Behavior of Traveling Waves at Fault Point

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In order to discriminate the surge characteristics for a case of breaking fault, generation phenomena and reflected traveling waves on transmission lines of finite length must be considered corresponding to various conditions of the transition point. This report presents the analytical method of reflection and refraction waves propagating along the transmission lines, at a fault point.

I. Introduction

When traveling waves on a multi-conductor system reach a fault point at which there are abrupt changes of circuit constants, as open or short-circuited, a part of the waves is reflected back on the lines, and a part may pass on to other sections of the system. The two waves to which it gives rise at the transition point, such as a fault point, are called the reflected and transmitted waves, respectively. Such waves are formed in accordance with Kirchhoff's law and they satisfy the differential equations of the lines.[1)]

This report shows how the behavior and reflection of multi-conductor waves at a point of breaking fault may be calculated.

II. Method of Calculation of Reflection or Refraction Waves at a Fault Point

Fig. 1 shows a system of n transmission line conductors, parallel each other. Suppose that the conductors from 1 to k are broken at a point P, conductors from $(m+2)$ to k

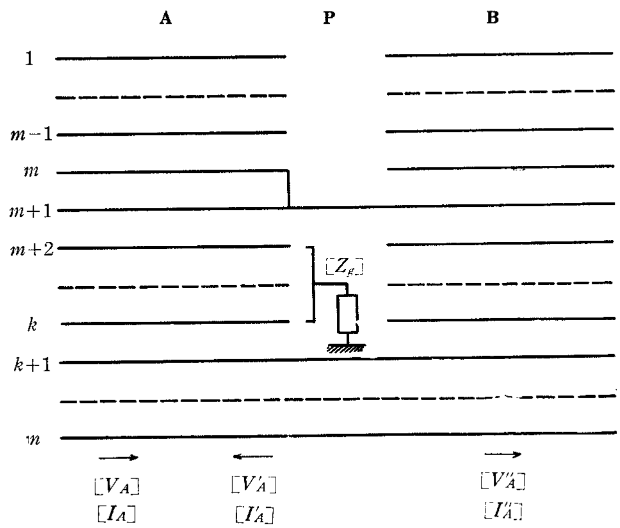

Fig. 1. General transition points on multi-conductor system.

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are terminated through series impedances.

When the incident wave V_A approaching along the line conductor reaches the transition point P, it will give rise to a wave V'_A reflected back on the line; transmitted wave V_B on the line. Let the transition point P be taken as the origin of coordinates, so that all the approaching incident waves are traveling in the positive direction. If all lines will be taken as ideal, or no-loss, the voltage and current waves are related by

$$[Y]\begin{pmatrix}[V_A]_{\substack{1\\m-1}}\\ [V_A]_{\substack{m\\m+1}}\\ [V_A]_{\substack{m+2\\k}}\\ [V_A]_{\substack{k+1\\n}}\end{pmatrix}=\begin{pmatrix}[I_A]_{\substack{1\\m-1}}\\ [I_A]_{\substack{m\\m+1}}\\ [I_A]_{\substack{m+2\\k}}\\ [I_A]_{\substack{k+1\\n}}\end{pmatrix} \tag{1}$$

$$-[Y]\begin{pmatrix}[V'_A]_{\substack{1\\m-1}}\\ [V'_A]_{\substack{m\\m+1}}\\ [V'_A]_{\substack{m+2\\k}}\\ [V'_A]_{\substack{k+1\\n}}\end{pmatrix}=\begin{pmatrix}[I'_A]_{\substack{1\\m-1}}\\ [I'_A]_{\substack{m\\m+1}}\\ [I'_A]_{\substack{m+2\\k}}\\ [I'_A]_{\substack{k+1\\n}}\end{pmatrix} \tag{2}$$

$$[Y]\begin{pmatrix}[V_B]_{\substack{1\\m-1}}\\ [V_B]_{\substack{m\\m+1}}\\ [V_B]_{\substack{m+2\\k}}\\ [V_B]_{\substack{k+1\\n}}\end{pmatrix}=\begin{pmatrix}[I_B]_{\substack{1\\m-1}}\\ [I_B]_{\substack{m\\m+1}}\\ [I_B]_{\substack{m+2\\k}}\\ [I_B]_{\substack{k+1\\n}}\end{pmatrix} \tag{3}$$

