A Simplified Analyzing Method for Operating Characteristics of Power Distribution System I

A Simplified Analyzing Method for Operating Characteristics of Power Distribution System-I

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In analyzing the operating characteristics of a power distribution system, for simplicity, the load is usually treated as a constant current one.

But, this treatment is not always suited to the analysis for the complicated system.

In this paper, the authors propose to treat the system as a combination of four terminal network, in which the load is regarded as an equivalent admittance.

According to this treatment, any system, such as tree-type, loop, network and the like, can be analyzed more systematically than the usual.

In Part-1, the analyzing method for the tree-type distribution system is described and it forms the foundation of the analysis for the more complicated system.

1. Introduction

The problem of power distribution is to design, construct, operate, and maintain a distribution system that will supply an adequate amount of electric power to the load area in consideration of the economical condition.

The effectiveness of design is measured in terms of voltage regulation, service continuity, flexibility, efficiency and cost.

While, no one type of distribution system can be applied economically in all load areas, because of differences in load densities, existing distribution plant, topography, and other local conditions.

Then, recently, the various kinds of distribution systems, such as loop, banking or network, which should meet the rapid growth of power demand are investigated and tried to be adopted in several areas.

But, the analysis of these systems is complicated and it is considerably difficult to anticipate the effectiveness of design.

So, the several means for it, such as the use of A. C. Calculating Board or Artificial Distribution System, and field tests on actual one, are employed, and also the approximate methods of calculation, one of which is described in this paper, are studied.

In the usual method of calculation, the load is treated as a constant current one.

But, this treatment is not always suited to the analysis for the complicated system.

Then, the authors regard the load as an equivalent admittance and attempt to treat the system as a combination of four terminal network.

By this method, any system can be analyzed more easily and systematically than the usual.

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Besides, in this paper, it is assumed that the load area is arranged in a rectangular pattern in view of the road section of our cities, on reference to the examples given by F. C. Van Wormer,(1) D.L.Hopkins, D. R. Samson,(2) W. J. Denton,(3) and that powers in its area are supplied through the three-phase, three-wire distribution lines and all balanced, so electric characteristics are analyzed on the basis of "per phase".

2. Determination of Constants of Four Terminal Network in a Distribution System

In the tree-type primary distribution system having a rectangular load area as shown in Fig. 1, if the load in it is uniformly distributed, and its load can be represented approximately by an equivalent admittance, the following equations will be adopted properly :

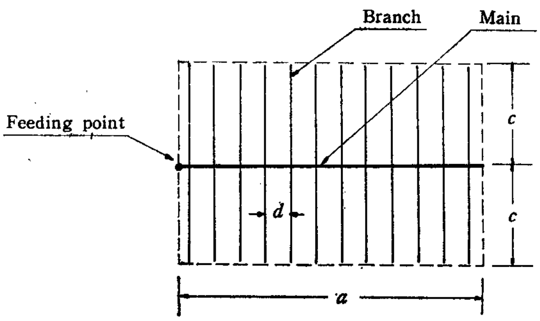

Fig. 1 A rectangular load area.

$$\dot{E}_x=\dot{D}_x\dot{E}_s-\dot{B}_x\dot{I}_s, \qquad \dot{I}_x=-\dot{C}_x\dot{E}_s+\dot{A}_x\dot{I}_s. \tag{1}$$

$$\dot{E}_x=\dot{A}_{a-x}\dot{E}_r+\dot{B}_{a-x}\dot{I}_r, \qquad \dot{I}_x=\dot{C}_{a-x}\dot{E}_r+\dot{D}_{a-x}\dot{I}_r. \tag{2}$$

$$\dot{E}_s=\dot{A}_a\dot{E}_r+\dot{B}_a\dot{I}_r, \qquad \dot{I}_s=\dot{C}_a\dot{E}_r+\dot{D}_a\dot{I}_r. \tag{3}$$

