



Frequency Characteristics of Three-phase Induction Motor

メタデータ	言語: eng 出版者: 公開日: 2010-04-05 キーワード (Ja): キーワード (En): 作成者: Ishizaki, Takemitsu, Tokuda, Tsutomu, Shimizu, Tatsuo メールアドレス: 所属:
URL	https://doi.org/10.24729/00008945

Frequency Characteristics of Three-phase Induction Motor

Takemitsu ISHIZAKI*, Tsutomu TOKUDA* and Tatsuo SHIMIZU*

(Received November 30, 1963)

The studies of the speed control of the induction motor driven by SCR converter and inverter have been extensively carried out, in recent years. But, as the fundamental research for the frequency characteristics of the induction motor which is employed for load driven by SCR must be carried out concurrently, the authors investigated the operating characteristics of the induction motor driven by adjustable frequency source with sinusoidal wave form. In this study, it has been found that these characteristics show the wide variety with the value of the ratio of source voltage to the frequency. Therefore a wide application will be developed in the fields where the d-c motors are used now.

1. Introduction

The speed of a-c motors such as the synchronous motor and the induction motor can be controlled smoothly over wide range by adjusting the frequency of the power source which is supplied to the motor. In recent years, several papers⁽¹⁾⁽²⁾⁽³⁾ dealt with the speed control of a-c motor driven by SCR converter and inverter. In these cases, the wave form of the source voltage which is supplied to the a-c motor by SCR is a very distorted one. But very little work has been done to estimate the characteristics of the induction motor driven by the sinusoidal wave source over the wide range of the frequency variations. The operating characteristics of the induction motor under the frequency control can be varied by the ratio of the voltage to the frequency.

This paper deals with the characteristics of the three-phase induction motor driven by widely adjustable frequency source with sinusoidal wave form.

2. Sample and measurement

2-1. Equivalent circuit of induction motor. Generally, the characteristics of the three-phase induction motor can be calculated by the equivalent circuit as shown in Fig. 1. All symbols indicated in this figure are on per-phase basis and expressed in stator terms. They are defined as follows :

- R_1 : stator resistance,
- X_1 : stator reactance,
- R_0 : resistance of exciting circuit,
- X_0 : reactance of exciting circuit,
- X_2 : rotor reactance (assumed equal to X_1),

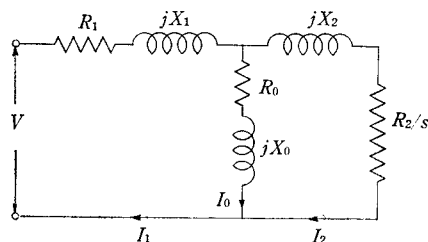


Fig. 1 The equivalent circuit of induction motor.

* Department of Electrical Engineering, College of Engineering.

R_2 : rotor resistance,

s : slip,

V : supply voltage,

I_1 , I_2 and I_0 : the stator, rotor and exciting currents respectively.

The value of R_1 , measured with d-c source, may be considered a constant regardless of frequency. The value of X_1 , X_2 , X_0 , R_0 and R_2 can be calculated by the data obtained in no-load and lock test of the induction motor.

2-2. Samples. Three motors, employed for the test, are enclosed and squirrel-cage type three-phase induction motors, manufactured by different makers respectively. The ratings of these motors are shown in Table 1.

Table 1. Ratings of sample motors.

No. of Sample	Cycle (cps)	Current (A)	Speed (rpm)	Class of Starting	Out put = 400(W) Voltage = 200(V) Voltage per phase=115.3(V) No. of poles = 4
I	50	2.0	1400	G	
	60	1.8	1690	G	
II	50	1.9	1420	J	
	60	1.8	1700	H	
III	50	2.0	1400	H	
	60	1.8	1690	H	

2-3. Measurement. For the adjustable frequency power source, a synchronous generator driven by Ward-Leonard system is employed. It has the symmetrical three-phase windings, and the wave form of its output voltage is sinusoidal. The ranges of frequency and voltage variation per phase are 10~90(cps) and 0~180(V), respectively.

