Foundations for the Boundary Problems of the Polyphase Transmission Lines Considering the Initial Conditions-II

Foundations for the Boundary Problems of the Polyphase Transmission Lines Considering the Initial Conditions–II

Masao KIDO* and Yoshio INAGAKI*

(Received June 30, 1964)

In this paper, physical concepts of the equations in the previous report will be clarified, noticing the conventional traveling wave theory have been based on symmetrical no-loss lines.

I Introduction

The general formulas for the voltages and currents on n-conductor system have been deduced in the previous report[1]. Generally speaking, the problem is to find voltage and current distributions after some specified disturbance, taking into account of arbitrary boundary conditions. But it is not easy to arrive at solutions usually, and the largest difficulties are encountered in the step of inverse Laplace transformation.

Thus, by way of illustration of physical concept, the conditions must be simplified, such as the conventional traveling wave theory have been based on symmetrical no-loss lines[2]. In this report, the application of the general equations derived is restricted to two-wire and three-wire circuits, since these very simple multi-conductor circuits adequately illustrate the methods of analysis with a minimum amount of algebraic exercise. Increasing the number of conductors involved merely magnifies the amount of algebra that must be done, without serving any other useful purpose.

II Application for Free Oscillations or Induced Lightning Surges

Suppose that the lines in two- or three-conductor system are closed by constant resistances R'''s and there are no impressed e.m.f.s at both ends. Such a system is shown in Fig. 1.

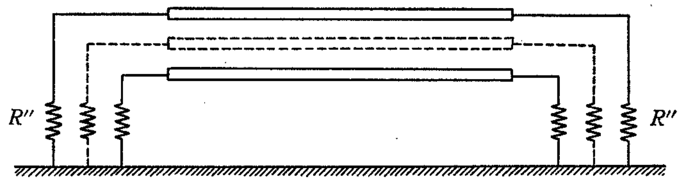

Fig. 1. Multi-conductor system grounded through resistances R'''s at both ends of the lines.

* Department of Electrical Engineering, College of Engineering.

All lines will be taken as ideal, that is, no-loss lines, so that it suffers no changes as a traveling wave moves along the line. Then, provided that the conductors are all of the same shape and symmetrically arranged, the matrices may be written[3], for two-conductor system,

$$\left.\begin{aligned} [L] &= \begin{bmatrix} L & M \\ M & L \end{bmatrix}, \quad [C] = \begin{bmatrix} C & C' \\ C' & C \end{bmatrix} \\ [R] &= [0], \qquad [G] = [0] \end{aligned}\right\} \tag{1}$$

Accordingly the latent roots of $[k]^2$ are

$$q_1 = \frac{s}{g_1}, \quad q_2 = \frac{s}{g_2} \tag{2}$$

where

$$g_1 = \frac{1}{\sqrt{(L+M)(C+C')}}, \quad g_2 = \frac{1}{\sqrt{(L-M)(C-C')}} \tag{3}$$

For three-conductor system, let

$$[L] = \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix}, \quad [C] = \begin{bmatrix} C & C' & C' \\ C' & C & C' \\ C' & C' & C \end{bmatrix} \tag{4}$$

Then, the latent roots of $[k]^2$ are

$$q_1 = \frac{s}{g_1}, \quad q_2 = \frac{s}{g_2} \tag{5}$$

in which

$$g_1 = \frac{1}{\sqrt{(L+2M)(C+2C')}}, \quad g_2 = \frac{1}{\sqrt{(L-M)(C-C')}} \tag{6}$$

Corresponding to g_1 and g_2, there are numerically two different values for velocity of propagation which satisfy the conditions for wave motion, and these values are given by (3) or (6). The waves having these velocities may be either zero-sequence components or positive-sequence components of moving waves.

Now, the first case to which this theory will be applied is "free oscillation" or "induced lightning surge"[2]. When a charged cloud approaches transmission lines and the cloud charge is suddenly removed by lightning discharge, the released bound charges on the lines become traveling waves. As an approximate analytical method[1),4], suppose that the initial voltage $E_{r,t=0}$ is uniformly distributed and there can be no current initially for the line wire r.

