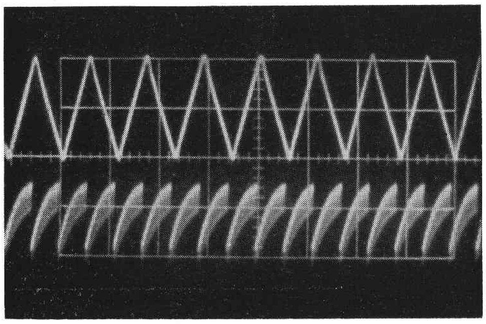

Fig. 6. Experimental results of triangular signal generation 3.4 (c/s).

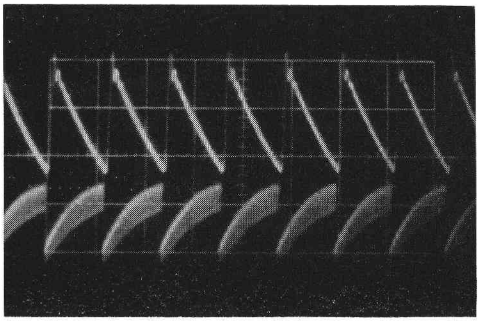

Fig. 7. Experimental results of saw-tooth signal generation 3.4 (c/s).

Fig. 6 and Fig. 7 show the experimental results.
Fig. 6 is for the case that the triangular output signal is obtained, and Fig. 7 is for the case of the saw-tooth output signal, for which this device operates in the leading edge modulation mode. In these Figures, the upper trace shows the waveform of the output signal and the lower trace shows the resultant waveform of the exponential input signal and the exponential specified bias. The recurring frequency of the input signal is 3.4 $(c/s)$ and that of the specified bias is 155 $(c/s)$. As the output filter, RC filter having the characteristics that the cutoff frequency is 18 $(c/s)$ and the slope of the attenuation is 40dB per octave is employed.

## 7. Conclusion

The method of generating a triangular output signal from an exponential input signal by using GCS amplifier is described. This is a kind of nonlinear waveform conversion by making use of the logarithmic characteristics between the input signal and the average value of the output signal.

The output signal of GCS amplifier is composed of D.C. component, odd-multiples of the input frequency and many other undesirable frequency components. These undesirable components concentrate in the vicinity of each even-multiple harmonics of anode source frequency according as the input frequency becomes lower. Therefore, the lower the input frequency becomes, the smaller the each energy of the undesirable components in the pass band becomes, and the more satisfactory triangular output signal is obtained.

Since the exponential input signal is employed to the device, the output signal can be generated in the lower frequency region than the most usual signal generator at present.

The theoretical frequency region of the signals generated by of this device is $0 \sim \omega/42\pi$ $(c/s)$.

And then, it is possible for this device to generate a sinusoidal wave by applying a sinusoidal anode source voltage instead of a rectangular one.

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