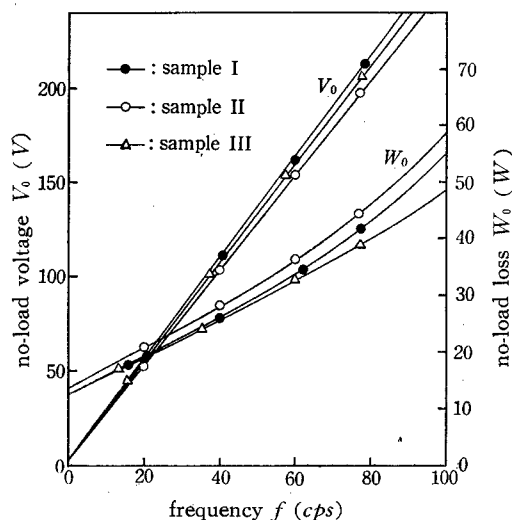


Fig. 2 Results of no-load test.

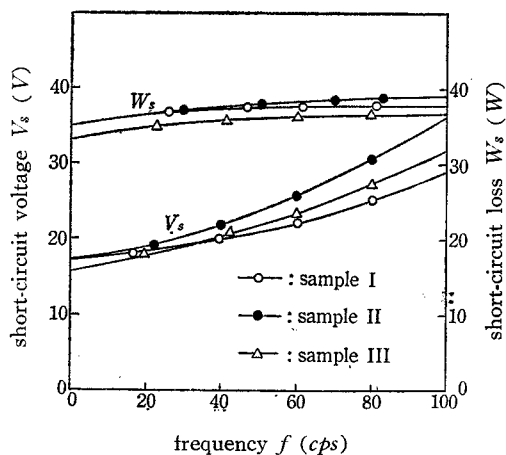


Fig. 3 Results of lock test.

No-load and lock test over the total range of frequency are carried out under constant exciting current and short-circuit current. The examples of the experimental results for samples I, II and III are shown in Figs. 2 and 3. Fig. 2 shows the results of no-load test under the constant exciting current $I_0=1.6$ (A). In this figure the curves of V_0 and W_0 represent the no-load voltage and the no-load loss vs. frequency characteristics, respectively. In the Fig. 3 showing the results of lock test under the constant short-circuit current $I_s=2.0$ (A), curves of V_s and W_s represent the short-circuit voltage and short-circuit loss vs. frequency characteristics, respectively.

Fig. 4 shows the equivalent circuit constants R_1 , X_1 , X_0 , R_0 , R_2 and X_2 (assumed equal to X_1) for sample I, calculated by the data obtained from the above tests and measurement of stator resistance. It is readily seen from Fig. 4 that the values of R_0 , X_0 , X_1 and X_2 are proportional to the frequency and R_2 is nearly equal to a constant over all frequencies. Then the equivalent circuit constants can be written as follows :

$$\begin{aligned} R_0 &= \alpha r_0, & X_0 &= \alpha x_0 \\ R_1 &= r_1, & X_1 &= \alpha x_1 \\ R_2 &= r_2, & X_2 &= \alpha x_2 \end{aligned} \quad (1)$$

where $\alpha = f/f_n$ is ratio of frequency f to the rated frequency f_n , and r_0 , x_0 , r_1 , x_1 , r_2 , x_2 are values of R_0 , X_0 , R_1 , X_1 , R_2 , X_2 at the rated frequency of 60 (cps). From $X_1 = \alpha x_1$, $X_2 = \alpha x_2$ and $X_0 = \alpha x_0$, it is seen that inductances of stator, rotor and exciting circuit are constant over all frequencies. From $R_0 = \alpha r_0$, no-load loss is clearly proportional to the frequency.

