Function Generator with Non-Ohmic Resistors

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Yasuhiro KOBAYASHI*, Tsutomu FUJIMURA* and Masao NAKAMURA*

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Abstract

Since the electronic analog computer came to be used as a useful system of computation, in order to solve a number of non-linear problems, the need for the universal function generators has steadily increased. Today, various devices are provided and used practically in the analog computing systems.[1)~14)] However, such function generator that the various abilities are displayed by a single set has not yet been developed.

The function generator described in this paper is of the above mentioned abilities, and its basic principle stems from the polynomial approximation. This generator is not only accurate and reliable but also economical so that it may be built any number of units in a system, if necessary.

Introduction

Diode universal function generator, [1),3),6),10),14)] so colled "Function Fitter", a useful apparatus for generating arbitrary function, is based on summing the output voltages of simple series-limiter or shunt-limiter circuits with diodes. Each diode is a kind of electronic switch acting automatically by the power of input voltages over the threshold level set previously corresponding to the desired function.

In this device, the desired function may be obtained with a high degree of accuracy by using a multiple channel of diode-limiters, but it is impossible to generate any function in a smooth curve, because any function is approximated by straight-line segments.

Meanwhile, the methods of generating any arbitrary functions taking advantage of the non-linearity of non-ohmic resistors have been researched by many workers. L. D. Kovach and W. Comley, being among them, provided some non-linear transfer functions and manufactured a multiplier using the "Silicon Carbide Varistor" with operational amplifiers.[2)5)9)] A. A. Maslov made some improvement on that multiplier in which the silicon carbide varistor is also used, however, the number of operational amplifiers is reduced.[4)] Then, T. Hirasa and M. Nakamura designed a function generator using silicon carbide varistors with a differential amplifier[8)] and used it as their useful tool for computing the transient stability of power system.

Silicon carbide varistor is characterized by a non-linear behaviour in that the current varies as some power of input voltage. This behaviour may be expressed in general by (1), where n is $2\sim4$ in all probability, and itself varies over a considerable range.

$$i = Ce^n \tag{1}$$

Where i : current through the varistor
e : input voltage
C: constant number

* Department of Electrical Engineering, College of Engineering

The value of n depends primarily upon the type of varistor. Of cource, this value may be varied to some extent by connecting some resistor in series or parallel, but by this procedure only, it is difficult to determine the precise value of this exponent, for instance, exactly n = 2, over greatly considerable range.

Therefore, to accomplish this purpose, it is necessary to provide and arrange properly the multiple channel consisting of several units, each of which is a proper combination of resistors and varistors.

The function generator described in this paper makes use of the above methods and simulates a desired function precisely, using the polynomial approximation in which the respective coefficients of its polynomial is determined so as to satisfy the value of that function.

Fundamental

1) Approximation of the functions and the output voltages of non-linear combinations

The work of generating a function starts from to approximate this function by a polynomial. Any function which is continuous in a cirtain interval may be approximated uniformly by polynomials in this interval. When a function does not be continuous it must be approximated by respective polynomials in the partial interval where it will be continuous, but sometimes, it may be replaced by an approximated function which is assumed to be reasonably continuous in that whole interval. Generally such efforts will not succeed in obtaining the proper approximation. If it is dared to do so for all over range, there might be never acquired the best approximation by a polynomial because of the fairly increased errors.

Its approximated polynomial $f(x)$ is shown in (2), when the function $F(x)$ is given in the interval $[a, b]$.

$$F(x) \simeq f(x) = \sum_{j=0}^{n} \alpha_j \cdot x^j$$

$$= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \cdots\cdots\cdots + \alpha_n x^n \qquad (2)$$

where $j = 0, 1, 2, \cdots\cdots, n$

$\alpha_0, \alpha_1, \alpha_2, \cdots\cdots, \alpha_n$: constant number.

Each value of the coefficient must be determined so carefully as to be obtained the best approximation. When acquiring the best approximation, the error function $\phi(x) = f(x) - F(x)$ has the values under the tolerance limit μ which is very small, and changes the sign beyond the partial neighbours in the interval. These are shown in (3).

$$\begin{aligned} |\phi(x_j)| &= \mu & (j = 0, 1, 2, \cdots\cdots, n+1) \\ \phi(x_j)\,\phi(x_{j+1}) &< 0 & (j = 0, 1, 2, \cdots\cdots, n\ ) \\ |\phi(x)| &\leqq \mu & \text{in } a \leqq x \leqq b \end{aligned} \qquad (3)$$

The error function in this case is shown in Fig. 1.

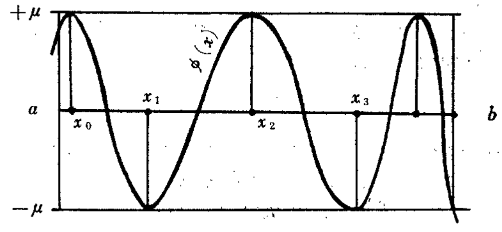


Fig. 1 Typical error curve in the best approximation

The next work is how to represent of a non-linear behaviour of combination consisting of some non-ohmic resistors and ordinary resistors. This non-linear behaviour, according to the above methods, may be approximated by a polynomial, too. The output voltage v of the combination is approximated by the polynomial $g(u)$ in the interval $[p, q]$ of the applied voltage u, as given in (4).

$$v \simeq g(u) = \sum_{j=0}^{n} \beta_j u^j$$
$$= \beta_0 + \beta_1 u + \beta_2 u^2 + \cdots\cdots + \beta_n u^n \qquad (4)$$

where $j = 0, 1, 2, \cdots\cdots, n$
$\beta_0, \beta_1, \beta_2, \cdots\cdots, \beta_n$: constant number

This equation may be rewritten as (5), transforming u to x and v to y

with $u = k_1 u_0 x$, $v = k_2 v_0 y$
where k_1, k_2 : constant number
u_0 : the unit value of u corresponding to x
v_0 : the unit value of v corresponding to y

$$y = \frac{v}{k_2 v_0} \simeq \frac{1}{k_2 v_0} g(u) = \frac{1}{k_2 v_0} g(k_1 u_0 x)$$
$$= \mathrm{K}_0 + \mathrm{K}_1 x + \mathrm{K}_2 x^2 + \cdots\cdots + \mathrm{K}_n x^n, \qquad (5)$$

then $\mathrm{K}_0 = \beta_0 \cdot 1 / k_2 v_0$
$\mathrm{K}_1 = \beta_1 \cdot k_1 u_0 / k_2 v_0$
$\mathrm{K}_2 = \beta_2 (k_1 u_0)^2 / k_2 v_0$
....
....
$\mathrm{K}_n = \beta_n (k_1 u_0)^n / k_2 v_0$.

