ANALYSIS OF THE BEHAVIOR OF KURTOSIS

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ABSTRACT

Among many dimensional and dimensionless amplitude parameters, Kurtosis(4-th normalized moment of probability density function) is recognized to be the sensitive good parameter for machine diagnosis. Kurtosis has a value of 3.0 under normal condition and the value generally goes up as the deterioration proceeds. But there are cases that kurtosis value goes up and then goes down when damages increase as time passes. In this paper, simplified calculation method of kurtosis is introduced for the analysis of impact vibration including affiliated impact vibration. Affiliated impact vibration is approximated by triangle and simplified calculation method is introduced. Varying the shape of triangle, various models are examined and above phenomenon is traced and its reason is clarified by the analysis.

Utilizing this method, the behavior of kurtosis is forecasted and analyzed while watching machine condition and correct diagnosis is executed.

Key Words : impact vibration, Kurtosis, deterioration

1. INTRODUCTION

In mass production firms such as steel making that have big equipments, sudden stops of production processes by machine failure cause great damages such as shortage of materials to the later processes, delays to the due date and the increasing idling time.

To prevent these troubles, machine diagnosis techniques play important roles. So far, Time Based Maintenance (TBM) technique has constituted the main stream of the machine maintenance, which makes checks for maintenance at previously fixed time. But it has a weak point that it makes checks at scheduled time without taking into account whether the parts still keeping good conditions or not. On the other hand, Condition Based Maintenance (CBM) makes maintenance checks by watching the condition of machines. Therefore, if the parts are still keeping good condition beyond

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its supposed life, the cost of maintenance may be saved because machines can be used longer than planned. Therefore the use of CBM has become dominant. The latter one needs less cost of parts, less cost of maintenance and leads to lower failure ratio.

However, it is mandatory to catch a symptom of the failure as soon as possible of a transition from TBM to CBM is to be made. Many methods are developed and examined focusing on this subject. In this paper, we propose a method for the early detection of the failure on rotating machines which is the most common theme in machine failure detection field.

So far, many signal processing methods for machine diagnosis have been proposed (Bolleter, 1998). As for sensitive parameters, Kurtosis, Bicoherence, Impact Deterioration Factor (ID Factor) were examined (Yamazaki, 1977; Maekawa et al.1997; Shao et al.2001; Song et al.1998; Takeyasu, 1987,1989). In this paper, we focus our attention to the index parameters of vibration.

Kurtosis is one of the sophisticated inspection parameters which calculates normalized 4th moment of Probability Density Function (PDF). Kurtosis has a value of 3.0 under normal condition and the value generally goes up as the deterioration proceeds.

But there were cases that kurtosis values went up and then went down when damages increased as time passed which were observed in our experiment in the past (Takeyasu,1987,1989).

In this paper, simplified calculation method of kurtosis is introduced for the analysis of impact vibration including affiliated impact vibration. Affiliated impact vibration is approximated by triangle and simplified calculation method is introduced.

Varying the shape of triangle, various models are examined and we try to clarify the reason of above stated phenomenon. If new model state the observed facts well, new method would be utilized effectively in making machine diagnosis.

We survey each index of deterioration in section 2. Simplified calculation method of Kurtosis including affiliated impact vibration is introduced in section 3. Numerical example is exhibited in section 4. Section 5 is a summary.

2. FACTORS FOR VIBRATION CALCULATION

In cyclic movements such as those of bearings and gears, the vibration grows larger whenever the deterioration becomes bigger. Also, it is well known that the vibration grows large when the setting equipment to the ground is unsuitable (Yamazaki, 1977). Assume the vibration signal is the function of time as $x(t)$. And also assume that it is a stationary time series with mean 0. Denote the probability density function of these time series as $p(x)$. Indices for vibration amplitude are as follows.

$$X_{root} = \left[\int_{-\infty}^{\infty} |x|^{\frac{1}{2}} p(x) dx \right]^2 \tag{1}$$

$$X_{rms} = \left[\int_{-\infty}^{\infty} x^2 p(x) dx \right]^{\frac{1}{2}} \tag{2}$$

$$X_{abs} = \int_{-\infty}^{\infty} |x| p(x) dx \tag{3}$$

$$X_{peak} = \lim_{n \to \infty} \left[\int_{-\infty}^{\infty} x^n p(x) dx \right]^{\frac{1}{n}} \tag{4}$$

These are dimensional indices which are not normalized. They differ by machine sizes or rotation frequencies. Therefore, normalized dimensionless indices are required. There are four big categories for this purpose.