Owing to the saturation of magnetic circuit, r_0 and x_0 vary with the exciting current. The saturation curve can be represented approximately by Fröhlich's equation at the rated frequency; i. e.,

$$V_0 = \frac{C_1 I_0}{C_2 + I_0}$$

where C_1 and C_2 are constants.

From the above eq., exciting impedance z_0 can be written as follow,

$$z_0 = z_{0n} + k (V_n - V_0) \quad (2)$$

where z_{0n} is exciting impedance at the rated voltage and the rated frequency, and k is a constant specified by the motor.

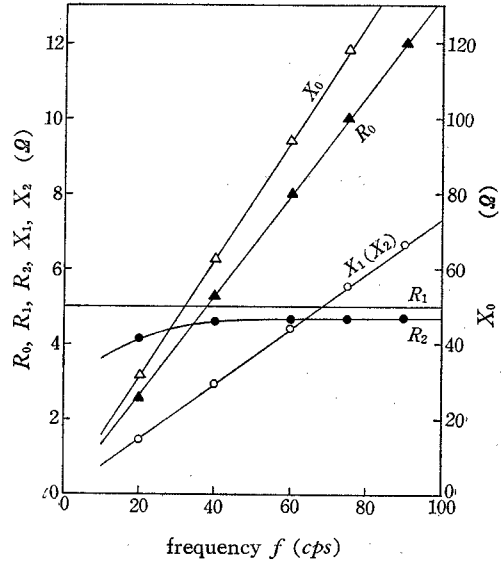


Fig. 4 The values of equivalent circuit constants for sample I.

Impedance z_0 and power factor $\cos\theta_0$ of exciting circuit for sample I obtained from the no-load test at the rated frequency f_n , are shown in Fig. 5. From this figure, it is seen that (i) z_0 can be represented by eq. (2) for $V_0 > 0.7 V_n$, (ii) $\cos\theta_0$ is nearly equal to a constant for $V_0 > 0.7 V_n$.

Then, r_0 and x_0 can be written as follows,

$$\begin{aligned} r_0 &= z_0 \cos\theta_0 = r_{0n} [1 + k(V_n - V_0)/z_{0n}] \\ x_0 &= z_0 \sin\theta_0 = x_{0n} [1 + k(V_n - V_0)/z_{0n}] \end{aligned} \quad (3)$$

where r_{0n} and x_{0n} are respectively resistance and reactance of exciting circuit at the rated voltage and the rated frequency.

The equivalent circuit constants R_0 , X_0 , R_1 , X_1 , R_2 and X_2 for samples II and III can be calculated by eqs. (1), (2) and (3), in the same manner as sample I. Table II shows these values at the rated frequency and rated voltage for samples I, II and III.

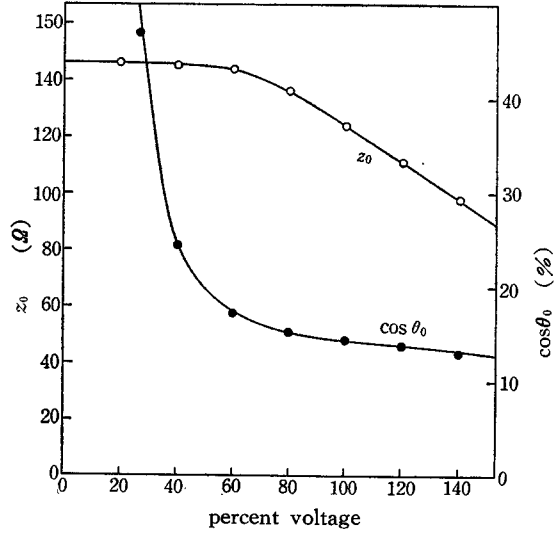


Fig. 5 z_0 and $\cos\theta_0$ vs. voltage curves.

Table 2. Equivalent circuit constants at the rated voltage and the rated frequency.