When there are the following relations between K_j and α_j ($j = 0, 1, 2, \cdots\cdots\cdots\cdots n$) respectively,

$$\mathrm{K}_0/\alpha_0 = \mathrm{K}_1/\alpha_1 = \mathrm{K}_2/\alpha_2 = \cdots\cdots = \mathrm{K}_n/\alpha_n = \gamma \qquad (6)$$

where γ: arbitrary number
in the interval $p/k_1 u_0 \leqq x \leqq q/k_1 k_0$,

the output signal which is proportional to the desired function (the true or approximated function) will be obtained as shown in (7).

$$\frac{1}{k_2 v_0} g(k_1 u_0 x) = \gamma \cdot f(x) \simeq y$$

The proportional output signal to the desired function that always meets the conditions as (6) may be obtained by summing the output signals of each non-linear combination channel connected inparallel, over all ranges of the input signal.

The details are the following.

The output signal of the r-th non-linear combination channel is given in (8), according to (5), in a cirtain interval. That is:

$$y_r = \sum_{j=0}^{n} (\mathrm{K}_j)_r x^j$$
$$= (\mathrm{K}_0) + (\mathrm{K}_1)x + (\mathrm{K}_2)x^2 + \cdots\cdots + (\mathrm{K}_n)x^n \qquad (8)$$

where $j = 0, 1, 2, \cdots\cdots\cdots\cdots\cdots, n$

If summing the respective output signals of the non-linear combination channels under the proper loading factors, and adding the bias from the external source, the total output as the following.

$$E_0+\sum_{r=1}^{n+1}\lambda_r y_r=E_0+\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_0)_r+\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_1)_r x+\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_2)_r x^2+\cdots\cdots$$

$$\cdots\cdots+\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_n)_r x^n \tag{10}$$

Then the values of loading factors (λ_1, λ_2, λ_3, ... ,λ_n) and the bias E_0 may be uniformly determined corresponding to (6), as shown in (10).

$$\frac{E_0+\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_0)_r}{\alpha_0}=\frac{\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_1)_r}{\alpha_1}=\frac{\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_2)_r}{\alpha_2}=\cdots\cdots\frac{\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_n)_r}{\alpha_n}=\delta \tag{10}$$

where δ : arbitrary number

Thus the total output which is proportional to the input signal may be obtained. As the result, the formula of the output signal comes to be approxi mated to the desired function under the condition as (10), and is shown in (11).

$$E_0+\sum_{r=1}^{n+1}\lambda_r y_r \simeq \delta\cdot f(x)$$

2) Loading factor in actual

Consider the ratio loading factor as defined by the following relation for the convenience of generating functions.

$$1>|\lambda_r'|=|\lambda_r|\Big/|\lambda_m|>0$$

where $r=1,2,3,\cdots\cdots\cdots\cdots,n,\ n+1$

λ_m: loading factor having the maximum absolute value

Thence, the total output voltage of the non-linear combination channels may be given in (12), from (5) to (11).

$$E_0'+\sum_{r=1}^{n+1}e_r\simeq E_0'+\sum_{r=1}^{n+1}\lambda_r'\,[g(u)]_r$$

$$\simeq\frac{k_2v_0}{\lambda_m}\left\{E_0+\sum_{r=1}^{n+1}\lambda_r y_r\right\}$$

$$=\frac{k_2v_0}{\lambda_m}\left\{E_0+\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_0)_r+\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_1)_r x+\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_2)_r x^2+\cdots\cdots+\cdots\cdots\sum_{r=1}^{n+1}\lambda_r(\mathrm{K}_n)_r x^n\right\}$$

$$=\frac{k_2v_0}{\lambda_m}\left\{\alpha_0+\alpha_1x+\alpha_2x^2+\cdots\cdots+\alpha_nx^n\right\}$$

$$=\delta'\cdot f(x) \tag{12}$$

where $E_0' = \frac{k_2 v_0}{\lambda_m} E_0$, $\delta' = \frac{k_2 v_0}{\lambda_m}$

In this equation, e_r is defined to be a output voltage of non-linear combination channel under a loading factor, as shown in Fig. 2, and this value is the amount of voltage drop across the resistor at the tap of potentiometer. In this case, if the muximum value of loading factor is of the reference value (equal to 1) and corresponds to the amount of resistance of potentiometer, the others may be proportional to the respective values at the taps. For the sake of convenience, the precise value of the loading factor may be determined by the value of the potentiometer dial.

