# Analytic Approximation of Tunnel Diode Static Characteristics

# Analytic Approximation of Tunnel Diode Static Characteristics*

Tamotsu KASAI** and Wataru SASAKI**

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The present paper describes a theoretical result on an approximate function derived by the authors concerning tunnel diode static characteristics.

The current-voltage static characteristics of tunnel diode have a peak point, a valley point and a point where the value of current is equal to that of the peak point.

The approximate function obtained expresses the currents and voltages at the above three points accurately, and has six constants which can be determined by the value of these currents and voltages. As a result of calculation, it is found that the degree of approximation of this function to the actual tunnel diode static characteristics is comparatively good.

Discussion is made on the error of the approximate function obtained as compared with those of the other approximate functions reported elsewhere.

## 1. Introduction

One simple approximation of tunnel diode static characteristics*** is shown in Fig. 1. This approximation is expressed by three straight lines in three domains, and it is frequently used in the analysis of the steady oscillation[2] or the like. This approximation, however, is of no avail in some cases; for example, the quantative analysis of the level deviation[3] in the output of a pulse compressor[4] containing tunnel diodes can't be performed by using it.

Hence, it is desirable that an approximate function not only has small errors to the actual diode characteristics, but also is expressed by a simple analytic function over all the ranges of working voltage.

The papers on the approximate functions of the diode characteristics seem to have been published by a few authors, and these functions usually have the constants determined by the actual diode characteristics. The constants of the approximate function derived by K. Tarnay[5] are determined by using the values of currents and voltages at the peak and the valley points of the actual tunnel diode. The degree of approximation of this function to the actual diode characteristics is comparatively good in the range of $V<V_v$, but is not good in the range of $V>V_v$. $V$ denotes the arbitrary voltage, and $V_v$ denotes the voltage at the valley point of the actual diode characteristics, and the peak and the valley points of this function do not agree well with those of the actual diode characteristics.

The approximate function derived by A. Ferendeci and W. H. Ko[6] can be expressed as the sum of two currents due to the tunnel effect and the ordinary diode action. The

* The outline of this paper was read at the Annual Joint Meeting of Four Institutes related with Electrical Engineers of Japan in 1963.[1)]

** Department of Electronic Engineering, College of Engineering.

*** In order to simplify, this term will be written as "diode characteristics" hereafter.

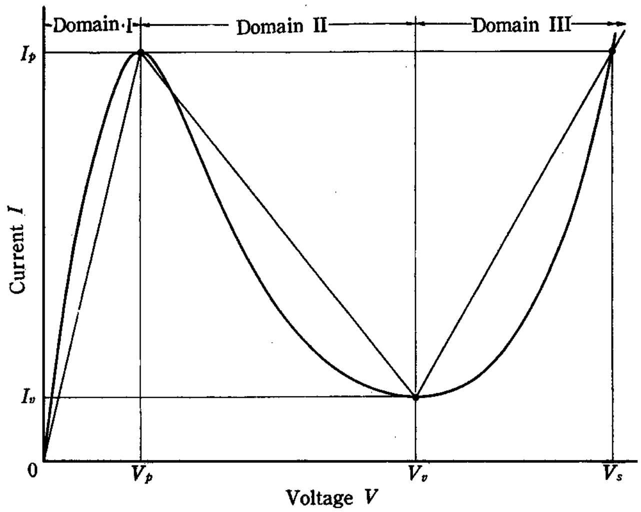


Fig. 1. Approximation by three straight lines of tunnel diode static characteristics.

constants of this function are determined by the measured values of current at low and high voltages. But even if the constants of this function are determined by the values measured very accurately, the peak and the valley points of this function do not always agree with those of the actual diode characteristics.

The outline of the actual diode characteristics is generally given by the current $I_p$ and the voltage $V_p$ at the peak point, the current $I_v$ and the voltage $V_v$ at the valley point and the voltage $V_s$ at the point where the value of current is equal to $I_p$. Consequently, it is desirable that the approximate function of the diode characteristics expresses these currents and voltages accurately.

On the basis of the standpoint mentioned above, the authors derived an approximate function, in which three points obtained theoretically agree very well with those of the corresponding actual diode characteristics, and this function has six constants determined by the values of $I_p$, $I_v$, $V_p$, $V_v$ and $V_s$.

To determine the constants of this function is a little troublesome compared with the other approximate methods, but if three of these constants are determined, the others can be determined easily.