$$[V_A]_{\substack{k+1\\n}}+[V'_A]_{\substack{k+1\\n}}=[V_B]_{\substack{k+1\\n}},\quad [I_A]_{\substack{k+1\\n}}+[I'_A]_{\substack{k+1\\n}}=[I_B]_{\substack{k+1\\n}} \tag{4}$$

$$[I_A]_{\substack{1\\m-1}}+[I'_A]_{\substack{1\\m-1}}=[I_B]_{\substack{1\\m-1}}=[0] \tag{5}$$

$$[V_A]_{\substack{m+2\\k}}+[V'_A]_{\substack{m+2\\k}}=[Z_g]\left([I_A]_{\substack{m+2\\k}}+[I'_A]_{\substack{m+2\\k}}\right),\quad [I_B]_{\substack{m+2\\k}}=[0] \tag{6}$$

$$\left.\begin{aligned}&V_{A,m}+V'_{A,m}=V_{A,m+1}+V'_{A,m+1}=V_{B,m+1}\\ &I_{A,m}+I_{A,m+1}+I'_{A,m}+I'_{A,m+1}-I_{B,m+1}=I_{B,m}=0\end{aligned}\right\} \tag{7}$$

in which $[Y]$ is the admittance matrix, whose elements Y_{rr} and Y_{rs} represent the self-admittance of conductor r and mutual-admittance between r and s, respectively. Suffices A and B denote the left-hand and right-hand directions of the point P. Convenient abbreviated notations used in foregoing analysis[2] will be also adopted in cases, where no confusion can arise.

Hereupon, partitioning $[Y]$ into sixteen submatrices in the same way of our previous report,[2] each of the submatrices will be represented by a single symbol.

$$[Y]=\begin{pmatrix}[Y_1] & [Y_2] & [Y_3] & [Y_4]\\ [Y_5] & [Y_6] & [Y_7] & [Y_8]\\ [Y_9] & [Y_{10}] & [Y_{11}] & [Y_{12}]\\ [Y_{13}] & [Y_{14}] & [Y_{15}] & [Y_{16}]\end{pmatrix} \tag{8}$$

And eliminating the current wave matrices from (1), (2), (3), (4), (5) and (7), we can readily obtain

$$
\begin{aligned}
&[Y_5][V_B]_{\substack{1\\m-1}}+[Y_6'][V_B]_{\substack{m\\m+1}}+[Y_7][V_B]_{\substack{m+2\\k}}\\
&=\begin{bmatrix}0&0\\1&1\end{bmatrix}\{[Y_5]([V_A]_{\substack{1\\m-1}}-[V_A']_{\substack{1\\m-1}})+2[Y_6][V_A]_{\substack{m\\m+1}}+[Y_7]([V_A]_{\substack{m+2\\k}}-[V_A']_{\substack{m+2\\k}})\}\\
&+\begin{bmatrix}-1&0\\1&0\end{bmatrix}[Y_8][V_A]_{\substack{k+1\\n}}-\begin{bmatrix}1&0\\1&2\end{bmatrix}[Y_8][V_A']_{\substack{k+1\\n}}
\end{aligned}
\tag{9}
$$

where

$$[Y_6']=[Y_6]+\begin{bmatrix}0&0\\0&2\end{bmatrix}$$

The equations (1), (2) and (5) give

$$
\begin{aligned}
&[Y_1]([V_A]_{\substack{1\\m-1}}+[V_A']_{\substack{1\\m-1}})+[Y_2]([V_A]_{\substack{m\\m+1}}+[V_A']_{\substack{m\\m+1}})+[Y_3]([V_A]_{\substack{m+2\\k}}+[V_A']_{\substack{m+2\\k}})\\
&+[Y_4]([V_A]_{\substack{k+1\\n}}+[V_A']_{\substack{k+1\\n}})=[0]
\end{aligned}
\tag{10}
$$

Substituting $[I_A]_{\substack{m+2\\k}}$, $[I_A']_{\substack{m+2\\k}}$ of (1) and (2) in (6), and making use of (7), there results

$$
\begin{aligned}
&[Z_g]\{[Y_9]([V_A]_{\substack{1\\m-1}}-[V_A']_{\substack{1\\m-1}})+[Y_{10}](2[V_A]_{\substack{m\\m+1}}-\begin{bmatrix}0&1\\0&1\end{bmatrix}[V_B]_{\substack{m\\m+1}})\\
&+[Y_{12}]([V_A]_{\substack{k+1\\n}}-[V_A']_{\substack{k+1\\n}})\}=([U]-[Z_g][Y_{11}])[V_A]_{\substack{m+2\\k}}\\
&+([U]+[Z_g][Y_{11}])[V_A']_{\substack{m+2\\k}}.
\end{aligned}
\tag{11}
$$