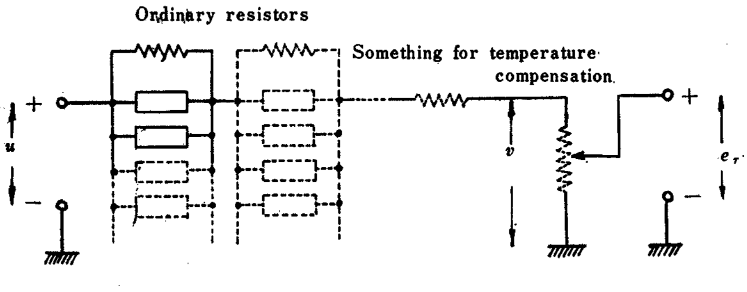


Fig. 2 A single non-linear combination channel consisting of non-ohmic resistors, ordinary resistors and a potentiometer (1000 Ω)

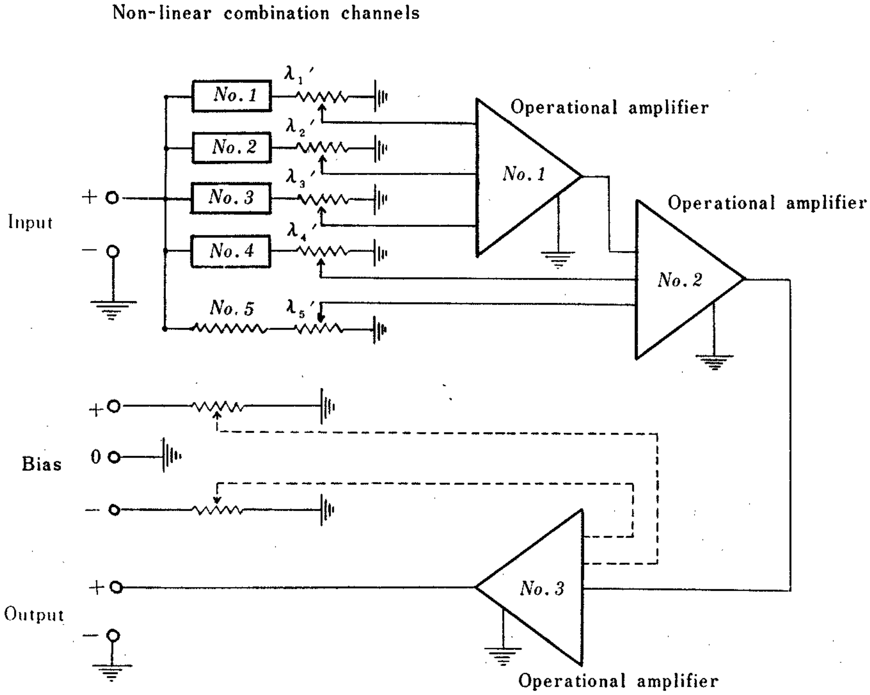


Fig. 3 Block diagram of the arbitrary function generator, having four non-linear combination channels one ordinary resistor channel and bias circits

Practice

1) Apparatus

A new apparatus for generating an arbitrary function has some non-linear combination channels and three operational amplifiers, as shown in Fig. 3. A single combination consists of some non-ohmic resistors and ordinary resistors, and has a potentiometer for giving the precise value of the loading factor. If desired, some thermally sensitive resistor may be added to the combination for the temperature compensation. These make one channel. The first operational amplifier is used for summing the output voltages of the non-linear combination channels under the positive loading factor, the second is for the negative loading factor and the third is for adjusting the amplitude of the output voltage.

2) Generation of the sine

In this case, the first, the sine-function must be properly approximated by a polynomial. The equation as given in (13) is one of the proper approximated polynomial of the sine in the interval $(0. + \pi)$.

$$a \sin x \simeq y = a\,(\mathrm{P}x + \mathrm{Q}x^3 + \mathrm{R}x^3 + \mathrm{S}x^4) \qquad (13)$$

with $\mathrm{P} = +3.087/\pi \qquad \mathrm{Q} = +0.567/\pi^2$

$\mathrm{R} = -7.308/\pi^3 \qquad \mathrm{S} = +3.654/\pi^4$

in $0 \leqq x \leqq +\pi.$

Then the full scale error varies only between + 0.04 and − 0.17 per cent. The error curve of this equation will be shown in Fig. 4.

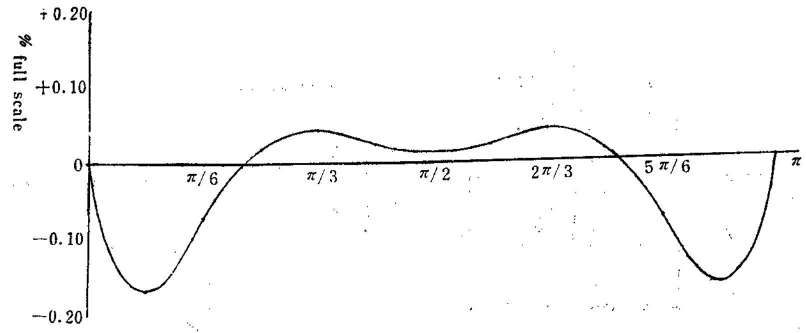


Fig. 4 Error curve in the approximation of sin x $(0 \leqq x \leqq +\pi)$ by the polynominal
$\sin x \simeq 3.087\, x/\pi + 0.567\, x^2/\pi^2 - 7.308\, x^3/\pi^3 + 3.654\, x^4/\pi^4$

The second, the non-linearity of the respective combination channels must be properly represented by the polynomial approximation for all over range of the input voltages. Each value of the coefficients, of course, may be determined by the same methods as aforementioned. Four non-linear combination channels and one ordinary resistor channel are provided in this device. Each non-linear combination has some silicon carbide varistors for taking advantage of their origin symmetry. Each behaviour of the non-linear combination channels is properly approximated by the polynomials as given in (15), when the input voltages varied between 0 and 40 volts. (See, Fig. 5.)