## 2. Conditions to determine the constants

The current-voltage characteristics of the approximate functions are expressed as follows:

$$I=f(V). \tag{1}$$

According to the actual diode characteristics, Eq. (1) must evidently have the peak point at $V=V_p$ and the valley point at $V=V_v$. Further, it is necessary that the value of Eq. (1) at $V=V_s$ is equal to the value of current at the peak point. Consequently, the conditions to determine the constants in Eq. (1) are expressed as follows:

$$\left.\begin{array}{lll} f(0)=0, & f(V_p)=I_p, & f(V_v)=I_v, \\ f(V_s)=I_p, & dI/dV|_{V=V_p}=0, & dI/dV|_{V=V_v}=0. \end{array}\right\} \quad (2)$$

## 3. Approximate function of tunnel diode static characteristics

The approximate function obtained is shown by a solid line in Fig. 2. This function is given by the product of three factors which are shown by the dotted lines in Fig. 2. Every curve in Fig. 2 shows only the form of each factor and does not show the relative value of each factor.

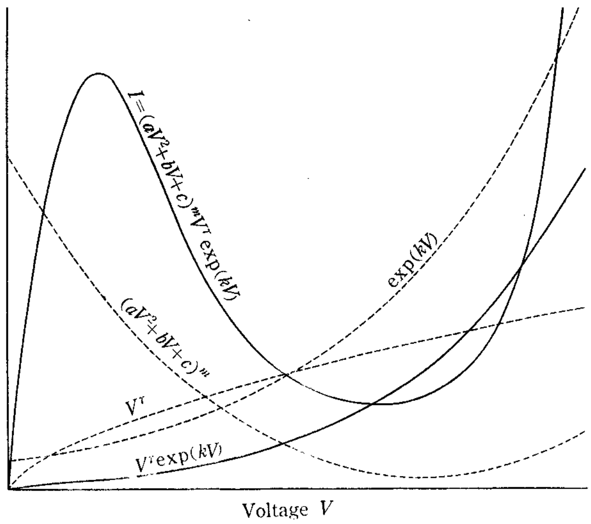


Fig. 2. Tunnel diode $I-V$ static characteristic and its three components.

Thus, the approximate function of Eq. (1) is expressed as follows:

$$I=(aV^2+bV+c)^m V^\gamma \exp(kV) \qquad (\gamma>0), \tag{3}$$

where $a$, $b$, $c$, $m$, $\gamma$ and $k$ are constants.

Evidently Eq. (3) satisfies the first condition of Eqs. (2). The six constants in Eq. (3), however, can't be determined by only the conditions of Eqs. (2), because the conditions of Eqs. (2), except the first condition, are less than the constants in Eq. (3) in number. Therefore, it is desirable to consider the functions which are given as follows:

$$I=(aV^2+bV+c)V^\gamma \exp(kV). \tag{4}$$

$$I=(aV^2+bV+c)^m V \exp(kV). \tag{5}$$

These functions, however, have some faults described below. First, in Eq. (4), the constants are able to be determined by the conditions of Eqs. (2). But the calculated value of current is a pretty larger than the measured value of current of the actual diode, because the value of $\gamma$ in this case is a pretty less than one. Second, in Eq. (5), the constants can't be determined so that it satisfies all the conditions of Eqs. (2), except a special case. This fact is described in Chapter 5.

For this reason, the authors abopted the function of Eq. (3). One of the constants in Eq. (3), however, must be determined regardless of the conditions of Eqs. (2), and so $m$ can be chosen for such a constant. According to Eq. (3), it seems that the curvature of this function in the range of $V_p < V < V_s$ increases according to the increase of $m$. Practically, however, when $m$ is over a certain value, the curvature of this function decreases conversely in the same range, because this curvature varies not only by $m$ but also by $\gamma$.

Well, when $m$ takes a proper value, $\gamma$ has a maximum value, and the larger the value of $\gamma$* becomes, the better the degree of approximation of Eq. (3) becomes. Therefore, it is desirable that the value of $m$ is determined so that $\gamma$ becomes maximum.

In this way, if $m$ is determined at first, $a$, $b$, $c$, $\gamma$ and $k$ will be determined by the conditions of Eqs. (2).