No. of Sample	r_1 (Ω)	$x_1 (=x_2)$ (Ω)	r_2 (Ω)	r_{0n} (Ω)	x_{0n} (Ω)	k (Ω/V)
I	5.0	4.4	4.6	18.4	124.7	0.73
II	5.3	3.7	3.8	12.9	120.3	0.64
III	5.7	3.0	3.8	17.6	121.7	0.97

3. Frequency characteristics of three-phase induction motor

The speed of the three-phase induction motor can be controlled smoothly over wide range by adjusting the frequency of the power source which is supplied to the motor. In this controlling system operating characteristics of the induction motor can be varied very widely with the ratio of source voltage to frequency.

Voltage V , supplied to the motor at any frequency, can be written by a general form as follow,

$$V = \alpha' V_n \quad (4)$$

where α' is a constant which is determined by the controlling method.

3-1. Approximate equivalent circuit. For simplicity, the equivalent circuit as shown in Fig. 1, is replaced by the approximate form shown in Fig. 6.

3-2. Frequency Characteristics. The current and the torque of the three-phase induction motor can be written from Fig. 6, as following eqs.,

$$\dot{I}_0 = \frac{\alpha^{r-1} V_n}{r_0 + jx_0} \quad (5)$$

$$\dot{I}_2 = \frac{s \alpha^r V_n}{(sr_1 + r_2) + js\alpha x} \quad (6)$$

$$\dot{I}_1 = \dot{I}_0 + \dot{I}_2 \quad (7)$$

$$T = \frac{3 I_2^2 r_2}{9.8 \omega s} = \frac{3 r_2 s \alpha^{2r-1} V_n^2}{9.8 \omega_n [(sr_1 + r_2)^2 + (s\alpha x)^2]} \quad (8)$$

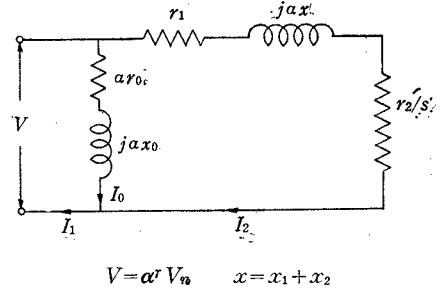


Fig. 6 The approximate equivalent circuit.

where $x = x_1 + x_2$,

$\omega_n = 2\pi f_n / p$: synchronous angular velocity at the rated frequency,

p : no. of pairs of poles,

$\omega = \alpha \omega_n$: synchronous angular velocity at any frequency.

The value of slip, s_m , at which the maximum torque T_m is developed, can be determined from $(dT/ds) = 0$. Therefore,

$$s_m = \frac{r_2}{\sqrt{r_1^2 + \alpha^2 x^2}} \quad (9)$$

Substituting this slip in eq. (8),

$$T_m = \frac{1.5 \alpha^{2r-1} V_n^2}{9.8 \omega_n [r_1 + \sqrt{r_1^2 + \alpha^2 x^2}]} \quad (10)$$

As the slip is unity at starting, the starting rotor current I_{2st} and the starting torque T_{st} can be reduced from eqs. (6) and (8) as follows,

$$I_{2st} = \frac{\alpha^r V_n}{\sqrt{(r_1 + r_2)^2 + \alpha^2 x^2}} \quad (11)$$

$$T_{st} = \frac{3 r_2 \alpha^{2r-1} V_n^2}{9.8 \omega_n [(r_1 + r_2)^2 + \alpha^2 x^2]} \quad (12)$$

Now, the frequency characteristic curves of the three-phase induction motor for different values of r can be plotted from the above equation. Exciting current vs. frequency characteristics for sample I plotted from eqs. (2) and (5) are shown in Fig. 7. From Fig. 7, it is seen that the exciting current is a

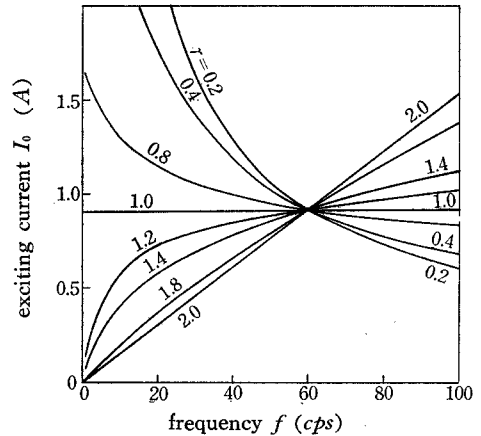


Fig. 7 Exciting current characteristic.