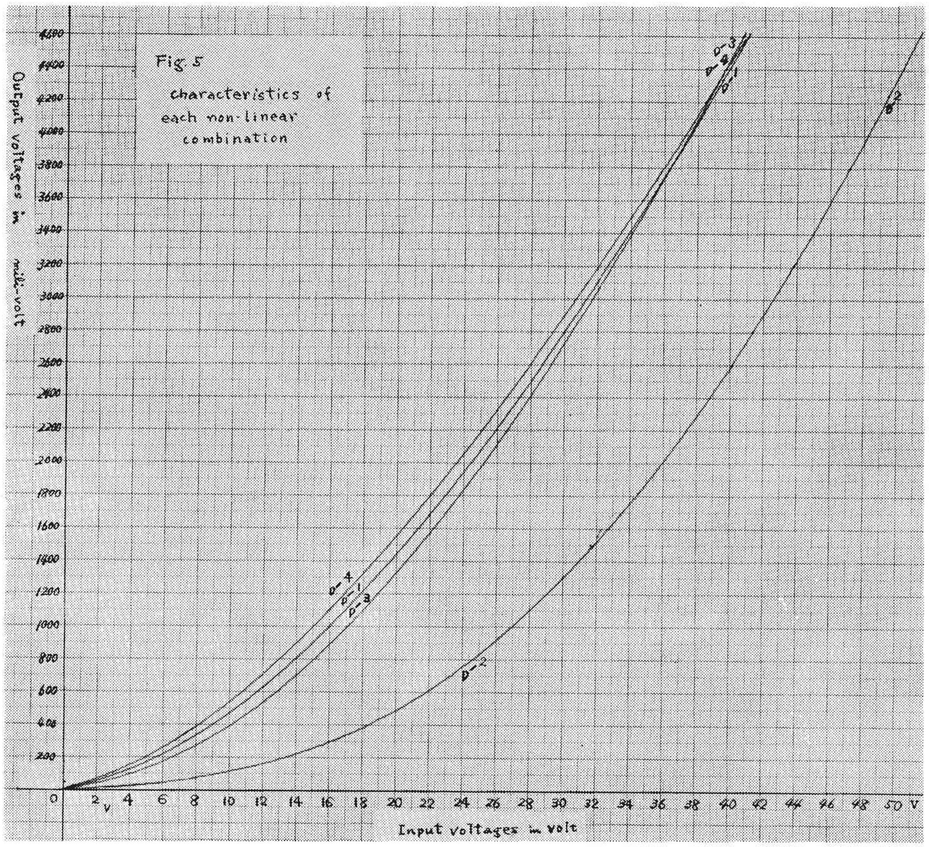


Fig. 5 Characteristics of these non-linear combinations

for no. 1,

$$v_1 = 2.091 \times 10^{-8}u + 2.908 \times 10^{-9}\, u^2 - 2.519 \times 10^{-11}u^3 + 8.601 \times 10^{-13}u^4 - 1.688 \times 10^{-14}u^5 \quad \text{(in volt)}$$

with an error less than 0.80 per cent of full scale for the range of an input of 40 volts peak, less than 0.05 per cent for less than 20 volts,

for no. 2,

$$v_2 = 3.323 \times 10^{-9}u + 5.416 \times 10^{-10}u^2 + 2.431 \times 10^{-11}u^3 + 1.698 \times 10^{-13}u^4 - 3.656 \times 10^{-15}u^5 \quad \text{(in volt)}$$

with an error less than 0.10 per cent of full scale for the range of an input of 40 volts peak, less than 0.05 per cent for less than 20 volts,

for no. 3,

$$v_3 = 1.540 \times 10^{-8}u + 1.889 \times 10^{-9}\, u^2 + 8.474 \times 10^{-11}u^3 - 3.315 \times 10^{-12}u^4 + 3.938 \times 10^{-14}u^5 \quad \text{(in volt)} \qquad (15)$$

with an error less than 0.80 per cent of full scale for the range of an input of 40 volts peak, less than 0.11 per cent for less than 20 volts.

for no. 4,

$$v_4 = 2.666 \times 10^{-8}u + 2.786 \times 10^{-9}\, u^2 + 1.427 \times 10^{-11}u^3 - 1.748 \times 10^{-12}u^4 + 2.531 \times 10^{-14}u \quad \text{(in volt)}$$

with an error less than 0.26 per cent of full scale for the range of an input of 40 volts peak, less than 0.21 per cent for less than 20 volts, and

for no. 5,

$$v_5 = 2.000 \times 10^{-7} u$$

with 0.10 per cent accuracy.

The third, the precise values of the loading factor must be determined. These may be obtained respectively by solving the linea requations as (17). Before this, it is convenient to transform the value of each coefficient in the approximated polynomial of the sine as given in (16), in order that the output voltage (in volt), which is obtained by summing every equation in (15) multiplied by the values of the respective loading factors, may become equal to the muximum value or the amplitude of sine when the input voltage of 18 volts applied to the circuit.

$$\left.\begin{array}{ll} P' = +3.087\,\kappa/36 & Q' = +0.567\,\kappa/36 \\ R' = -7.308\,\kappa/36 & S' = +3.654\,\kappa/36 \end{array}\right\} \quad (16)$$

where κ: arbitrary coefficient

therefore,

$$\left.\begin{array}{l} +20.91\,\lambda_1 + 3.323\,\lambda_2 + 15.30\,\lambda_3 + 26.66\,\lambda_4 + 200.0\,\lambda_5 = P' \times 10^9 \\ +29.08\,\lambda_1 + 5.416\,\lambda_2 + 18.89\,\lambda_3 + 27.86\,\lambda_4 = Q' \times 10^{10} \\ -2.519\,\lambda_1 + 2.431\,\lambda_2 + 8.474\,\lambda_3 + 1.427\,\lambda_4 = R' \times 10^{11} \\ +8.601\,\lambda_1 + 1.698\,\lambda_2 - 33.15\,\lambda_3 - 17.48\,\lambda_4 = S' \times 10^{13} \\ -16.88\,\lambda_1 - 3.655\,\lambda_2 + 39.38\,\lambda_3 + 25.31\,\lambda_4 = 0 \end{array}\right\} \quad (17)$$

Hence,

$$\left.\begin{array}{ll} \lambda_1' = \lambda_1/\lambda_4 = -0.3989 & \lambda_2' = \lambda_2/\lambda_4 = +0.0440 \\ \lambda_3' = \lambda_3/\lambda_4 = -0.8096 & \lambda_4' = \lambda_4/\lambda_4 = +1.0000 \\ \lambda_5' = \lambda_5/\lambda_4 = +0.0885 & \end{array}\right\} \quad (18)$$

And the last is the calculation of the amplitude of this approximated sine. This may be obtained by summing the respective output voltages of the non-linear combination channels under the respective loading factors, when the input voltage of 18 volts is applied there. These are -0.490 for no. 1, $+0.017$ for no. 2, -0.898 for no. 3, $+1.330$ for no. 4 and $+0.318$ for no. 5 (each in volt).