Further, $dI/dV$ is expressed as follows from Eq. (3).

$$dI/dV = \{m(2aV+b)V + (\gamma+kV)(aV^2+bV+c)\} \times (aV^2+bV+c)^{m-1} V^{\gamma-1} \exp(kV) \tag{6}$$

## 4. Equations to determine the constants in Eq. (3)

The equations to determine every constant in Eq. (3) can be derived from Eqs. (2), (3) and (6). Thus, $m$, $\gamma$ and $k$ are connected with each other by the following equations.

$$(V_s - V_v)\{(V_v - V_p)(V_s - V_p)(\gamma + kV_p)/m - V_p(V_v + V_s - 2V_p)\} g^{1/m}(V_p, I_p, \gamma, k) + V_p(V_s - V_p)^2 g^{1/m}(V_v, I_v, \gamma, k) - V_p(V_v - V_p)^2 g^{1/m}(V_s, I_p, \gamma, k) = 0 \tag{7}$$

$$V_v(V_s - V_v)^2 g^{1/m}(V_p, I_p, \gamma, k) - (V_s - V_p)\{(V_v - V_p)(V_s - V_v)(\gamma + kV_v)/m - V_v(2V_v - V_p - V_s)\} g^{1/m}(V_v, I_v, \gamma, k) - V_v(V_v - V_p)^2 g^{1/m}(V_s, I_p, \gamma, k) = 0, \tag{8}$$

where

$$g^{1/m}(V, I, \gamma, k) = [I/\{V^\gamma \exp(kV)\}]^{1/m}. \tag{9}$$

As aforesaid, it is desirable that the value of $m$ is chosen so that the value of $\gamma$ becomes maximum. Therefore, it is necessary that the value of $m$ should be determined first, as shown in Chapter 5. From Fig. 3, it is evident that the value of $m$ may take the integer nearest to the correct value.

Thus, if the value of $m$ is determined, the values of $\gamma$ and $k$ will be determined by Eqs. (7) and (8). Further, the values of other constants are determined by the following equations which have $m$, $\gamma$ and $k$.

$$a = g^{1/m}(V_p, I_p, \gamma, k)/\{(V_v - V_p)(V_s - V_p)\} - g^{1/m}(V_v, I_v, \gamma, k)/\{(V_v - V_p)(V_s - V_v)\} + g^{1/m}(V_s, I_p, \gamma, k)/\{(V_s - V_p)(V_s - V_v)\} \tag{10}$$

$$b = -(V_v + V_s) g^{1/m}(V_p, I_p, \gamma, k)/\{(V_v - V_p)(V_s - V_p)\}$$

* This value, generally, is less than one.

$$+(V_s+V_p)g^{1/m}(V_p,\ I_p,\ \gamma,\ k)/\{(V_v-V_p)(V_s-V_v)\}$$
$$-(V_p+V_v)g^{1/m}(V_s,\ I_p,\ \gamma,\ k)/\{(V_s-V_p)(V_s-V_v)\} \quad (11)$$

$$c=V_vV_sg^{1/m}(V_p,\ I_p,\ \gamma,\ k)/\{(V_v-V_p)(V_s-V_p)\}$$
$$-V_sV_pg^{1/m}(V_v,\ I_v,\ \gamma,\ k)/\{(V_v-V_p)(V_s-V_v)\}$$
$$+V_pV_vg^{1/m}(V_s,\ I_p,\ \gamma,\ k)/\{(V_s-V_p)(V_s-V_v)\} \quad (12)$$

## 5. Example

We now pick np SONY 1T1103 tunnel diode* which has the following standard values.

$I_p=2.025$ mA, $I_v=0.393$ mA, $V_p=75$ mV,
$V_v=360$ mV, $V_s=500$ mV

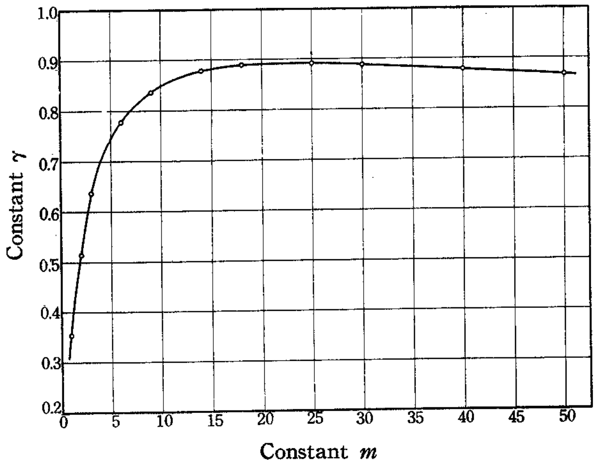


Fig. 3. Constant $\gamma$ vs. constant $m$ curve.