On the other hand, from (1)～(7) following simultaneous equations can be derived:

$$
\left.
\begin{aligned}
&[Y_1][V_B]_{\substack{1\\m-1}}+[Y_3][V_B]_{\substack{m+2\\k}}+[Y_4][V_A']_{\substack{k+1\\n}}=-[Y_2][V_B]_{\substack{m\\m+1}}-[Y_4][V_A]_{\substack{k+1\\n}}\\
&[Y_9][V_B]_{\substack{1\\m-1}}+[Y_{11}][V_B]_{\substack{m+2\\k}}+[Y_{12}][V_A']_{\substack{k+1\\n}}=-[Y_{10}][V_B]_{\substack{m\\m+1}}-[Y_{12}][V_A]_{\substack{k+1\\n}}\\
&[Y_{13}][V_B]_{\substack{1\\m-1}}+[Y_{15}][V_B]_{\substack{m+2\\k}}+[Y_{16}][V_A']_{\substack{k+1\\n}}=[Y_{13}]([V_A]_{\substack{1\\m-1}}-[V_A']_{\substack{1\\m-1}})\\
&\quad+2[Y_{14}][V_A]_{\substack{m\\m+1}}-[Y_{14}'][V_B]_{\substack{m\\m+1}}+[Y_{15}]([V_A]_{\substack{m+2\\k}}-[V_A']_{\substack{m+2\\k}})
\end{aligned}
\right\}
\tag{12}
$$

in which

$$[Y_{14}']=[Y_{14}]\begin{bmatrix}1&1\\0&2\end{bmatrix}$$

Noticing that $[Y_1]$, $[Y_{11}]$ and $[Y_{16}]$ are square and solving these three simultaneous equations, we can obtain

$$
\left.
\begin{aligned}
&[V_B]_{\substack{1\\m-1}}=[\xi_1]+[\lambda_1][V_A']_{\substack{1\\m-1}}+[\lambda_2][V_B]_{\substack{m\\m+1}}+[\lambda_3][V_A']_{\substack{m+2\\k}}\\
&[V_A']_{\substack{k+1\\n}}=[\xi_2]+[\lambda_4][V_A']_{\substack{1\\m-1}}+[\lambda_5][V_B]_{\substack{m\\m+1}}+[\lambda_6][V_A']_{\substack{m+2\\k}}\\
&[V_B]_{\substack{m+2\\k}}=[\xi_3]+[\lambda_7][V_A']_{\substack{1\\m-1}}+[\lambda_8][V_B]_{\substack{m\\m+1}}+[\lambda_9][V_A']_{\substack{m+2\\k}}
\end{aligned}
\right\}
\tag{13}
$$

where

$$
\begin{aligned}
[\xi_1]=&([Y_1]-[Y_3][Y_{11}]^{-1}[Y_9])^{-1}\{([Y_3][Y_{11}]^{-1}[Y_{10}]-[Y_2])[V_A]_{\substack{m\\m+1}}\\
&+([Y_3][Y_{11}]^{-1}[Y_{12}]-[Y_4])[V_A]_{\substack{k+1\\n}}+([Y_3][Y_{11}]^{-1}[Y_{12}]-[Y_4])[\xi_2]\}
\end{aligned}
$$

$$[\xi_2]=([\mu_3][\mu_1]^{-1}[\mu_2]-[\mu_4])^{-1}\big[\{([Y_{13}]-[\mu_3][\mu_1]^{-1}[Y_9])[Y_1]^{-1}[Y_2]+[\mu_3][\mu_1]^{-1}[Y_{10}]$$
$$+2[Y_{14}]\}[V_A]_{\substack{m\\m+1}}+\{([Y_{13}]-[\mu_3][\mu_1]^{-1}[Y_9])[Y_1]^{-1}[Y_4]+[\mu_3][\mu_1]^{-1}[Y_{12}]\}[V_A]_{\substack{k+1\\n}}$$
$$+[Y_{13}][V_A]_{\substack{1\\m-1}}+[Y_{15}][V_A]_{\substack{m+2\\k}}\big]$$