A. Normalized root mean square value
B. Normalized peak value
C. Normalized moment
D. Normalized correlation among frequency domain

A. Normalized root mean square value
a.Shape Factor : SF

$$SF = \frac{X_{rms}}{\overline{X}_{abs}} \tag{5}$$

($\overline{X}_{abs}$: mean of the absolute value of vibration)
B. Normalized peak value
b.Crest Factor : CrF

$$CrF = \frac{X_{peak}}{X_{rms}} \tag{6}$$

(X_{peak} : peak value of vibration)
c.Clearance Factor : ClF

$$ClF = \frac{X_{peak}}{X_{root}} \tag{7}$$

d.Impulse Factor : IF

$$IF = \frac{X_{peak}}{\overline{X}_{abs}} \tag{8}$$

e.Impact Deterioration Factor : ID Factor

$$ID = \frac{X_{peak}}{X_c} \tag{9}$$

(X_c : vibration amplitude where the curvature of PDF becomes maximum)

C. Normalized moment

f.Skewness : SK

$$SK = \frac{\int_{-\infty}^{\infty} x^3 p(x)dx}{\left[\int_{-\infty}^{\infty} x^2 p(x)dx\right]^{\frac{3}{2}}} \tag{10}$$

g.Kurtosis : KT

$$KT = \frac{\int_{-\infty}^{\infty} x^4 p(x)dx}{\left[\int_{-\infty}^{\infty} x^2 p(x)dx\right]^{2}} \tag{11}$$

D. Normalized correlation in the frequency domain

h.Bicoherence

Bicoherence means the relationship of a function at different points in the frequency domain and is expressed as

$$Bic,_{xxx}(f_1, f_2) = \frac{B_{xxx}(f_1, f_2)}{\sqrt{S_{xx}(f_1)\cdot S_{xx}(f_2)\cdot S_{xx}(f_1+f_2)}} \tag{12}$$

Here

$$B_{xxx}(f_1, f_2) = \frac{X_T(f_1)\cdot X_T(f_2)\cdot X_T^*(f_1+f_2)}{T^{\frac{3}{2}}} \tag{13}$$

means Bispectrum and

$$X_T(t) = \begin{cases} x(t) & (0 < t < T) \\ 0 & (else) \end{cases}$$

T : Basic Frequency Interval

$$X_T(f) = \int_{-\infty}^{\infty} X_T(t) e^{-j2\pi ft} dt \tag{14}$$

$$S_{xx}(f) = \frac{1}{T} X_T(f) X_T^*(f) \tag{15}$$

Range of Bicoherence satisfies

$$0 < Bic,_{xxx}(f_1, f_2) < 1 \tag{16}$$

When there exists a significant relationship between frequencies f_1 and f_2, Bicoherence is near 1 and otherwise comes close to 0.

These indices are generally used in combination and machine condition is judged totally. Among them, Kurtosis is recognized to be superior index (Noda, 1987) and many researches on this have been made (Maekawa et al.1997; Shao et al.2001; Song et al.1998).

Judging from the experiment we made in the past, we may conclude that Bicoherence is also a sensitive good index (Takeyasu, 1989, 1989).

In Maekawa et al.(1997), ID Factor is proposed as a good index. In this paper, we focusing on the indices of vibration amplitude, simplified calculation method of Kurtosis including affiliated impact vibration is introduced.

Varying the shape of triangle, various models are examined.

3. SIMPLIFIED CALCULATION METHOD OF KURTOSIS

3.1. Several facts on Kurtosis

KT is transformed into the one for discrete time system as

$$KT = \frac{\int_{-\infty}^{\infty} x^4 p(x) dx}{\left[\int_{-\infty}^{\infty} x^2 p(x) dx\right]^2} = \lim_{N \to \infty} \frac{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \overline{x}\right)^4}{\left\{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \overline{x}\right)^2\right\}^2} \tag{17}$$

Here,

$\{x_i\}: i = 1,2,\cdots,N$

are the discrete signal data.

$\bar{x}$ is an average of $\{x_i\}$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Here the variance, the mean, *KT* of N amount of data are stated as

$\sigma_N{}^2$, $\bar{x}_N$, KT_N

3.2. Simplified Calculation Method of Kurtosis

When there arise failures on bearings or gears, peak value arises cyclically. In the early stage of the defect, this peak signal usually appears clearly. Generally, defects will injure another bearing or gears by contacting the inner covering surface as time passes. When defects grow up, affiliated impact vibration arises.

Assume that the peak signal which has p times magnitude from normal sgnals asises during m times measurement of samplings. As for determining sampling interval, sampling theorem is well known (Tokumaru et al.1982). But in this paper, we do not pay much attention on this point in order to focus on proposed theme.

Suppose that affiliated vibration can be approximated by triangle and set sampling count as d, then we can assume following triangle model(Fig.1).