Then, the line voltage follows from Eqs. (20) in previous report I, upon integrating with respect to x. The inverse transformation yielding the result as a function of time is, for line wire r,

$$\begin{aligned} E_r = (\rho_0+\rho_r)H(t) - \frac{1}{2}\sum_{n=0}^{\infty}\Big[\rho_0(1-\sigma_1)\sigma_1^n\Big\{H\Big(t-\frac{nl+x}{g_1}\Big) + H\Big(t-\frac{\overline{n+1}\cdot l-x}{g_1}\Big)\Big\} \\ +\rho_r(1-\sigma_2)\sigma_2^n\Big\{H\Big(t-\frac{nl+x}{g_2}\Big) + H\Big(t-\frac{\overline{n+1}\cdot l-x}{g_2}\Big)\Big\}\Big] \end{aligned} \tag{7}$$

in which

$$\sigma_1=\frac{R''-z_1}{R''+z_1},\qquad \sigma_2=\frac{R''-z_2}{R''+z_2} \tag{8}$$

$$z_1=\sqrt{\frac{L+M}{C+C'}},\quad z_2=\sqrt{\frac{L-M}{C-C'}}\qquad \text{for two-conductor system}$$

$$z_1=\sqrt{\frac{L+2M}{C+2C'}},\quad z_2=\sqrt{\frac{L-M}{C-C'}}\qquad \text{for three-conductor system}$$

$$H\left(t-\frac{\delta}{g}\right)=\begin{cases}0, & t<\dfrac{\delta}{g}\\ 1, & t\geqq\dfrac{\delta}{g}\end{cases}$$

In the following, illustration of physical concepts of Eq. (7) will be clarified.

$(\rho_0+\rho_r)H(t)$ denotes, obviously, the voltage wave distributed initially along the line wire r. Therefore, noticing only the voltage waves of ρ_0 having the velocity g_1 and rewriting them,

$$\begin{aligned}\rho_0\cdot H(t)-\frac{1}{2}\sum_{n=0}^{\infty}\rho_0(1-\sigma_1)\sigma_1^n\left\{H\left(t-\frac{nl+x}{g_1}\right)+H\left(t-\frac{\overline{n+1}\cdot l-x}{g_1}\right)\right\}\\ =\frac{\rho_0}{2}\left\{H(t)+(-1+\sigma_1)H\left(t-\frac{x}{g_1}\right)+(-\sigma_1+\sigma_1^2)H\left(t-\frac{l+x}{g_1}\right)+\cdots\cdots\right\}\\ +\frac{\rho_0}{2}\left\{H(t)+(-1+\sigma_1)H\left(t-\frac{l-x}{g_1}\right)+(-\sigma_1+\sigma_1^2)H\left(t-\frac{2l-x}{g_1}\right)+\cdots\cdots\right\}\end{aligned} \tag{9}$$

$H\left(t-\frac{nl+x}{g_1}\right)$ and $H\left(t-\frac{\overline{n+1}\cdot l-x}{g_1}\right)$ mean that the functions start and traveling waves reach the point x at $t=\frac{nl+x}{g_1}$ and $t=\frac{(n+1)l-x}{g_1}$, respectively. If a transmission line of surge impedance z_1 is closed by a resistance R'', the reflected voltage wave is equal in magnitude to σ_1-times of the incident wave[3),4)]. Thus, inspection of (9) shows that the initial voltage distribution, represented by $\rho_0\cdot H(t)$, is propagated as traveling waves and each may consist of forward waves and backward waves moving in opposite directions, which are equal in magnitude and of the same shape, i.e. $\rho_0/2$.

$-\frac{\rho_0}{2}\cdot H\left(t-\frac{x}{g_1}\right)$ means the component moving along the line and vanishing in the positive direction of x. When the wave reaches the line end, it is refracted accordance with the above-mentioned principle. $\sigma_1\cdot\frac{\rho_0}{2}\cdot H\left(t-\frac{x}{g_1}\right)$ is the forward wave, which was reflected as soon as the phenomena starts. $-\sigma_1\frac{\rho_0}{2}\cdot H\left(t-\frac{l+x}{g_1}\right)$ is the wave which was suffered with reflection at $x=0$ and vanished in the positive direction of x, however $\sigma_1^2\frac{\rho_0}{2}\cdot H\left(t-\frac{l+x}{g_1}\right)$ denotes the forward wave reflected back at both line ends.