where

$\dot{E}_s$, $\dot{I}_s$: phase voltage and line current at the sending end,

$\dot{E}_r$, $\dot{I}_r$: phase voltage and line current at the receiving end,

$\dot{E}_x$, $\dot{I}_x$: phase voltage and line current at any point,

$$\left.\begin{aligned}\dot{A}_x=\dot{D}_x &= \cosh\dot{\gamma}x,\\ \dot{B}_x &= \dot{Z}_\omega\sinh\dot{\gamma}x,\\ \dot{C}_x &= (1/\dot{Z}_\omega)\sinh\dot{\gamma}x,\end{aligned}\right\}$$
: general circuit constants from the sending end to x,

$$\left.\begin{aligned}\dot{A}_a=\dot{D}_a &= \cosh\dot{\gamma}a,\\ \dot{B}_a &= \dot{Z}_\omega\sinh\dot{\gamma}a,\\ \dot{C}_a &= (1/\dot{Z}_\omega)\sinh\dot{\gamma}a,\end{aligned}\right\}$$
: general circuit constants from the sendinig end to the receiving end,

$$\left.\begin{array}{l}\dot{A}_{a-x}=\dot{D}_{a-x}=\cosh\dot{\gamma}\,(a-x),\\ \dot{B}_{a-x}=\dot{Z}_{\omega}\sinh\dot{\gamma}\,(a-x),\\ \dot{C}_{a-x}=(1/\dot{Z}_{\omega})\sinh\dot{\gamma}\,(a-x),\end{array}\right\}$$: general circuit constants from x to the receiving end,

- a : length of line from the sending end to the receiving end,
- x : length of line from the sending end to x,
- $\dot{\gamma}$: propagation constant $=\sqrt{\dot{z}\dot{y}}$,
- $\dot{Z}_{\omega}$: characteristic impedance $=\sqrt{\dot{z}/\dot{y}}$,
- $\dot{z}$: impedance of line in ohm per kilometer,
- $\dot{y}$: load admittance in mho per kilometer.

Then, the authors attempt to use the above equations by the following reductions.

(1) Load admittance $\dot{y}$

The admittance $\dot{y}$ which represents the load admittance can be expressed as follows,

$$\dot{y} \simeq k\delta\dot{D}/3E^2 = k\delta\dot{D}/V^2 \text{ (mho/km)} \tag{4}$$

where

- δ : width of area in kilometer,
- $\dot{D}$: vector quantity of load-density given in planning in volt-ampere per square kilometer,
- E, V : phase and line voltage of line in volt ($V=\sqrt{3}E$)
- k : a constant of area, for the main line in Fig. 1,
  - $k=2$, for the main line in Fig. 1.
  - $k=1$, for the branch line in Fig. 1,

Concerning the load-density $\dot{D}$ and the phase voltage E in Eq. (4), the following convenient methods are adopted.

Strictly speaking, the load-density $\dot{D}$ is involved the losses of line, but, in time of planning, the consumer's loads and the losses of line can not be distinguished and the losses of line are far less than the consumer's loads. So, in calculating the load admittance $\dot{y}$, these conditions of $\dot{D}$ are ignored.

The load admittance $\dot{y}$ is changed by the phase voltage E at load point, too, but, for simplicity, it is assumed that $\dot{y}$ is a constant load admittance calculated by a represented phase voltage E_n. The voltage E_n is examined in 3-(2).

(2) Approximate solution of the general circuit constants

As the length of distribution line is comparatively short, the general circuit constants A, B, C and D, can be expressed sufficiently accurately by the following Eq. (5) and, moreover, the phase difference between E_s and E_r is so small for distribution line that the absolute values of the general circuit constants are approximately given as the following Eq. (6).