When $d = 1$, the peak signal which has p times magnitude from normal signals arises.

When $d = i$, the peak signal which has $p - (i-1)\dfrac{p-1}{q}$ times magnitude from normal signals arises ($i = 1,\cdots,q$).

When $d \geq q+1$, normal signal.

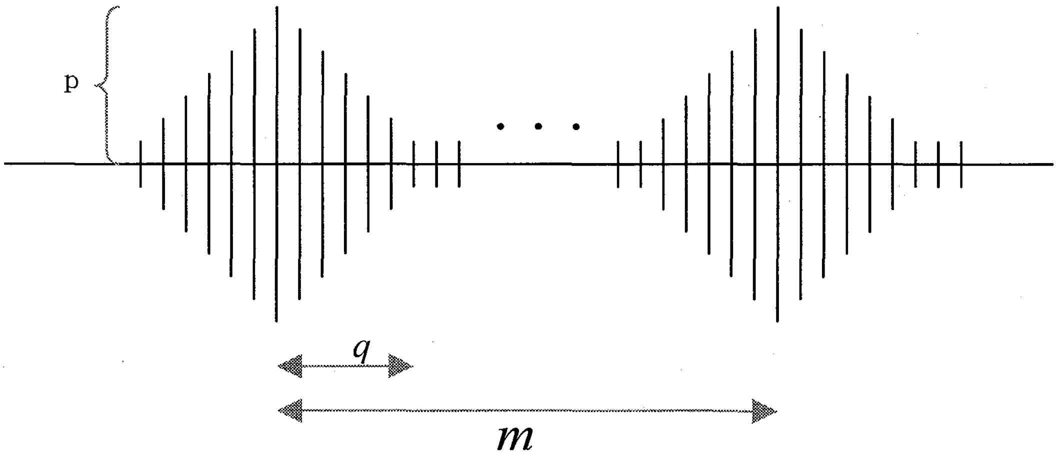

Fig1. Impact vibration and affiliate vibration

Let $\sigma_N{}^2$ state as $\overline{\sigma_N}^2$ when impact vibration occurs.

As for 4th moment and Kurtosis, let them state as $\overline{MT_N}(4)$, $\overline{KT_N}$ in the same way.

$\overline{\sigma_N}^2$ can be calculated as follows.

$$
\begin{aligned}
\overline{\sigma_N}^2 &= \frac{1}{m-1}\sum_{i=1}^{m}\left(x_i-\bar{x}\right)^2 \\
&= 2\left[\sum_{i=1}^{q}\left\{p-(i-1)\frac{(p-1)}{q}\right\}^2\right]\frac{\sigma_N{}^2}{m-1}+(m-1-2q)\frac{\sigma_N{}^2}{m-1} \qquad (18)\\
&= \sigma_N{}^2+\frac{\sigma_N{}^2}{m-1}(q+1)(p-1)\left\{2+\frac{(p-1)(2q+1)}{3q}\right\}
\end{aligned}
$$

As for $\overline{MT_N}(4)$, utilizing

$$\sum_{i=1}^{n} i^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

$$\sum_{i=1}^{n} i^4 = \frac{n}{30}(n+1)(2n+1)\left(3n^2+3n-1\right)$$

$\overline{MT_N}(4)$ can be calculated as follows.

$$\overline{MT_N(4)} = \frac{1}{m-1}\sum_{i=1}^{m}\left(x_i - \bar{x}\right)^4$$
$$= \frac{2}{m-1}\left[\sum_{i=1}^{q}\left\{p-(i-1)\frac{(p-1)}{q}\right\}^4\right]MT_N(4) + \frac{m-1-2q}{m-1}MT_N(4)$$
$$= \left[1 + \frac{2}{m-1}(q+1)(p-1)\left\{\frac{1}{30}(p-1)^3\frac{1}{q^3}(2q+1)(3q^2+3q-1) + (p-1)^2\frac{1}{q}(q+1) + (p-1)\frac{1}{q}(2q+1) + 2\right\}\right]MT_N(4) \tag{19}$$

Then we get $\overline{KT_N}$ as

$$\overline{KT_N} = \frac{1 + \frac{2}{m-1}(q+1)(p-1)\left\{\frac{1}{30}(p-1)^3\frac{1}{q^3}(2q+1)(3q^2+3q-1) + (p-1)^2\frac{1}{q}(q+1) + (p-1)\frac{1}{q}(2q+1) + 2\right\}KT_N}{\left[1 + \frac{1}{m-1}(q+1)(p-1)\left\{\frac{2q+1}{3q}(p-1)+2\right\}\right]^2} \tag{20}$$

Here we introduce the following number. Each index is compared with normal index as follows.