Similary, $(-1+\sigma_1)\frac{\rho_0}{2}H\left(t-\frac{l-x}{g_1}\right)$ is the backward wave which may consist of the wave, vanished in negative direction of x with no reflection, and the one reflected back at $x=l$. $(-\sigma_1+\sigma_1^2)\frac{\rho_0}{2}H\left(t-\frac{2l-x}{g_1}\right)$ represents the wave, vanished in negative direction suffering with reflection, and the wave reflected at both ends. Continuing this process, history of the wave easily will be seen, from physical considerations of Eq. (9), that is, one can find where it came from and just what other waves went into its composition, by reflections at both ends of the lines.

In general, there can exist simultaneously on each conductor of two- or three-conductor system two pairs of traveling waves of different velocities, g_1, g_2, and, thus each pair consists of forward and backward waves, as those aforementioned. In this case, the initial voltage distributions are propagated as traveling waves, moving along the lines and repeating the reflections at both ends, after the phenomena starts. The total potential at any point at any instant of time is the superposition of all the waves which have arrived at that point until that instant of time.

Returning to the equation of type (7) and calculating the line current at point x, there is

$$I_r = \frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{\rho_0}{z_1}(1-\sigma_1)\sigma_1^n\left\{-H\left(t-\frac{nl+x}{g_1}\right)+H\left(t-\frac{\overline{n+1}\cdot l-x}{g_1}\right)\right\}\right.$$
$$\left.+\frac{\rho_r}{z_2}(1-\sigma_2)\sigma_2^n\left\{-H\left(t-\frac{nl+x}{g_2}\right)+H\left(t-\frac{\overline{n+1}\cdot l-x}{g_2}\right)\right\}\right] \tag{10}$$

It can be noticed upon comparison (7) and (10) that

$$\text{Line voltage} = \text{Initial distribution} + z_1\times(\text{Forward current wave components})$$
$$+z_2\times(\text{Backward current wave components}) \tag{11}$$

that is, the ratios of voltage to current for forward wave or backward wave—the surge impedances—are constants. They represent the zero-sequence and positive-sequence surge impedances, corresponding the different propagation velocity, respectively, since they have the dimension of *ohms*. Therefore, ρ/z implies the current waves associated with the voltage waves moving on the conductors, owing to the initial distributions[3),4)].

When the surges are impressed on the conductors, they are resolved into one pair of equal waves of the same polarity and one pair of equal waves of opposite polarity. The voltage and current waves are replicas of each other, but while the forward voltage and current waves are of the same sign, the backward voltage and current waves are of opposite sign. Physical concepts of (7) and (10), or (11), can be readily seen from the above point of view.

The fact that the solutions (7) and (10) contain the waves having velocities g_1 and g_2 does not mean that both waves must be actually present. If the equal initial voltages are distributed along the lines, the waves having the velocity g_2 are unnecessary in the solutions.

For an example, assuming that the lines are distortionless and have decrement factor of $\varepsilon^{-\nu t}$ with respect to time[3),4)], the voltage and current waves on the conductors may be written as follows:

$$E_r = \rho_0\varepsilon^{-\nu_1 t}\left[H(t)-\frac{1}{2}\sum_{n=0}^{\infty}(1-\sigma_1)\sigma_1^n\left\{H\left(t-\frac{nl+x}{g_1}\right)+H\left(t-\frac{\overline{n+1}\cdot l-x}{g_1}\right)\right\}\right] \tag{12}$$

$$I_r = \frac{\rho_0}{2z_1}\varepsilon^{-\nu_1 t}\sum_{n=0}^{\infty}\left\{-H\left(t-\frac{nl+x}{g_1}\right)+H\left(t-\frac{\overline{n+1}\cdot l-x}{g_1}\right)\right\} \tag{13}$$

in which

$$\left.\begin{aligned} \nu_1 &= \frac{R+R'}{L+M} \quad \text{for two-conductor system} \\ \nu_1 &= \frac{R+2R'}{L+2M} \quad \text{for three-conductor system} \end{aligned}\right\} \tag{14}$$

and the resistance matrix of the multi-conductor system is defined as

$$[R] = \begin{bmatrix} R & R' \\ R' & R \end{bmatrix} \quad \text{or} \quad [R] = \begin{bmatrix} R & R' & R' \\ R' & R & R' \\ R' & R' & R \end{bmatrix}$$