First, the relation between the constants $\gamma$ and $m$, which are connected with each other by Eqs. (7) and (8), is shown in Fig. 3. From this figure, the value of $m$ is chosen as follows:

$m=25.$

Then, the values of $\gamma$ and $k$ are determined as follows by Eqs. (7) and (8).

$\gamma=0.890578,\quad k=46.9944$

Second, from Eqs. (10), (11) and (12), the values of $a$, $b$ and $c$ are determined as follows:

$a=1.73304,\quad b=-2.01000,\quad c=0.884196.$

By the above calculation, in conclusion, the approximate function of this diode is expressed by the following equation.

$$I=(1.73304V^2-2.01000V+0.884196)^{25}\ V^{0.890578}\exp(46.9944V), \quad (13)$$

* This diode was generously supplied by Dr. T. Komoto of Sony Corp..

where $I$ is expressed in amperes and $V$ is expressed in volts. The calculated result of Eq. (13) is shown by a bold solid line in Fig. 4.

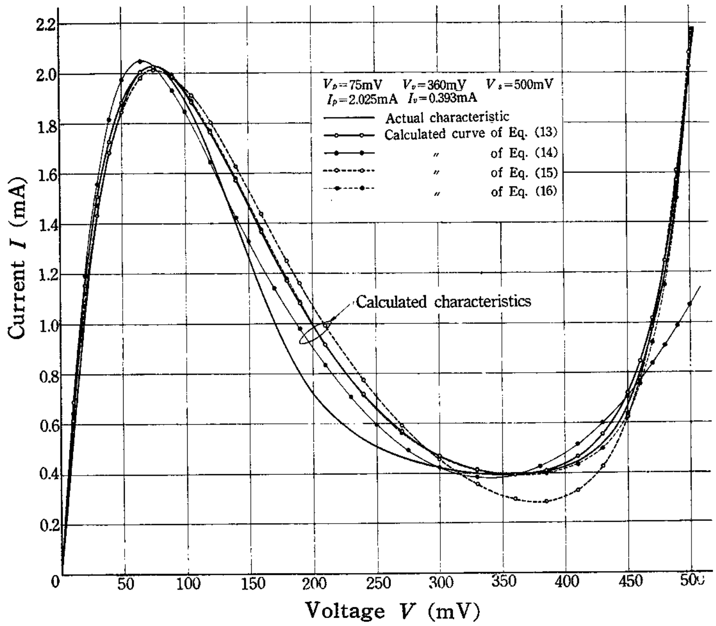


Fig. 4. Actual and calculated characteristics of SONY 1T 1103 germanium tunnel diode.

It is found that the value of $\gamma$ is less than one. This fact shows that every constant in the approximate function of Eq. (5) is unable to be determined so that it satisfies all the conditions of Eqs. (2).

## 6. Comparisons among several approximate functions

This chapter describes the comparisons among the approximate functions obtained and the others.

It is assumed that $I_p$, $I_v$, $V_p$, $V_v$ and $V_s$ have the values shown in Chapter 5. In this case, the approximate function derived by K. Tarnay[5] is expressed as follows at 25°C.

$$I=2.66489\times10^{-2}(0.36-V)^2\tanh(19.4721V)+1.09167\times10^{-3}V \tag{14}$$

The calculated result of Eq. (14) is shown by a fine solid line in Fig. 4. From this curve, it is seen that the peak point is not at $V=V_p$ and the valley point is not at $V=V_v$ also. Further, the error of this approximate function to the actual diode characteristics in the range of $V>V_v$ is comparatively large.

The approximate function derived by A. Ferendeci and W. H. Ko[6] is expressed as follows:

$$I=7.03545\times10^{-2}V\exp(-12.8590V)+2.89235\times10^{-9}\{\exp(26.9180V)-1\}. \tag{15}$$

The calculated result of Eq. (15) is shown by a bold dotted line in Fig. 4. The error of this approximate function in the range of $0< V< V_p$ is smaller than that of the approximate function of Eq. (13). This difference arises from the facts that Eq. (13) has the factor $V^\gamma$ and $\gamma$ is less than one.