$$\left.\begin{aligned} &\dot{A}_x=\dot{D}_x\simeq 1+(\dot{\gamma}x)^2/2, \quad \dot{B}_x\simeq \dot{z}x[1+(\dot{\gamma}x)^2/6], \\ &\dot{C}_x\simeq \dot{y}x[1+(\dot{\gamma}x)^2/6], \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} &|\dot{A}_x|=|\dot{D}_x|\simeq 1+yZx^2/2, \quad |\dot{B}_x|\simeq zx(1+yZx^2/6), \\ &|\dot{C}_x|\simeq yx(1+yZx^2/6). \end{aligned}\right\} \tag{6}$$

where

y : absolute value of load admittance in mho per kilometer $(y=k\delta D/3E^2)$,

$z=\sqrt{r^2+X^2}$: absolute value of line impedance in ohm per kilometer,

r : resistance of line in ohm per kilometer,

X : reactance of line in ohm per kilometer,

$Z=r\cos\theta+X\sin\theta$: effective impedance of line in ohm per kilometer,

$\cos\theta$: power factor of load.

Henceforth, by the Eqs. (5) and (6), the simplified analysis will be proceeded.

3. A Simplified Analysis of the Tree-Type Distribution System

(1) Voltage and current of the tree-type circuit

Consider the tree-type primary circuit having a rectangular area and an uniform load distribution as shown in Fig. 2.

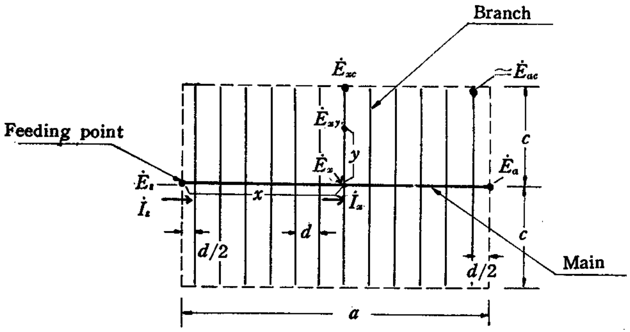

Fig. 2 A rectangular load area with uniform load distribution

In this figure ;

$\dot{E}_s$, $\dot{I}_s$: phase voltage and line current at the feeding point,

$\dot{E}_a$, $\dot{I}_a$: phase voltage and line current at the end of main line,

$\dot{E}_x$, $\dot{I}_x$: phase voltage and line current at the point, where is at x on main from the feeding point,

$\dot{E}_{xy}$, $\dot{I}_{xy}$: phase voltage and line current at the point, where is at x on main and at y on branch,

$\dot{E}_{ac}$(or $\dot{E}_{xc}$) : phase voltage at the end of branch, where is at a (or x) on main,

$\dot{\gamma}_1$, $\dot{\gamma}_2$: propagation constants of main and branch,

$\dot{Z}_{\omega 1}$, $\dot{Z}_{\omega 2}$: characteristic impedances of main and branch,

$E_n(V_n)$, $E'_n(V'_n)$: represented phase voltages (line voltages),

$\dot{z}_1$, $\dot{z}_2$: impedances of main and branch per kilometer,

$\dot{y}_1$, $\dot{y}_2$: load admittances of main and branch per kilometer,

Z_1, Z_2 : effective impedances of main and branch per kilometer.

(a) Current of the feeding point.

Put $\dot{I}_a = 0$ and using Eq. (3), the current of the feeding point, $\dot{I}_s$, is calculated as follows,

$$\dot{I}_s=\dot{C}_a\cdot\dot{E}_a, \quad (7), \qquad \dot{E}_a\doteq(1/\dot{A}_a)\cdot\dot{E}_s, \quad (8), \qquad \text{therefore} \quad \dot{I}_s=(\dot{C}_a/\dot{A}_a)\cdot\dot{E}_s. \quad (9)$$

Referring to Eq.(6), the absolute value of the current, I_s, is obtained as follows,

$$I_s=|\dot{I}_s|=|\dot{C}_a||\dot{E}_s|/|\dot{A}_a|\simeq E_s y_1 a[1+(y_1Z_1/6)a^2]\,[(1-(y_1Z_1/2)a^2],$$

where

$$\frac{1}{|\dot{A}_a|}\simeq\frac{1}{1+(y_1Z_1/2)a^2}\simeq1-\frac{y_1Z_1}{2}a^2.$$

As the second terms in parentheses are comparatively less than the first term (= 1), the multiplication of the second terms can be neglected.