These solutions hold rigorously only at small values of t, but under actual conditions this transient is usually over within a millisecond, and thus steady state has gained any headway. Therefore, in a case of this kind much time is saved by using the Eqs. (12) and (13) directly, rather than reducing from the general equations, by way of exact calculations.

III Breaking Faults or Switching Surges

Fig. 2 illustrates generators of voltage E_{ir} and load connected to the lines. The circuit is normally energized and carrying load until breaking faults suddenly occur. The faults

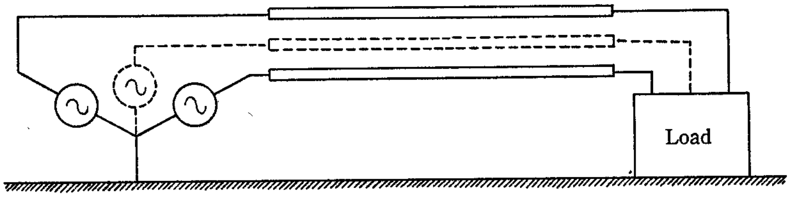

Fig. 2. Transmission system just before breaking faults.

then correspond to the opening of switches in electrical circuit. The opening of the switches changes the circuit so that new distributions of currents and voltages are brough out. These redistributions are accompanied in general by a transient period during which the resultant currents and voltages may momentarily be relatively high.

There are sinusoidal and symmetrical voltages and currents distributed along each conductors in such a system, just before the occurrence of a system fault or opening the switches in the electrical network.

Let, for the r-th line,

$$E_{r,t=0} = A_r \sin(\phi x+\alpha_r)$$

$$\frac{dI_{r,t=0}}{dx} = B_r \sin(\phi x+\beta_r)$$

In previous report were given the general relations for instantaneous voltages and currents, including the transients as well as the foundamental-frequency components. Using the generalized method for distortionless lines, the following equations are obtained:

$$E_r = \varepsilon^{-\nu_2 t}\Big[\Big\{A_r \sin(\phi x+\alpha_r)\cos g_2\phi t - \frac{B_r z_2}{\phi}\sin(\phi x+\beta_r)\sin g_2\phi t\Big\}H(t)$$
$$+\sum_{n=0}^{\infty}(-1)^n\Big[\Big[V_r \sin\Big\{g_2\phi\Big(t-\frac{2nl+x}{g_2}\Big)-\varphi_r\Big\}+E_{ir}\Big]H\Big(t-\frac{2nl+x}{g_2}\Big)$$
$$+\Big[V_r \sin\Big\{g_2\phi\Big(t-\frac{2\cdot\overline{n+1}\cdot l-x}{g_2}\Big)-\varphi_r\Big\}+E_{ir}\Big\}H\Big(t-\frac{2\cdot\overline{n+1}\cdot l-x}{g_2}\Big)$$
$$-V_r'\sin\Big\{g_2\phi\Big(t-\frac{\overline{2n+1}\cdot l-x}{g_2}+\varphi_r'\Big\}H\Big(t-\frac{\overline{2n+1}\cdot l-x}{g_2}\Big)$$
$$+V_r'\sin\Big\{g_2\phi\Big(t-\frac{\overline{2n+1}\cdot l+x}{g_2}\Big)+\varphi_r'\Big\}H\Big(t-\frac{\overline{2n+1}\cdot l+x}{g_2}\Big)\Big]\Big] \qquad (15)$$