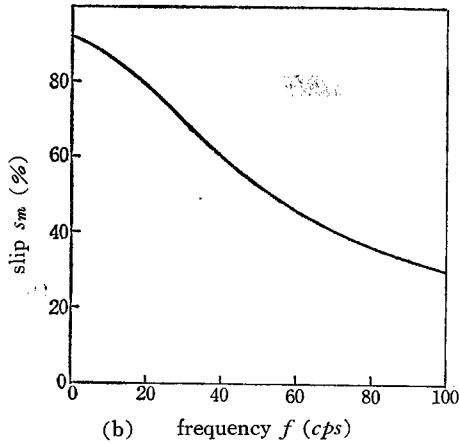
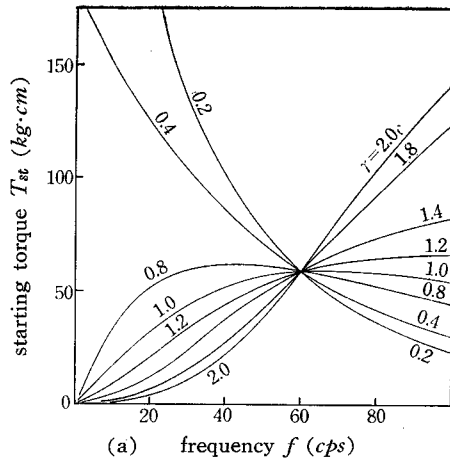
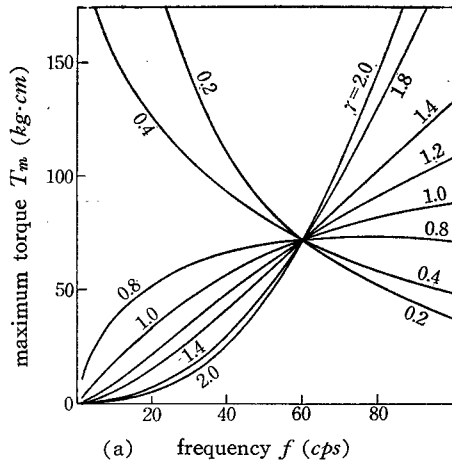


Fig. 8 T_m and s_m vs. f characteristics.

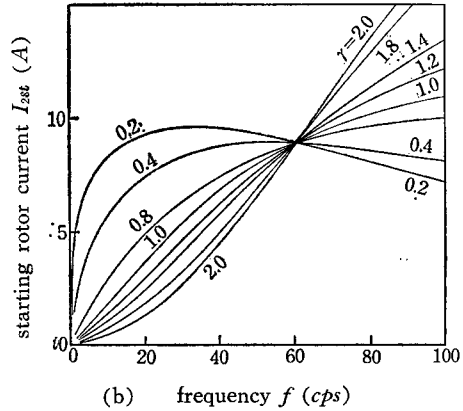


Fig. 9 Starting characteristics.

constant at any frequency for $r=1$, but it increases with the increase of frequency for $r>1$ and on the contrary it decreases with the increase of frequency for $r<1$.

Frequency characteristics of the maximum torque and the slip corresponding to it, plotted from eqs. (9) and (10), are shown in Fig. 8. Fig. 8 (a) and (b) show the maximum torque and the corresponding slip vs. frequency curves for sample I. From eq. (9), it is easily seen that s_m is not a function of r , but can be determined only by α (i. e. frequency). Neglecting the stator resistance r_1 in eq. (9), following relation can be deduced.

$$s_m = \frac{1}{\alpha} \frac{r_2}{x} \quad \text{or} \quad \alpha s_m = \frac{r_2}{x} \quad (13)$$

Hence, it can be mentioned that slip s_m is inversely proportional to frequency, or slip speed $2\pi f s_m / p$ is a constant at any frequency.