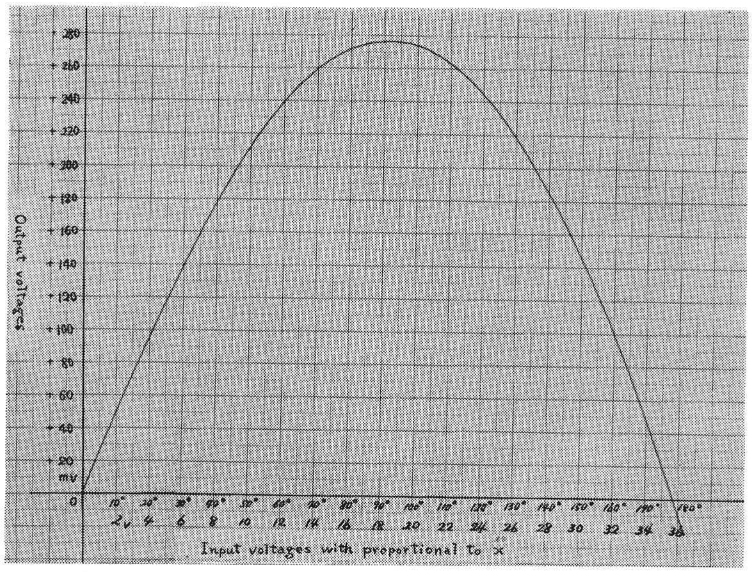
Thus the amplitude of the approximated sine results in -0.276 volt for inputs of ± 36 volts peak.

The error in this case is less than 0.36 per cent of full scale in the interval $(-5\pi/9, +5\pi/9)$.

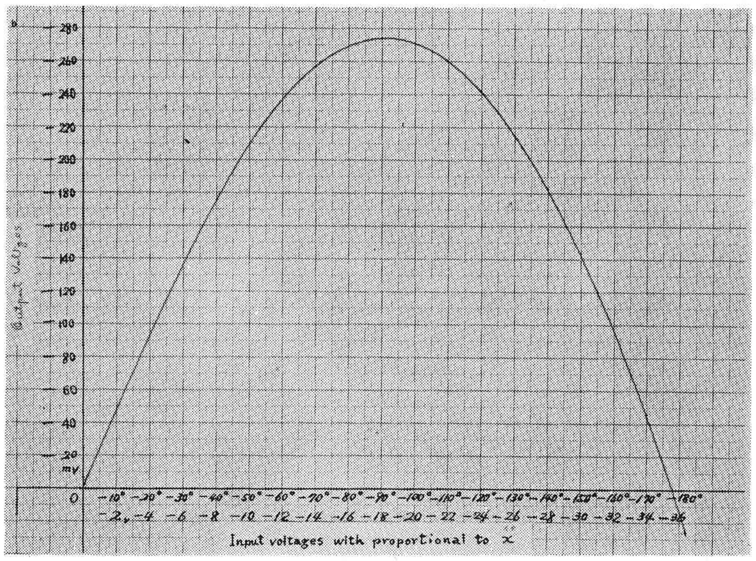
An empirical result is shown in Fig. 6-*a*, 6-*b* and its error curve is shown in Fig. 6-*c*, 6-*d*.

Fig. 6 Empirical result in the generation of sin x ($-\pi \leqq x \leqq +\pi$)

a) Output voltage for positive input
sin x ($0 \leqq x \leqq +\pi$)



b) Output voltage for negative input
sin x ($-\pi \leqq x \leqq 0$)



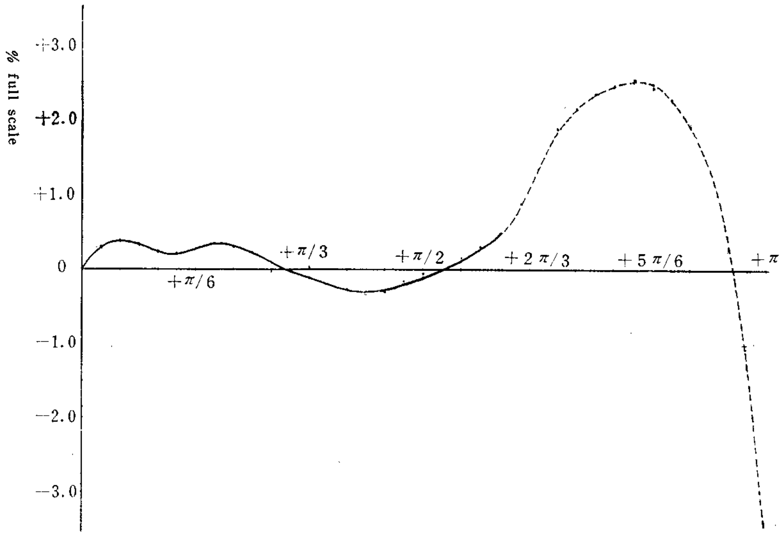


Fig. 6–*c* Error curve of empirical output for positive input †

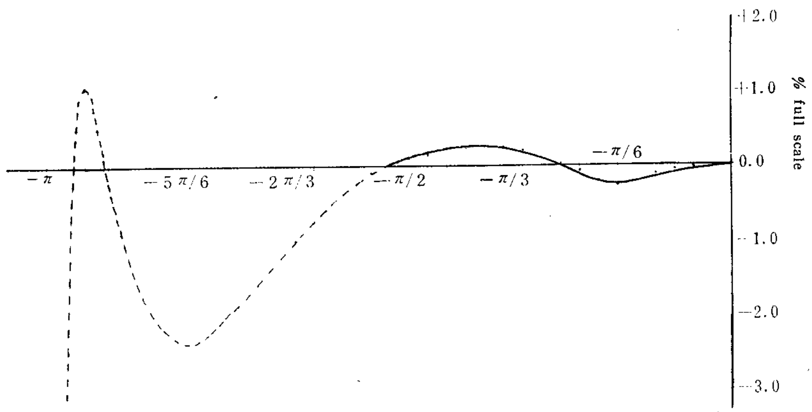


Fig. 6–*d* Error curve of empirical output for negative input †

† Empirical error shown in Fig. 6-*c*, 6-*d* may be reduced by the precise measurement of the characteristics of non-linear combinations and proper temperature copensation.