$$I_r = -\frac{1}{z_2}\varepsilon^{-\nu_2 t}\Big[\Big\{A_r \cos(\phi x+\alpha_r)\sin g_2\phi t + \frac{B_r z_2}{\phi}\cos(\phi x+\beta_r)\cos g_2\phi t\Big\}H(t)$$
$$-\sum_{n=0}^{\infty}(-1)^n\Big[\Big[V_r \sin\Big\{g_2\phi\Big(t-\frac{2nl+x}{g_2}\Big)-\varphi_r\Big\}+E_{ir}\Big]H\Big(t-\frac{2nl+x}{g_2}\Big)$$
$$-\Big[V_r \sin\Big\{g_2\phi\Big(t-\frac{2\cdot\overline{n+1}\cdot l-x}{g_2}\Big)-\varphi_r\Big\}+E_{ir}\Big]H\Big(t-\frac{2\cdot\overline{n+1}\cdot l-x}{g_2}\Big)$$
$$+V_r'\sin\Big\{g_2\phi\Big(t-\frac{\overline{2n+1}\cdot l-x}{g_2}\Big)+\varphi_r'\Big\}H\Big(t-\frac{\overline{2n+1}\cdot l-x}{g_2}\Big)$$
$$+V_r'\sin\Big\{g_2\phi\Big(t-\frac{\overline{2n+1}\cdot l+x}{g_2}\Big)+\varphi_r'\Big\}H\Big(t-\frac{\overline{2n+1}\cdot l+x}{g_2}\Big)\Big]\Big] \qquad (16)$$

where

$$\nu_2=\frac{R-R'}{L-M},\quad E_{ir}H\Big(t-\frac{\delta}{g}\Big)=E_{ir}\Big(t-\frac{\delta}{g}\Big)\cdot H\Big(t-\frac{\delta}{g}\Big)$$

$$V_r=\Big\{(\eta_r A_1\sin\alpha_1+\zeta_r A_2\sin\alpha_2+\phi_r A_3\sin\alpha_3)^2+\Big(\frac{z_2}{\phi}\Big)^2(\eta_r B_1\sin\beta_1+\zeta_r B_2\sin\beta_2$$
$$+\phi_r B_3\sin\beta_3)^2\Big\}^{1/2}$$

$$\varphi_r=\tan^{-1}\{\phi(\eta_r A_1\sin\alpha_1+\zeta_r A_2\sin\alpha_2+\phi_r A_3\sin\alpha_3)/z_2(\eta_r B_1\sin\beta_1+\zeta_r B_2\sin\beta_2$$
$$+\phi_r B_3\sin\beta_3)\}$$

$$V_r'=[\{\eta_r A_1\cos(\phi l+\alpha_1)+\zeta_r A_2\cos(\phi l+\alpha_2)+\phi_r A_3\cos(\phi l+\alpha_3)\}^2$$
$$+(z_2/\phi)^2\{\eta_r B_1\cos(\phi l+\beta_1)+\zeta_r B_2\cos(\phi l+\beta_2)+\phi_r B_3\cos(\phi l+\beta_3)\}^2]^{1/2}$$

$$\varphi_r'=\tan^{-1}\frac{z_2\{\eta_r B_1\cos(\phi l+\beta_1)+\zeta_r B_2\cos(\phi l+\beta_2)+\phi_r B_3\cos(\phi l+\beta_3)\}}{\phi\{\eta_r A_1\cos(\phi l+\alpha_1)+\zeta_r A_2\cos(\phi l+\alpha_2)+\phi_r A_3\cos(\phi l+\alpha_3)\}}$$

$$\left.\begin{aligned}&\eta_1=\zeta_2=1/2\\&\zeta_1=\eta_2=-1/2,\quad \phi_1=\phi_2=0\end{aligned}\right\}\quad \text{two-conductor system}$$

$$\left.\begin{aligned}&\eta_1=\zeta_2=\phi_3=2/3\\&\zeta_1=\phi_1=\eta_2=\phi_2=\eta_3=\zeta_3=-1/3\end{aligned}\right\}\quad \text{three-conductor system}$$

In above equations, $A_r\sin(\phi x+\alpha_r)$ and $-(B_r/\phi)\sin(\phi x+\beta_r)$ represent the sum of the initially distributed voltages and currents along the lines, respectively. If $\phi=0$ and $B_r=0$, then (15) and (16) reduce to (12) and (13), respectively.

IV Numerical Examples

In this chapter, detailed numerical examples are presented on calculations of induced lightning surges and breaking faults. To estimate the potentials and currents in such cases it is not permissible to assume arbitrary values for any of the parameters in the equations which violate the known confines of any of the other.