$$[\xi_3]=([\mu_1]-[\mu_2][\mu_4])^{-1}[\mu_3]^{-1}\big[\{([\mu_2][\mu_4]^{-1}[Y_{13}]-[Y_9])[Y_1]^{-1}[Y_2]+[Y_{10}]$$
$$+2[\mu_2][\mu_4]^{-1}[Y_{14}]\}[V_A]_{\substack{m\\m+1}}+\{([\mu_2][\mu_4]^{-1}[Y_{13}]-[Y_9])[Y_1]^{-1}[Y_4]+[Y_{12}]\}[V_A]_{\substack{k+1\\n}}$$
$$+[\mu_2][\mu_4]^{-1}([Y_{13}][V_A]_{\substack{1\\m-1}}+[Y_{15}][V_A]_{\substack{m+2\\k}})\big]$$

$$[\lambda_1]=[\lambda'][\lambda_4],\qquad [\lambda_2]=[\lambda'][\lambda_5],\qquad [\lambda_3]=[\lambda'][\lambda_6],\quad [\lambda_4]=[\lambda''][Y_{13}]$$
$$[\lambda_5]=[\lambda''][Y'_{14}],\quad [Y_6]=[\lambda''][Y_{15}],\quad [\lambda_7]=[\lambda'''][Y_{13}],\quad [\lambda_8]=[\lambda'''][Y'_{14}]$$
$$[\lambda_9]=[\lambda'''][Y_{15}],\quad [\lambda']=([Y_1]-[Y_3][Y_{11}]^{-1}[Y_9])^{-1}([Y_3][Y_{11}]^{-1}[Y_{12}]-[Y_4])$$
$$[\lambda'']=([\mu_4]-[\mu_3][\mu_1]^{-1}[\mu_2])^{-1},\quad [\lambda''']=([\mu_2][\mu_4]^{-1}[\mu_3]-[\mu_1])^{-1}[\mu_2][\mu_4]^{-1}$$
$$[\mu_1]=[Y_9][Y_1]^{-1}[Y_3]-[Y_{11}],\quad [\mu_2]=[Y_9][Y_1]^{-1}[Y_4]-[Y_{12}]$$
$$[\mu_3]=[Y_{13}][Y_1]^{-1}[Y_3]-[Y_{15}],\quad [\mu_4]=[Y_{13}][Y_1]^{-1}[Y_4]-[Y_{16}]$$

Therefore, substituting (13) in (9), (10) and (11) and rewriting them, we get finally

$$\left.\begin{aligned}
&[\nu_1][V'_A]_{\substack{1\\m-1}}+[\nu_2][V_B]_{\substack{m\\m+1}}+[\nu_3][V'_A]_{\substack{m+2\\k}}=[\zeta_1]\\
&[\nu_4][V'_A]_{\substack{1\\m-1}}+[\nu_5][V_B]_{\substack{m\\m+1}}+[\nu_6][V'_A]_{\substack{m+2\\k}}=[\zeta_2]\\
&[\nu_7][V'_A]_{\substack{1\\m-1}}+[\nu_8][V_B]_{\substack{m\\m+1}}+[\nu_9][V'_A]_{\substack{m+2\\k}}=[\zeta_3]
\end{aligned}\right\}\qquad(14)$$