Discussion

1) Polynomial approximation versus straight-line approximation

There is no need to dwell upon the advantages of making uses of polynomials in the approximation of any function which is continuous in a cirtain interval.

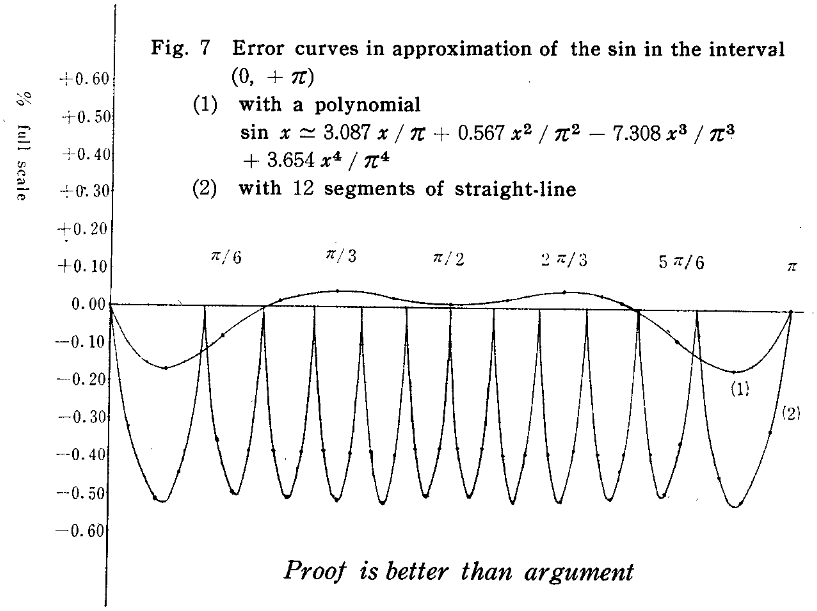


Fig. 7 Error curves in approximation of the sin in the interval $(0, +\pi)$
(1) with a polynomial
$\sin x \simeq 3.087\, x / \pi + 0.567\, x^2 / \pi^2 - 7.308\, x^3 / \pi^3 + 3.654\, x^4 / \pi^4$
(2) with 12 segments of straight-line

Proof is better than argument

2) Value of output voltage

There are some difficulties in the mehtods of generating arbitrary functions making use of the polynomial approximation. These are 1) low output voltage, 2) necessity of solving the determinants to determine the precise values of the loading factor for the desired function and 3) inability of predicting the values of output voltages for variable inputs. Regarding to 1), by selecting the non-linear combinations suitable for the respective purposes, and to 2), by providing some samples of non-linear combination channels for the functions, these may be improved. The problem of 3) will be avoidable as long as taking advantage of the polynomial approximation methods of generating the arbitrary functions.

3) Temperature compensation

Non-ohmic resistance has a large temperature coeffiicient in general. A source of error not yet considered in generating functions is due to the temperature coefficient of the non-obmic resistor. Exessive current through the non-obmic resistor will cause internal temperature rise, and result in an undesirable variation of the characteristic of the unit. It must be restrained as small as possible.

Silicon carbide varistor, having a large negative temperature coefficient, may be compensated over reasonable range of temperature by adding something whose temperature coefficient has the value of opposite sign. Posistor which is a kind of thermister, will serve this purpose because of the large positive temperature coefficient.

Using the units enclosed in a temperature-controlled environment may also bring about good results.

Once the non-linear combination has been properly compensated for the variations of characteristics, there is available a suitable device for generating functions.

In the device presented here, the current is controlled by an ordinary resistor connected in series, in order to keep the current under a lower level, and the amount of resistance which will be added must be determined so as not to disturb the non-linear behavionr of the combination. A unit embodied is packaged. The octal-base enclosures for radio frequency crystals are ideal for this purpose. (See, Photo. 1)

Moreover, a suitable thermistor being there, temperature-compensation for the silicon carbide varistor would be acomplished.

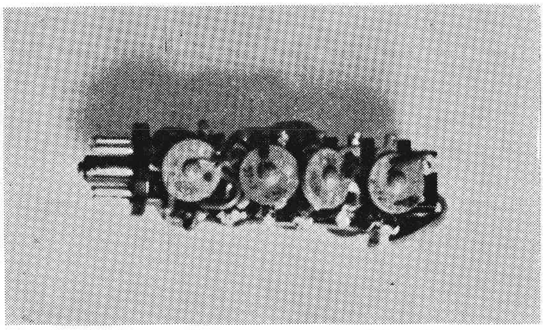


Photo. 1-*a* Non-linear combinations

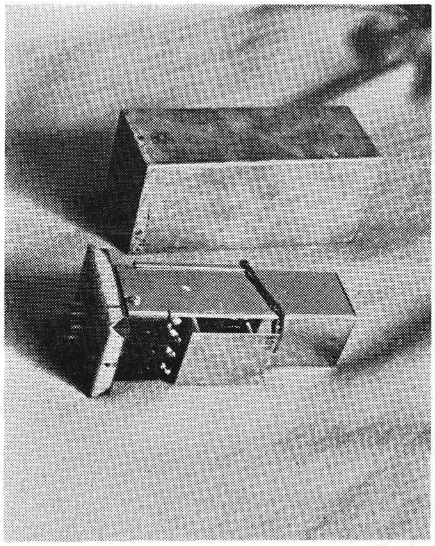


Photo. 1-*b* Temperature-controlled enclosure

4) Versatility of silicon carbide varistor

Silicon carbide varistor may be used to generate functions because of its non-linearity and non-polarity. The most advantageous behaviour of the silicon carbide varistor is due to origin symmetry. The behaviour is greatly convenient in generating functions with origin symmetry, rather than axis symmetry. The versatility of taking use of the silicon carbide varistor stems from its unique characteristic that an algebraic function, which ought to be axis symmetry, is presented as origin symmetry. Several examples of this relation is shown in Fig. 8-*a*, 8-*b*.