Then

$$I_s\simeq E_s y_1 a[1-(1/3)y_1Z_1a^2]. \tag{10}$$

According to the method of F.C.Van Wormer,(1) $I_s=E_s y_1 a$ (put $E_n=E_s$).

This equation is derived by treating the load as a constant current one and the phase voltage at any point on line as an invariant.

In this paper, treating the load as a constant admittance, the load current is changed by the phase voltage at load point.

The result arisen from the the difference of the assumption for load is found as the second term in Eq. (10).

However, considering the general consumer's load, it seems to be appropriate that the load is treated as a constant admittance.

(b) Power of the feeding point (apparent power).

Power of the feeding point W_1 is derived as follows,

$$\dot{W}_1=3\dot{E}_s\dot{I}_s=3\dot{C}_a E_s^2/\dot{A}_a.$$

Thus

$$W_1\doteq|\dot{W}_1|=3|\dot{C}_a|E_s^2/|\dot{A}_a|\simeq3E_s^2 y_1 a[1-(1/3)y_1Z_1a^2] \tag{11}$$

or, on reference to Eq. (4), putting $y_1=2cD/3E_n^2$,

$$W_1=\left(2cDa-\frac{4c^2Z_1a^3}{9E_n^2}\right)\left(\frac{E_s}{E_n}\right)^2. \tag{12}$$

(c) Voltage, current and voltage drop at x from the feeding point. Put $\dot{I}_a=0$ and using Eq.(2), the following equations are obtained.

$$\dot{E}_x=\dot{A}_{a-x}\dot{E}_a, \quad (13) \qquad \dot{I}_x=\dot{C}_{a-x}\cdot\dot{E}_a. \quad (14)$$

From Eq. (8),

$$\dot{E}_x=\dot{A}_{a-x}\dot{E}_s/\dot{A}_a. \tag{15}$$

Then

$$E_x=|\dot{E}_x|=|\dot{A}_{a-x}|\,|\dot{E}_s|/|\dot{A}_a|\simeq E_s[1-(y_1Z_1/2)(2a-x)x]. \tag{16}$$

The voltage drop may be expressed in percentage of the voltage at the feeding point as follows,

$$\%E=\frac{E_s-E}{E_s}\times 100.$$

So, from Eq. (16),

$$\%E_x=\frac{E_s-E_x}{E_s}\times 100\simeq\frac{y_1Z_1}{2}(2a-x)x\times 100. \tag{17}$$

Next, from Eqs. (8) and (14),

$$I_x=|\dot{C}_{a-x}|\,|\dot{E}_s|/|\dot{A}_a|\simeq E_sy_1(a-x)[1-(y_1Z_1/6)(2a^2+2ax-x^2)]. \tag{18}$$

Thus, from the equations (16)~(18) the approximate values of E_x, $\%E_x$ and I_x are given as the function of distance from the feeding point.

(b) Voltage and voltage drop at the end of main.

Put $x=a$ and using the equations (16)~(18),

$$E_a\simeq E_s[1-(y_1Z_1/2)a^2], \tag{19}$$

$$\%E_a\simeq(y_1Z_1/2)a^2\times 100=(cDZ_1/V_n^2)a^2\times 100. \tag{20}$$

Giving the load density D in kVA/km^2 and the line voltage V in kV,

$$\%E_a\simeq(cDZ_1/10V_n^2)a^2 \tag{21}$$

The mean voltage drop in percent of main is derived as follows,

$$\%E_{a\text{mean}}=\left[\frac{1}{a}\int_0^a\frac{y_1Z_1}{2}(2a-x)x\,dx\right]\times 100=\frac{2}{3}\left(\frac{y_1Z_1}{2}a^2\right)\times 100=\frac{2}{3}\times(\%E_a). \tag{22}$$

This Eq. (22) shows that the mean voltage drop of main is equal to two-thirds of total voltage drop of main.