in which

$$[\nu_1]=[Y_1]+[Y_4][Y_4],\quad [\nu_3]=[Y_3]+[Y_4][\lambda_6]$$
$$[\nu_2]=2[Y_2]\begin{bmatrix}0&1\\0&1\end{bmatrix}+[Y_4][\lambda_5]$$
$$[\nu_4]=[Y_5][\lambda_1]+[Y_7][\lambda_7]+\begin{bmatrix}0&0\\1&1\end{bmatrix}[Y_5]+\begin{bmatrix}1&0\\1&2\end{bmatrix}[Y_8][\lambda_4]$$
$$[\nu_5]=[Y_5][\lambda_2]+[Y'_6]+[Y_7][\lambda_8]+\begin{bmatrix}1&0\\1&2\end{bmatrix}[Y_8][\lambda_5]$$
$$[\nu_6]=[Y_5][\lambda_3]+[Y_7][\lambda_9]+\begin{bmatrix}0&0\\1&1\end{bmatrix}[Y_7]+\begin{bmatrix}1&0\\1&2\end{bmatrix}[Y_8][\lambda_6]$$
$$[\nu_7]=[Z_g][Y_9]+\{[U]+[Z_g]([Y_{11}]+[Y_{12}])\}[\lambda_7]$$
$$[\nu_8]=[Z_g][Y_{10}]\begin{bmatrix}0&1\\0&1\end{bmatrix}+\{[U]+[Z_g]([Y_{11}]+[Y_{12}])\}[\lambda_8]$$
$$[\nu_9]=\{[U]+[Z_g]([Y_{11}]+[Y_{12}])\}[\lambda_9]$$
$$[\zeta_1]=-[Y_1][V_A]_{\substack{1\\m-1}}-[Y_3][V_A]_{\substack{m+2\\k}}-[Y_4]([V_A]_{\substack{k+1\\n}}+[\xi_2])$$
$$[\zeta_2]=-[Y_5][\xi_1]-[Y_7][\xi_3]+\begin{bmatrix}0&0\\1&1\end{bmatrix}\{[Y_5][V_A]_{\substack{1\\m-1}}+2[Y_6][V_A]_{\substack{m\\m+1}}+[Y_7][V_A]_{\substack{m+2\\k}}\}$$
$$+\begin{bmatrix}-1&0\\1&0\end{bmatrix}[Y_8][V_A]_{\substack{k+1\\n}}-\begin{bmatrix}1&0\\1&2\end{bmatrix}[Y_8][\xi_2]$$
$$[\zeta_3]=[Z_g]([Y_9][V_A]_{\substack{1\\m-1}}+2[Y_{10}][V_A]_{\substack{m\\m+1}})+[Y_{12}][V_A]_{\substack{k+1\\n}}$$

$$-([U]-[Z_g][Y_{11}])[V_A']_{\substack{m+2\\k}}-\{[U]+[Z_g]([Y_{11}]+[Y_{12}])\}[\xi_3]$$

Since, $[\nu_1]$, $[\nu_5]$ and $[\nu_9]$ are necessarily square and of different order, it follows that

$$\left.\begin{aligned}[V_A']_{\substack{1\\m-1}}&=([\rho_1]-[\rho_2][\rho_4]^{-1}[\rho_3])^{-1}([\sigma_1]-[\rho_2][\rho_4]^{-1}[\sigma_2])\\ [V_A']_{\substack{m+2\\k}}&=([\rho_4]-[\rho_3][\rho_1]^{-1}[\rho_2])^{-1}([\sigma_2]-[\rho_3][\rho_1]^{-1}[\sigma_1])\end{aligned}\right\}\quad(15)$$

where

$$[\rho_1]=[\nu_1]-[\nu_2][\nu_5]^{-1}[\nu_4],\quad [\rho_2]=[\nu_3]-[\nu_2][\nu_5]^{-1}[\nu_6]$$

$$[\rho_3]=[\nu_7]-[\nu_8][\nu_5]^{-1}[\nu_4],\quad [\rho_4]=[\nu_9]-[\nu_8][\nu_5]^{-1}[\nu_6]$$

$$[\sigma_1]=[\zeta_1]-[\nu_2][\nu_5]^{-1}[\zeta_2],\quad [\sigma_2]=[\zeta_3]-[\nu_8][\nu_5]^{-1}[\zeta_2]$$

Using the relations reduced above, the reflected voltage waves at a fault point can be obtained in matrix form.

From (7) and (14)

$$[V_A']_{\substack{m\\m+1}}=\begin{bmatrix}0&1\\0&1\end{bmatrix}[\nu_5]^{-1}\{[\zeta_2]-[\nu_4][V_A']_{\substack{1\\m-1}}-[\nu_6][V_A']_{\substack{m+2\\k}}\}-[V_A]_{\substack{m\\m+1}}\quad(16)$$

According to (13) and (14)

$$[V_A']_{\substack{k+1\\n}}=[\xi_2]+[\lambda_5][\nu_5]^{-1}[\zeta_2]+([\lambda_4]-[\lambda_5][\nu_5]^{-1}[\nu_4])[V_A']_{\substack{1\\m-1}}$$

$$+([\lambda_6]-[\lambda_5][\nu_5]^{-1}[\nu_6])[V_A']_{\substack{m+2\\k}}\quad(17)$$

It is evident that the order in which the unknown are found is immaterial, but in the studies of surges the voltage matrix $[V_A']$ is usually the first to be calculated, and from it the other quantities are readily found if needed. If in a particular instance it is desired to find only the transmitted waves, it must go through all the calculations outlined above.