From Fig. 8 (a), it is seen that the maximum torque increases with the increase of

frequency for $\tau \geq 1.0$ and contrarily it decreases with the increase of frequency for $\tau \leq 0.8$.

Starting characteristics for sample I, plotted from eqs. (11) and (12), are shown in Fig. 9. Fig. 9 (a) and (b) show starting torque and starting rotor current vs. frequency curves for different values of τ . From Fig. 9, it can be mentioned that starting torque increases with the increase of frequency for $\tau \geq 1.2$, and contrarily it decreases with the increase of frequency for $\tau \leq 0.8$.

The fact that operating characteristics of the three-phase induction motor with respect to the frequency vary extensively with the value of τ (i.e. ratio of voltage to frequency), can be seen from Figs. 7, 8 and 9.

These great variety of characteristics of the induction motor driven by adjustable frequency source produce many fascinating results. Because of the flexibility of these characteristics, a wide application will be found in the fields where d-c motors are used now. The problem of finding a most suitable control which can fit the operating characteristics to the speed and torque requirements of the driven load, is how to select the value of τ . For heigher starting torque requirement the value of τ must be small, and on the contrary for lower starting current requirement the value of τ must be large.

The torque and current vs. speed characteristics of the induction motor with respect to controlling method which is determined by value of τ , will be described in the following sections.

3-3. Controlling method 1

($V = \alpha V_n$). In this method, the supply voltage to the induction motor must be widely regulated in proportion to the frequency of the power source. Putting $\tau = 1$, torque and current vs. speed characteristics for different values of frequency can be plotted from eqs. (5), (6), (7) and (8). Fig. 10 shows the characteristics for sample I. In this figure, thick and thin lines represent torque and current curves respectively, and dotted line shows exciting current at the synchronous speed.

From Fig. 10, it can be mentioned that starting current decreases with the decrease of frequency and contrarily the starting torque is nearly equal to a constant with respect to frequency, i. e.,

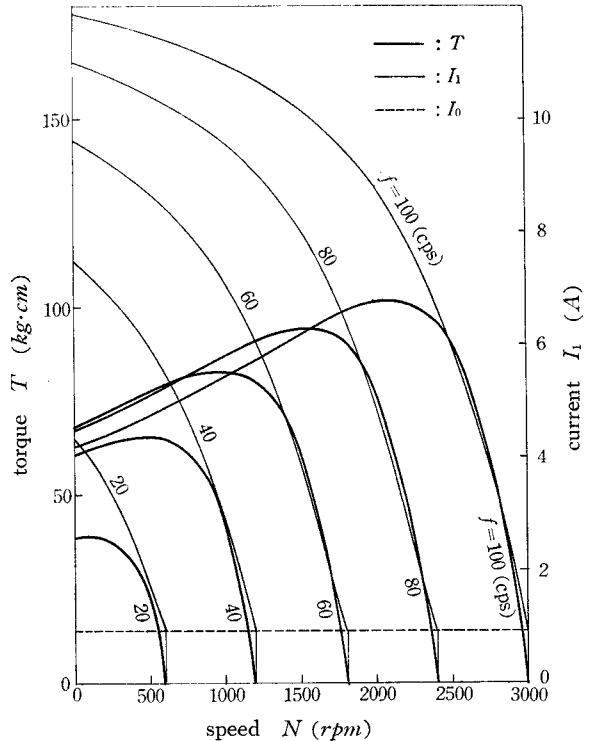


Fig. 10 Torque and current vs. speed characteristics at $V = \alpha V_n$ for different values of frequency.

$$[I_{1st}]_{40} \approx \frac{2}{3} [I_{1st}]_{100}$$

$$[T_{st}]_{40} \approx [T_{st}]_{100}$$

where $[I_{1st}]_{40}$ and $[I_{1st}]_{100}$: starting currents at 40 and 100 (cps),
 $[T_{st}]_{40}$ and $[T_{st}]_{100}$: starting torque at 40 and 100 (cps).