$$F_a = \frac{P_{abn}}{P_{nor}} \tag{21}$$

Here,

P_{nor} : Index at normal condition

P_{abn} : Index at abnormal condition

We get F_a as

$$Fa = \frac{\overline{KT_N}}{KT_N}$$
$$= \frac{1 + \frac{2}{m-1}(q+1)(p-1)\left\{\frac{1}{30}(p-1)^3\frac{1}{q^3}(2q+1)(3q^2+3q-1) + (p-1)^2\frac{1}{q}(q+1) + (p-1)\frac{1}{q}(2q+1) + 2\right\}}{\left[1 + \frac{1}{m-1}(q+1)(p-1)\left\{\frac{2q+1}{3q}(p-1)+2\right\}\right]^2} \tag{22}$$

4. NUMERICAL EXAMPLE

If the system is under normal condition, we may suppose $p(x)$ becomes a normal distribution function. Under this condition, KT is always

$KT = 3.0$

Under the assumption of 3, let $m = 12$. Considering the case $S = 2,3,\ldots,6$ and $q = 1,2,3,4$, we obtain Table 1 from the calculation of (22).

Table 1. F_a by the variation of p, q

		p					
		1	**2**	**3**	**4**	**5**	**6**
q	**1**	1.0	1.561	2.580	3.409	3.978	4.361
	2	1.0	1.421	2.030	2.477	2.775	2.988
	3	1.0	1.320	1.709	1.971	2.142	2.251
	4	1.0	1.235	1.482	1.644	1.749	1.821
	5	1.0	1.157	1.311	1.412	1.480	1.527

As $KT_N \cong 3.0$, we show Table 2 as an approximation of $\overline{KT_N}$ by multiplying 3.0 for each item of Table 1.

Table 2. $\overline{KT_N}$ for each case

		p					
		1	**2**	**3**	**4**	**5**	**6**
q	**1**	3.0	4.683	7.740	10.227	11.934	13.083
	2	3.0	4.263	6.090	7.431	8.325	8.964
	3	3.0	3.960	5.127	5.913	6.426	6.753
	4	3.0	3.705	4.336	4.932	5.247	5.463
	5	3.0	3.470	3.935	4.238	4.440	4.582

As p increases, F_a and $\overline{KT_N}$ increase.

On the other hand, F_a and $\overline{KT_N}$ decrease as q increases when p is the same.

When damages increase or transfer to another place, peak level grows up and affiliated impact vibration spread.

This means that $\overline{KT_N}$ value shift from left-hand side upwards to right-hand side downwards in Table 2.

For example, following transition of $\overline{KT_N}$ can be supposed.

When $q=1,\ p=1,$ $\overline{KT_N}=3.0$

When $q=1,\ p=2,$ $\overline{KT_N}=4.683$

When $q=4,\ p=3,$ $\overline{KT_N}=4.336$

When $q=5,\ p=6,$ $\overline{KT_N}=4.582$

We made experiment in the past (Takeyasu(1987),Takeyasu(1989)). Summary of the experiment is as follows. Pitching defects are pressed on the gears of small testing machine.

Small defect condition: Pitching defects pressed on 1/3 gears of the total gear.
Middle defect condition: Pitching defects pressed on 2/3 gears of the total gear.
Big defect condition: Pitching defects pressed on whole gears of the total gear.

RMS and Kurtosis in this case are exhibited in Table 3.

Table 3. Experiment Result

	Kurtosis	RMS
Normal	2.961	289.212
Small Defect	3.747	671.175
Middle Defect	2.970	833.592
Big Defect	3.310	855.375

RMS values grow up as damages increase.

Kurtosis value responds to the damage in the small defect level. But it is rather close to normal level under middle and goes up again in big defect.

We thought damages became rounded, so Kurtosis had fallen.

Considering the above stated model which includes the affiliated impact vibration, we can explain the case that Kurtosis is big initially and then fall and goes up again.

Though the score may differ by the adjustment of parameter, we can analyze the behavior of Kurtosis principally by utilizing this simplified model and calculation method.

We can easily calculate (20) watching the waveform at the maintenance site, and we can get much more correct estimation of Kurtosis than the method presented by Takeyasu et al. 2003.

5. CONCLUSION

We proposed a simplified calculation method of Kurtosis for the analysis of impact vibration including affiliated impact vibration. Affiliated impact vibration was approximated by triangle and simplified calculation method was introduced.

Varying the shape of triangle, various models were examined and the phenomenon that Kurtosis went up and down as the deterioration proceeded was traced and its reason was clarified by the analysis.

Utilizing this method, the behavior of Kurtosis would be forecasted and analyzed while watching machine condition and correct diagnosis would be executed.

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