III. Special Cases

By way of illustration of the general equations of reflection and refraction, consider three phase system, where two incoming lines are broken and one of them is bussed to the third line. Then

$$[Z_g]=[\infty]$$

$$m=2$$

$$[Y]=\begin{pmatrix}Y_{11}&Y_{12}&Y_{13}\\Y_{21}&Y_{22}&Y_{23}\\Y_{31}&Y_{32}&Y_{33}\end{pmatrix}$$

Neglecting from $(m+2)$ to n and substituting these equations in (9)∼(17), there results

$$V_{1,B}=(Y_{12}Y_{23}-Y_{13}Y_{22})\cdot\theta$$

$$V_{2,B}=(Y_{13}Y_{21}-Y_{11}Y_{23})\cdot\theta$$

$$V_{3,B}=(Y_{11}Y_{22}-Y_{12}Y_{21})\cdot\theta$$

in which

$$\theta=\frac{2\{(\delta_1-Y_{32})V_{2,A}+(\delta_2-Y_{33})V_{3,A}\}}{Y_{31}(Y_{12}Y_{23}-Y_{13}Y_{22})+Y_{32}(Y_{13}Y_{21}-Y_{11}Y_{23})+(\delta_1+\delta_2)(Y_{11}Y_{22}-Y_{12}Y_{21})}$$

$$\delta_1=Y_{22}+2Y_{32}-Y_{12}(Y_{21}+Y_{31})/Y_{11}$$

$$\delta_2=Y_{23}+2Y_{33}-Y_{13}(Y_{21}+Y_{31})/Y_{11}$$

Thus we can obtain the solutions for a given problem by an ingenious interpretation of the above solutions. However, it is more convenient to use a following method so as to avoid complicated mathematical calculations in order to obtain the reflected- and refracted-waves.

To show our conventional method, let us start with Fig. 2. Now, two incoming lines in four conductor system are broken and one of them is bussed to the third line. When an incident wave e_r reaches the transition point, it is reflected and refracted. Throughout this chapter, unprimed quantity, such as e_r, will denote incident wave; single-primed quantity, such as e_r', reflected wave; and double-primed quantity, such as e_r'', transmitted wave.

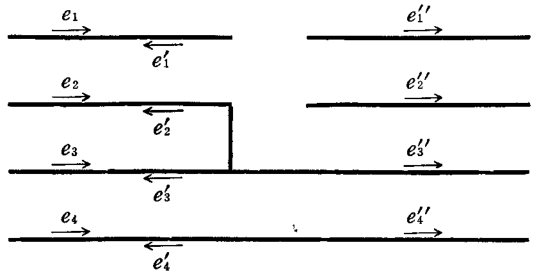

Fig. 2. Reflected and transmitted waves on four-conductor system.

Then, in accordance with Kirchhoff's laws, the voltage waves are related by the surge admittances of the lines.

$$Y_{11}(e_1-e_1')+Y_{12}(e_2-e_2')+Y_{13}(e_3-e_3')+Y_{14}(e_4-e_4')=0$$

$$(Y_{21}+Y_{31})(e_1-e_1')+(Y_{22}+Y_{32})(e_2-e_2')+(Y_{23}+Y_{33})(e_3-e_3')+(Y_{24}+Y_{34})(e_4-e_4')$$
$$=Y_{31}e_1''+Y_{32}e_2''+Y_{33}e_3''+Y_{34}e_4''$$

$$Y_{41}(e_1-e_1')+Y_{42}(e_2-e_2')+Y_{43}(e_3-e_3')+Y_{44}(e_4-e_4')$$
$$=Y_{41}e_1''+Y_{42}e_2''+Y_{43}e_3''+Y_{44}e_4''$$