As an example, let

$$l/g_1 = 0.1 \text{ ms}, \qquad l/g_2 = 1 \text{ ms}$$
$$z_1 = 500\,\Omega, \qquad z_2 = 400\,\Omega$$

A large number of specific cases calculated from the equations are obtained, of which several representative examples are shown in Fig. 3∼Fig. 5. In these figures, the values corresponding to no-loss lines are plotted in solid curves and those corresponding to distortionless lines are plotted in dashed curves.

There is one aspect of the results worth pointing out. Inspection of these curves in Figs. 3 and 4 shows that for values of t in this range there is little difference, regardless of existence of line losses, between the previous method in report I and the method in this paper. This fact tolds us that the attenuation and distortion due to earth resistivity can not be ignored, but asymmetry in arrangement of the conductors are of little consequence. For this reason, this method of solutions is able to be adapted to engineering calculations. To determine the wave shape with respect to time at any point x on the line prohibits the use of the exact solutions for rapid engineering calculations.

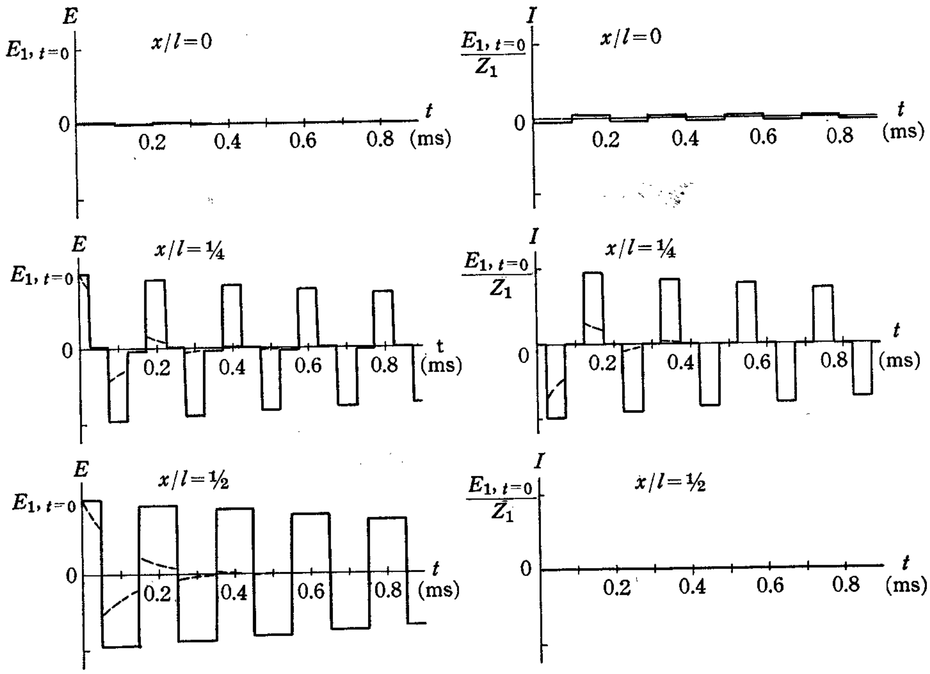

Fig. 3. Potential and current curves of lines as functions of time for $R''=10\,\Omega$.