(e) Voltage and current at the point, where is at x on main and at y on branch.
Putting $\dot{I}_{xc} = 0$,

$$\dot{E}_{xy} = \dot{A}_{c-y} \cdot \dot{E}_{xc} = (\dot{A}_{c-y}/\dot{A}_c)\dot{E}_x = (\dot{A}_{c-y} \cdot \dot{A}_{a-x}/\dot{A}_c \cdot \dot{A}_a)\dot{E}_s, \tag{23}$$

$$\dot{I}_{xy} = \dot{C}_{c-y} \cdot \dot{E}_{xc} = (\dot{C}_{c-y} \cdot \dot{A}_{a-x}/\dot{A}_c \cdot \dot{A}_a)\dot{E}_s. \tag{24}$$

(f) Voltage and voltage drop at the last end of the distribution line.
Putting $\dot{I}_{ac} = 0$,

$$\dot{E}_{ac} = (1/\dot{A}_c)\dot{E}_a = \dot{E}_s/\dot{A}_c\dot{A}_a, \tag{25}$$

$$E_{ac} = |\dot{E}_{ac}| \simeq E_s[1-(y_1Z_1/2)a^2][1-(y_2Z_2/2)c^2]$$
$$\simeq E_s[1-(y_1Z_1/2)a^2-(y_2Z_2/2)c^2], \tag{26}$$

$$\%E_{ac} \simeq [(cDZ_1/V_n^2)a^2 + (dDZ_2/2V_n'^2)c^2] \times 100, \tag{27}$$

where $$y_2 = dD/V_n'^2. \tag{28}$$

With the same representation as Eq. (21),

$$\%E_{ac} = (cDZ_1/10V_n^2)a^2 + (dDZ_2/20V_n'^2)c^2. \tag{29}$$

In this equation, the first term shows the voltage drop in percent of main and the second term the voltage drop in percent of branch.

(2) The represented phase voltage for load admittance

Here, the represented phase voltage used for determination of load admittances (y_1 and y_2) is considered.

In Eq. (12), the apparent power of feeding point was given as follows,

$$W_1 = \left(2cDa - \frac{4c^2D^2Z_1a^3}{9E_n^2}\right)\left(\frac{E_s}{E_n}\right)^2. \tag{30}$$

Meanwhile, in planning, the power is conceived as $2cDa$.
That is $W_1 = 2cDa$.
Then, equating these two equations,

$$W_1 = 2cDak^2\left(1 - \frac{2cDZ_1a^2}{9E_n^2}\right) = 2cDa, \tag{31}$$

in which $k = E_s/E_n$.

From this equation

$$k^2\left(1 - \frac{2cDZ_1a^2}{9E_n^2}\right) = 1. \tag{32}$$

Hence, minding that

$$y_1=\frac{2cD}{3E_n^2}\,,\quad k=\frac{E_s}{E_n}\simeq\frac{1}{1-\frac{1}{6}y_1Z_1a^2}\simeq 1+\frac{1}{6}\,y_1Z_1a^2\,. \tag{33}$$

Therefore

$$\%E_n=1/3(y_1Z_1/2)a^2\times 100)=1/3(\%E_a). \tag{34}$$

This result means that the represented voltage of main is the voltage at the point where the voltage drop is equal to one-thirds of the total voltage drop of main. But, in planning, the voltage drop of main is actually unknown, and further, there is a problem left how to consider the represented voltage of branch.

Then, it becomes necessary to estimate the appropriate values of represented voltages of main and branch.

Now, for the purpose of this estimation, preliminarily, the next two questions will be solved on reference to the method of F. C. Van Wormer.

(i) Relation between main and branch to have a maximum load area within a limited voltage drop.

From Eq. (29),

$$\%E_{ac}=K_1ca^2+K_2c^2, \tag{35}$$

where $K_1=DZ_1/10V_n^2$, $K_2=dDZ_2/20V_n'^2$.