$$Y_{11}e_1''+Y_{12}e_2''+Y_{13}e_3''+Y_{14}e_4''=0$$

$$Y_{21}e_1''+Y_{22}e_2''+Y_{23}e_3''+Y_{24}e_4''=0$$

$$e_2+e_2'=e_3+e_3'=e_3''$$

$$e_4+e_4'=e_4'' \qquad (18)$$

The solution of these simultaneous equations gives

$$\begin{aligned}
e_1' &= \{f(Y_{33}'Y_{44}' - Y_{34}'Y_{43}') + g(Y_{14}Y_{43}' - Y_{13}'Y_{44}') + h(Y_{13}'Y_{34}' - Y_{14}Y_{33}')\}/\varDelta \\
e_2' &= -e_2 + e_3 + e_3' \\
e_3' &= \{f(Y_{34}'Y_{41} - Y_{31}'Y_{44}') + g(Y_{11}Y_{44}' - Y_{14}Y_{41}) + h(Y_{14}Y_{31}' - Y_{11}Y_{34}')\}/\varDelta \\
e_4' &= \{f(Y_{31}'Y_{43}' - Y_{33}'Y_{41}) + g(Y_{13}'Y_{41} - Y_{11}Y_{43}') + h(Y_{11}Y_{33}' - Y_{13}'Y_{31}')\}/\varDelta
\end{aligned} \tag{19}$$

in which

$$\begin{aligned}
&Y_{13}' = Y_{12} + Y_{13}, \quad Y_{31}' = Y_{21} + Y_{31} \\
&Y_{33}' = Y_{22} + Y_{23} + \kappa_1 Y_{31} + (1+\kappa_3) Y_{32} + 2Y_{33} \\
&Y_{34}' = Y_{24} + \kappa_2 Y_{31} + \kappa_4 Y_{32} + 2Y_{34} \\
&Y_{43}' = \kappa_1 Y_{41} + (1+\kappa_3) Y_{42} + 2Y_{43} \\
&Y_{44}' = \kappa_4 Y_{41} + \kappa_4 Y_{42} + 2Y_{44} \\
&\varDelta = Y_{11}(Y_{33}'Y_{44}' - Y_{34}'Y_{43}') + Y_{31}'(Y_{14}Y_{43}' - Y_{13}'Y_{44}') + Y_{41}(Y_{13}'Y_{34}' - Y_{14}Y_{33}') \\
&f = Y_{11}e_1 + 2Y_{12}e_2 + (Y_{13} - Y_{12})e_3 + Y_{14}e_4 \\
&g = (Y_{21} + Y_{31})e_1 + 2(Y_{22} + Y_{32})e_2 - \{Y_{22} - Y_{23} + \kappa_1 Y_{31} + (1+\kappa_3) Y_{32}\}e_3 \\
&\qquad + (Y_{24} - \kappa_2 Y_{31} - \kappa_4 Y_{32})e_4 \\
&h = Y_{41}e_1 + 2Y_{42}e_2 - \{\kappa_1 Y_{41} + (1+\kappa_3) Y_{42}\}e_3 - (\kappa_2 Y_{41} + \kappa_4 Y_{42})e_4 \\
&\kappa_1 = \kappa(Y_{12}Y_{23} - Y_{13}Y_{22}), \qquad \kappa_2 = \kappa(Y_{12}Y_{24} - Y_{14}Y_{22}) \\
&\kappa_3 = \kappa(Y_{13}Y_{21} - Y_{11}Y_{23}), \qquad \kappa_4 = \kappa(Y_{14}Y_{21} - Y_{11}Y_{24}) \\
&\kappa = 1/(Y_{11}Y_{22} - Y_{12}Y_{21})
\end{aligned}$$

Thus, if incident voltage waves are given at the transition point and all reflected voltage waves are known, then the transmitted waves can be readily found from above equations.

Of course, in a case of this kind much time is saved by writing the transition point equations directly, rather than reducing from the general equations. The derivation of the general equations is principally of value in serving as a model for procedure. We must find the most profitable method to solve each case.

IV. Conclusions

The equations in this report hold rigorously only at $t=0$ or no-loss lines, but under actual conditions the electrostatic transient is usually over within a fraction of a micro-second, and thus our method would no doubt be of much use from an engineering point of view.

The circuit shown in Fig. 1 is a representative of those that simulate many actual conditions. In performing reductions from the general equations, great care must be taken in evaluating the order of matrices.

Most of actual faults encountered in transmission system reduce to relatively simple combinations under surge conditions. For this reason, it is quicker and there is less chance for error if the results are derived directly from the transition point equations of the actual case, as described in Chapter III.

References

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