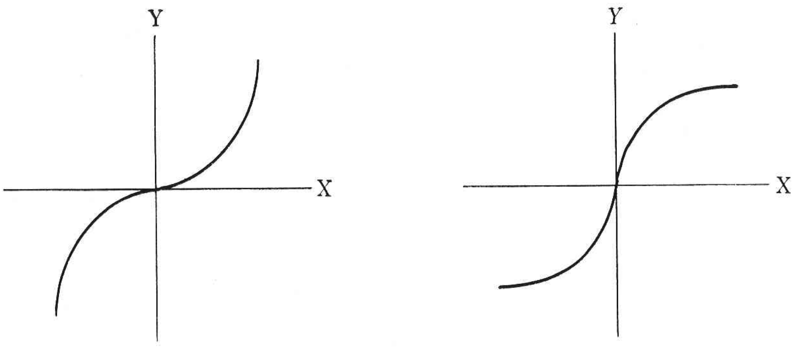
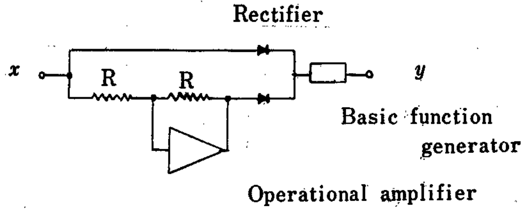


Fig. 8-*a* $y = x|x|$ Fig. 8-*b* $y|y| = x$

The basic output voltages of the function generator using silicon carbide varistor with origin symmetry

The axis symmetry required of any function may be obtained by applying the absolute value of the inputs variable, or of the outputs. These applications are shown in Fig. 9-*a* ~ 9*h*.



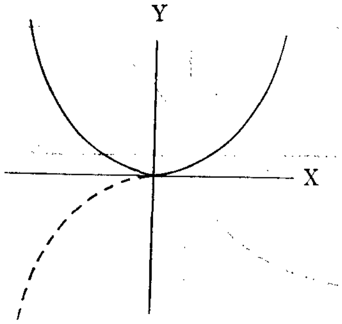


Fig. 9-*a* $y = x^2$

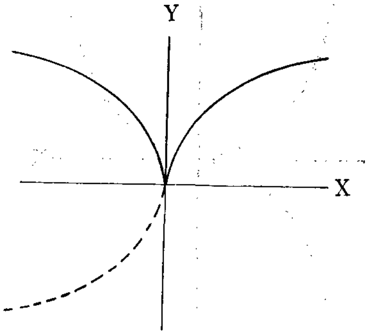
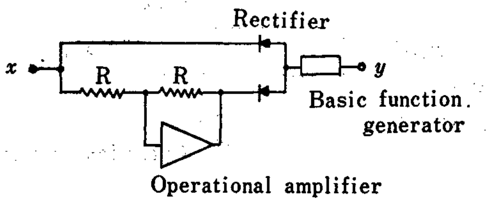


Fig. 9-*b* $y = \sqrt{|x|}$

The functions with positive Y-axis symmetry using Silicon Carbide Varistor obtained by applying the absolute value of the input variable are acquired by the transposition of each curve in the 1st-quarter to the 2nd.



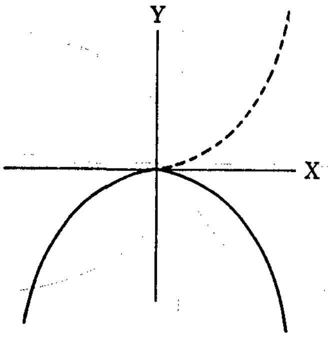


Fig. 9-*c* $y = -x^2$

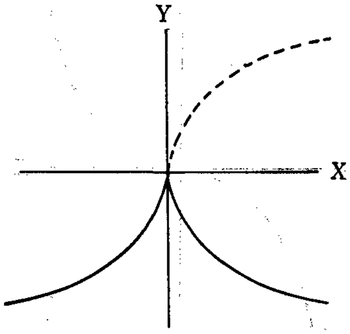
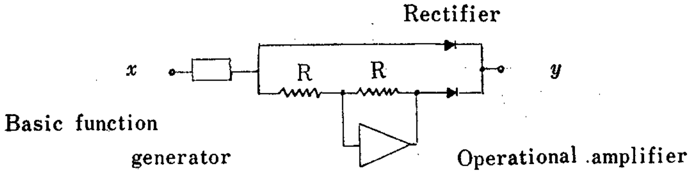


Fig. 9-*d* $-y = \sqrt{|x|}$

The functions with negative Y-axis symmetry using Silicon Carbide Varistor obtained by applying the absolute value of the input variable are acquired by the transposition of each curve in the 3rd-quarter to the 4th.