Rewritting this equation,

$$a=\left(\frac{\%E_{ac}-K_2c^2}{K_1c}\right)^{0.5}. \tag{36}$$

And load area, A, is equal to $(2ac)$,
then

$$A=\left[\frac{4c(\%E_{ac})-4K_2c^3}{K_1}\right]^{0.5}. \tag{37}$$

The condition to have a maximum load area is obtained as follows,

$$\frac{dA}{dc}=\frac{1}{2K_1}\left[\frac{4c(\%E_{ac})-4K_2c^3}{K_1}\right]^{-0.5}\cdot\left[4(\%E_{ac})-12K_2c^2\right]=0\,.$$

Therefore $4(\%E_{ac})-12K_2c^2=0$,

or $$K_2c^2=\%E_{ac}/3\,. \tag{38}$$

The above condition shows that a maximum load area is obtained in the case that the total voltage drop to far end of distribution line as shown in Fig. 2 is shared in the proportion of two-thirds on main to one-thirds on branch.

(ii) Relation between main and branch to give a minimum voltage drop with an allowable current.

The total load of area is given as follows,

$$W = AD = 2acD, \tag{39}$$

hence

$$a = W/2cD. \tag{40}$$

From Eqs. (35) and (40),

$$\%E_{ac} = K_1 W^2/4cD^2 + K_2 c^2. \tag{41}$$

The condition to give a minimum voltage drop when c is variable is obtained as follows,

$$\frac{d(\%E_{ac})}{dc} = -\frac{K_1 W^2}{4D^2c^2} + 2K_2 c = 0 \tag{42}$$

$$K_1 W^2/4D^2c^2 = 2K_2 c\,. \tag{43}$$

Multiplied by c,

$$K_1 W^2/4D^2 c = 2K_2 c^2. \tag{44}$$

While, $K_1 W^2/4D^2 c = K_1 c a^2 = \%E_a =$ voltage drop of main, and

$K_2 c^2 =$ voltage drop of branch.

In conclusion, the same condition is obtained as in Case (i).

Now, on the basis of the above results, the adequate values of the represented voltages of main and branch are determined as follows :

For main ;

If the total voltage drop to far end of distribution line is given as ε in percent, the following equations are obtained.

$$\%E_{ac} = \varepsilon, \qquad \%E_a = (2/3)\varepsilon,$$

From Eq. (34), $\%E_n = 1/3 \cdot (\%E_a) = \frac{2}{9}\varepsilon$,

hence, minding that

$k = E_s/E_n \simeq 1 + (\%E_n)/100$ from Eq. (33),

$k \simeq 1 + (2/900)\varepsilon$.

Therefore, the represented phase voltage, used for determination of y_1, is obtained as follows,

$$E_n = (1/k)/E_s \simeq [1 - (2/900)\varepsilon]E_s \tag{45}$$

For branch ;

For simplicity, assume that the phase voltage of each tap is equal to the mean value given by Eq. (22),
that is,

$\%E_{a\text{mean}}=2/3(\%E_a)=(4/9)\varepsilon.$

And, referring to Eq. (38),

(voltage drop in percent of branch) $=(1/3)\varepsilon$.

Then, the represented phase voltage drop in percent for branch only is derived, referring to Eq. (34) for main, as $(1/3)\varepsilon\times(1/3)=(1/9)\varepsilon$.

Consequently, the represented phase voltage, used for determination of y_2, is obtained as follows,

$$\%E'_n=(4/9)\varepsilon+(1/9)\varepsilon=(5/9)\varepsilon.$$

Hence, $k=1+(5/900)\varepsilon,$

and, $$E'_n=(1/k)E_s\simeq[1-(5/900)\varepsilon]E_s \tag{46}$$

The above equations may be applied in case of an idealized distribution system. But, considering that it is an aim in planning to gain a minimum voltage drop or a maximum load area under the limited condition, it seems that the represented voltages give by Eqs. (45) and (46), may be a good guide to the determination of actual load admittance.

In this case, of course, the adequate value of ε must be assumed.

(3) Resistance loss of the tree-type distribution system

The resistance loss of a three-phase line is three times the product of the resistance of one conductor and the square its current. Then, the resistance losses of the distribution line as shown in Fig. (2) are given by the following equations, approximately.