On the starting of motor, the frequency of the source is set at 20 (cps). As the motor is speeded up, the frequency may be gradually increased to 100 (cps). Thus motor starting is carried out smoothly at low current.

Neglecting the stator resistance r_1 in eq. (10), following relation can be deduced

$$T_m = \frac{1.5}{9.8 \omega_n} \frac{V_n^2}{x} \quad (14)$$

Hence, the maximum torque is a constant at any frequency.

3-3. Controlling method 2 ($V = \alpha^{0.4} V_n$). In this method, the value of α is equal to 0.4. Fig. 11 shows the characteristics for sample I. In this figure, thick and thin lines represent the torque and current curves respectively, and dotted line shows exciting current at the synchronous speed. From Fig. 11, it can be mentioned that the maximum and starting torques increase with the decrease of frequency, and contrarily the starting

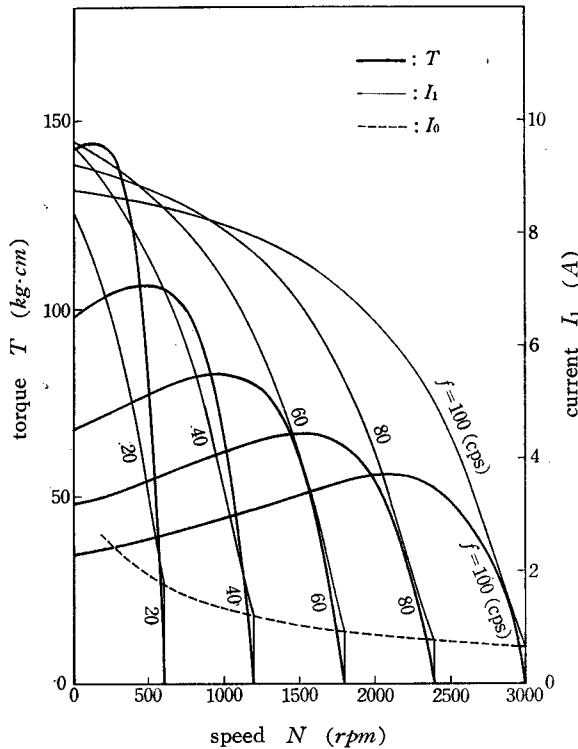


Fig. 11 Torque and current vs. speed characteristics at $V = \alpha^{0.4} V_n$ for different values of frequency.

current is nearly equal to a constant with respect to frequency, i. e.,

$$[I_{st}]_{20} \simeq [I_{st}]_{100}$$

$$[T_{st}]_{20} \simeq 4[T_{st}]_{100}$$

Therefore, in this controlling method, instantaneous starting of motor will be possible.

4. Summary

First, it is described in this paper that the equations representing the equivalent circuit constants of the motor can be written by simplified forms, such as equations (1), (2) and (3). These equations are verified by no-load and lock test of the sample motors over the wide range of frequency.

Next, this paper shows the frequency characteristic curves plotted using the above equations. These characteristics of the induction motor vary widely with the value of r (i. e., ratio of voltage to frequency).

This variety of characteristics of the motor driven by adjustable frequency source can produce very facinating results in the application of the induction motor.

References

- 1) S. Miyairi and Y. Tanehiro; JIEE of Japan **82**, 1741 (1962).
- 2) E. Hayasi and I. Zyûri; Toshiba Review **16**, 561 (1961).
- 3) T. Ishizaki, T. Tokuda and S. Ishikawa; Journal of the Japan Association of Automatic Control Engineers **7**, 453 (1963).