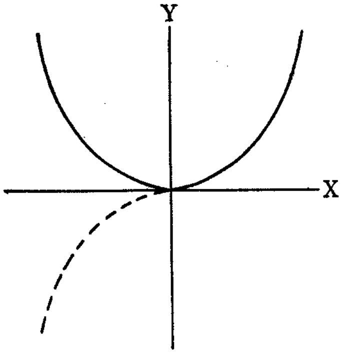


Fig. 9-*e* $y = x^2$

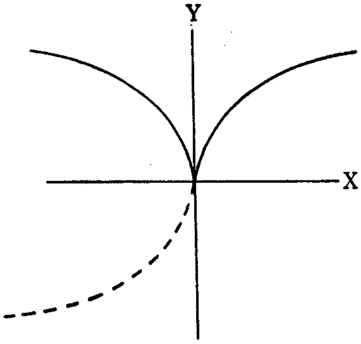
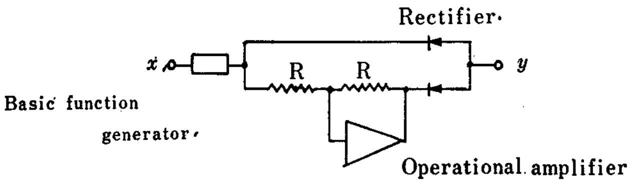


Fig. 9-*f* $-y = \sqrt{|x|}$

The functions with positive Y-axis symmetry using Silicon Carbide Varistor obtained by applying the absolute value of the output variable are acquired by the transposition of each curve in the 3rd-quarter to the 2nd.



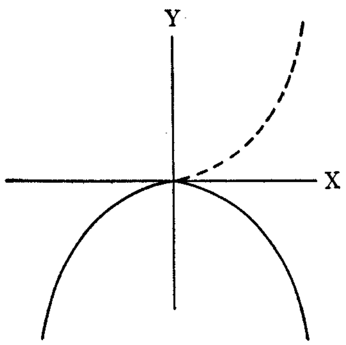


Fig. 9-*g* $y = -x^2$

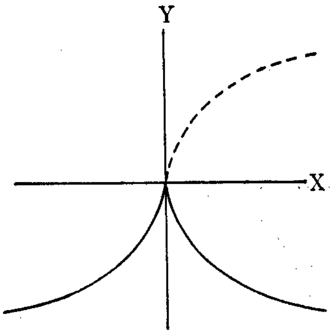


Fig. 9-*h* $-y = \sqrt{|x|}$

The functions with negative Y-axis symmetry using Silicon Carbide Varistor obtained by applying the absolute value of the output variable are acquired by the transposition of each curve in the 1st-quarter to the 4th.

Real silicon carbide varistor, however, may represent a slight rectification effect displayed by materials. If a single silicon carbide varistor is used for generating origin symmetric functions, the full scale error due to this effect will be above 3 per cent. This source of error may be practically eliminated by connecting a number of silicon carbide varistors in parallel so that the rectification effects of the elements in the combination may cancel one another.

5) Other applications

The square and the square root circuits will provide "Multiplier" making use of the so called "quarter-square" identity, and "Square root of a sum of squares", and these circuits may ae also useful for "Signal Expander", "Signal Suppressor" and so on.

In other interesting applications taking advantage of the non-linearity and origin symmetry of the silicon carbide varistor, there are "Frequency multiplier" and "Phase-sensitive modulator", etc.

For these purposes, modufied non-linear combinations having the most suitable behaviour can be provided.

Further investigations of these applications will be described in the next opportunity.

Conclusion

A new method of generating arbitrary functions is just established, as aforementioned, which is based on summing the output voltages of non-linear combination channels for all over range of the inputs. This method brings about good results in generating any function, whenever it is properly approximated by polynomials in the desired interval where it is continuous.

New device has excellent characteristics. These are 1) possibilities of generating arbitrary functions with any commercially available non-ohmic resistors, 2) n-times continuous differenciability, without any cusp or break point, 3) a high degree of accuracy and reliability, 4) common use for usual electronic analog computers whether low speed type's or high speed type's, with same degree of accuracy and reliability, 5), generation of origin symmetric functions by using silicon carbide varistors, 6) unnecessities of intericated systems, 6) non-expensive, and others.

Authors believe that this is a noticeable device in generating arbitrary functions for an electronic analog computing system.

Supplement

Further improvement in an approximation of the sine-function by a polynomial was recently introduced by authors. That is,

$$\sin x \simeq 3.101\, x / \pi + 0.4850\, x^2 / \pi^2 - 7.160\, x^3 / \pi^3 + 3.582\, x^4 / \pi^4$$

$$(0 \leqq x \leqq +\pi)$$

The error of this equation varies only between -0.10 and $+0.08$ per cent of full scale, and the error curve is shown Fig. 10.

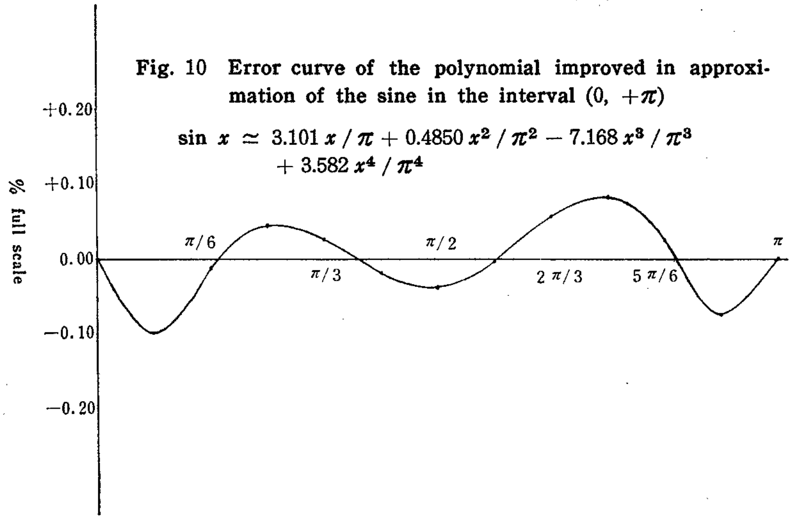


Fig. 10 Error curve of the polynomial improved in approximation of the sine in the interval (0, $+\pi$)

$$\sin x \simeq 3.101\, x/\pi + 0.4850\, x^2/\pi^2 - 7.168\, x^3/\pi^3 + 3.582\, x^4/\pi^4$$

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