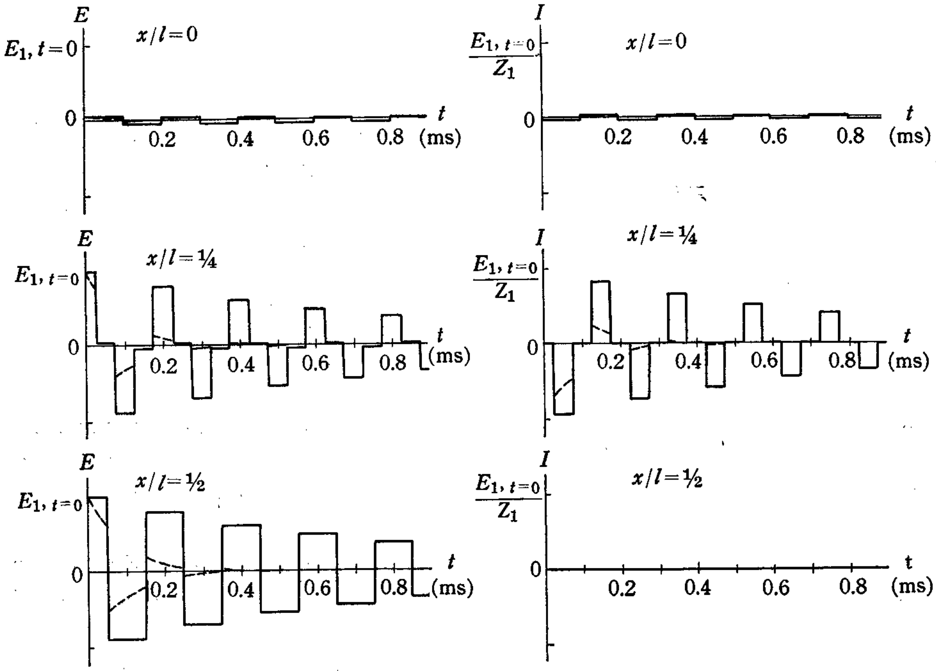

Fig. 4. Potential and current curves of lines as functions of time for $R''=30\ \Omega$.

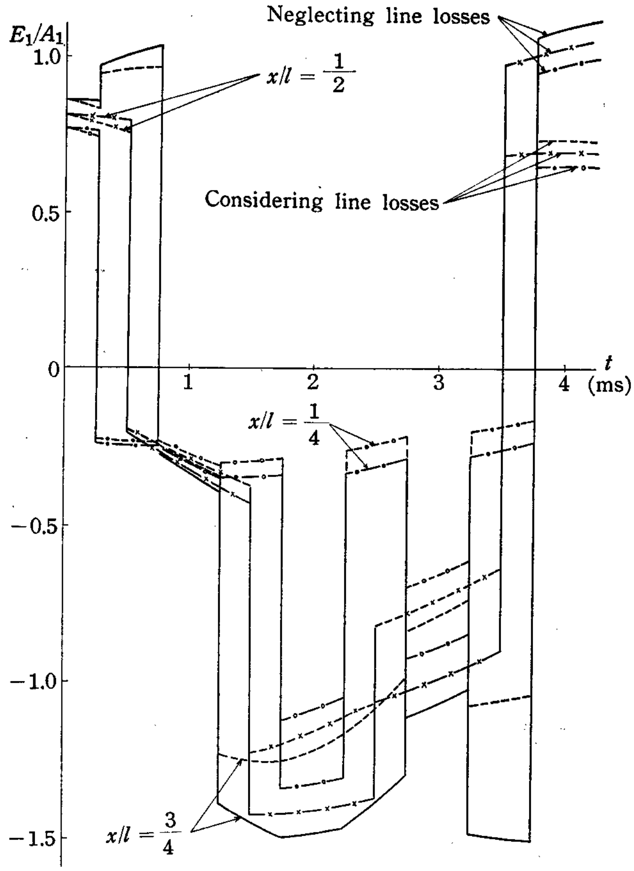

Fig. 5. Overvoltages due to breaking faults.

V Conclusions

In this paper, the most simple problems of two- or three-conductor system were dealt with. More generally, it will be readily proved that there can exist simultaneously on each conductor of a n-phase system two pairs of traveling waves of different velocities and each pair consists of forward and backward waves. They start out together and repeat the reflections at both ends of the lines infinitely.

Previous authors[2)~4)] investigated this transient problems ignoring either initial conditions or boundary conditions. However, it should be remarked that erroneous results will be gained unless both conditions are taken into account.

References

1) M. Kido, Foundations for the Boundary Problems of the Polyphase Transmission Lines Considering the Initial Conditions-I, Bulletin of University of Osaka Prefecture, Vol. 12, No. 2, (1964).
2) L. V. Bewley, Traveling Waves on Transmission Systems, (1951).
3) S. Hayashi, Surges on Transmission Systems, (1955).
4) Y. Moriwaki, Calculation method for Transient Phenomena, (1950).