(a) The resistance loss of main, (L_1).

$$L_1=3\int_0^a|I_x|^2r_1\cdot dx. \tag{47}$$

From Eq. (18),

$$L_1\simeq3\int_0^a\left[E_sy_1(a-x)\left\{1-\frac{y_1Z_1}{6}(2a^2+2ax-x^2)\right\}\right]^2r_1\cdot dx$$

$$=(E_ny_1a)^2r_1a[1-(4/5)y_1Z_1a^2](E_s/E_n)^2,$$

or

$$L_1\simeq\frac{4c^2D^2a^3}{3V_n^2}\cdot r_1\cdot\left(1-\frac{4}{5}\cdot\frac{2cD}{V_n^2}Z_1a^2\right)\left(\frac{E_s}{E_n}\right)^2, \tag{48}$$

where $y_1=2cD/V_n^2$.

(b) The total loss of all branches, (L_2).

The resistance loss of one branch line, l_2, is given as follows,

$$l_2=3\int_0^c|I_{xy}|^2\cdot r_2\cdot dy. \tag{49}$$

Referring to Eqs. (16) and (24),

$$|I_{xy}|\simeq\left\{1-\frac{y_1(2a-x)Z_1x}{2}\right\}y_2(c-y)\left\{1-\frac{y_2Z_2}{6}(2c^2+2cy-y^2)\right\}E_s. \tag{50}$$

From the above two equations (49) and (50),

$$l_2\simeq E_s^2\left\{1-\frac{y_1(2a-x)Z_1x}{2}\right\}^2\cdot(y_2c)^2\cdot\left(1-\frac{4}{5}y_2Z_2c^2\right)\cdot(r_2c). \tag{51}$$

The number of branches is derived from Fig. 2 as $2n=2a/d$, where, "n" is number of branches of one side of main.

The distances from the feeding point to each tap are shown as

$$x=d/2,\quad 3d/2,\cdots\cdots\cdots\cdots\cdots\cdots,\ (2n-1)d/2.$$

Then, the total resistance loss of all branches, L_2, is given as follows,

$$\begin{aligned}L_2&=2\sum_{n=1}^{n=a/d}l_2=2\left[(y_2c)^2\left(1-\frac{4}{5}y_2Z_2c\right)r_2c\cdot\sum_{n=1}^{n=a/d}\left\{1-\frac{y_1(2a-x)Z_1x}{2}\right\}^2E_s^2\right]\\&=2(E_sy_2c)^2a/d\left\{1-y_1Z_1\frac{(8a^2+d^2)}{12}\right\}\left(1-\frac{4}{5}y_2Z_2c^2\right)r_2c\\&=\frac{2(a/d)}{3}\frac{d^2D^2c^3}{V_n'^2}r_2\left\{1-\frac{2cD}{V_n^2}Z_1\frac{(8a^2+d^2)}{12}\right\}\left(1-\frac{4dD}{5V_n'^2}Z_2c^2\right)\left(\frac{E_s}{E_n'}\right)^2.\end{aligned} \tag{52}$$

Thus, the total resistance loss of the system, L, is

$$L=L_1+L_2 \tag{53}$$

Put $V_s=V_n=V_n'$ ($E_s=E_n=E_n'$) neglecting the second terms in parentheses of the equations (48) and (52), these agree with the results of the simplified solutions of D. L. Hopkins and D.R. Samson.(1)

4. Conclusion

The merits of the above-mentioned method are as follows:

(1) By representing the load as an equivalent admittance, the tree-type distribution system with the uniformly distributed load can be treated as a simple four terminal network.

(2) By estimating rationally the value of the above admittance, the electric characteristics of the system can be fairly precisely derived.

(3) By treating the whole distribution system as a combination of simple four terminal network, it is expected that the complicated system, such as loop, banking or network and the like, may be systematically analyzed.

About the problems in the last item, the authors wish to describe in the next opportunity.

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(3) W. J. Denton, Power Apparatus and Systems, p. 485 (1955).