When the authors read the outline of this paper at the Annual Joint Meeting of Four Institutes related with Electrical Engineers of Japan, it was found that one approximate function* had been used in the paper of H. R. Kaupp and D. R. Crosby.[7] This approximate function has five constants, and the values of these constants were calculated by a computer.

The authors determined the constants of this approximate function by using the conditions of Eqs. (2), and obtained the approximate function as follows:

$$\begin{aligned} I=&(7.32790\times10^{-2}V+4.25427\times10^{-2}V^3-4.25890V^5\\ &+56.5619V^7)\exp\{-V/(7.5\times10^{-2})\}\\ &+5.36456\times10^{-12}\{\exp(39V)-1\}. \end{aligned} \tag{16}$$

The calculated result of Eq. (16) is shown by a fine dotted line in Fig. 4.

The approximate functions of Eqs. (15) and (16) have the first terms different in form which control the current due to the tunnel effect, but the second terms are of the same type; that is, as to the first terms, Eq. (15) includes a factor of the first degree of $V$ and Eq. (16) includes a factor of the seventh degree of $V$. Generally, the larger the degree of $V$ becomes, the better the approximation becomes. This fact is shown by the curves of Eqs. (15) and (16) in Fig. 4.

To calculate the error between the approximate functions and the actual diode characteristics the following formula is used.

$$\varepsilon=(I_c-I_a)/I_a=I_c/I_a-1, \tag{17}$$

where $\varepsilon$ is the error, $I_c$ is the calculated value of current of the approximate function and $I_a$ is the value of current of the actual tunnel diode.

The calculated results of Eq. (17) are shown by four curves in Fig. 5.

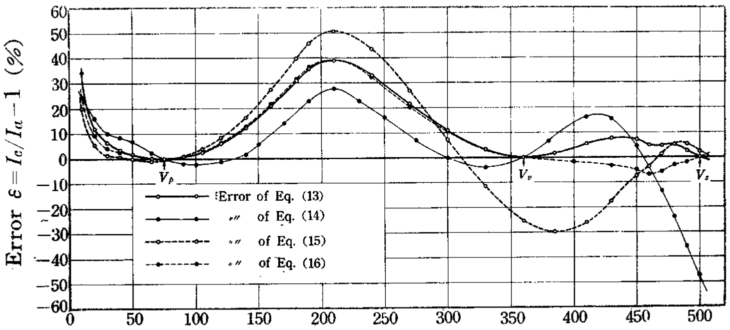


Fig. 5. Per cent errors between actual and calculated characteristics of SONY 1T 1103 germanium tunnel diode.

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* The authors knew this function on an advice of Dr. K. Fushimi of Electrical Communications Laboratory.

## 7. Discussions

It is observed that the degree of approximation of Eq. (13) is not good in the part of negative resistance as shown in Fig. 4. Consequently, to improve the degree of approximation in this part, it is desirable to find a better function in future. However, it seems that such a function will be more complicated than the present function.

The approximate function of Eq. (14) has the constants which satisfy the three conditions, $f(0)=0$, $f(V_p)=I_p$ and $f(V_v)=I_v$, but this function does not have the peak point at $V=V_p$ and the valley point at $V=V_v$ in general. Further, the value of current of this function at $V=V_s$ is not equal to the current $I_p$ at the peak point.

If it is assumed that this function is expressed by only the first term, it is certain that the value of current of such a function at $V=V_v$ becomes zero. To prevent this contradiction, the second term of Eq. (14) should be added so that the value of current at $V=V_v$ becomes $I_v$, and so the value of constant of this term becomes equal to $I_v/V_v$.

This approximate function can be modified so that the value of current at $V=V_s$ becomes $I_p$. In this case, the second term of Eq. (14) is replaced by an exponential form of $\alpha\{\exp(\beta V)-1\}$, and the constants $\alpha$ and $\beta$ are determined so that the value of current becomes $I_v$ at $V=V_v$ and $I_p$ at $V=V_s$. This modified function satisfies three conditions, $f(0)=0$, $f(V_v)=I_v$ and $f(V_s)=I_p$, and it is certain that the error of this function in the range of $V>V_v$ is smaller than that of the approximate function of Eq. (14).

In the approximate function of Eq. (15), the first and the second terms express the currents due to the tunnel effect and the ordinary diode action respectively, and the two constants of each term can be determined by the measured values of currents at low and high voltages, because the values of the second term at low voltage and the first term at high voltage become nearly zero. Consequently, in order to determine the four constants of this function, it is necessary that the measured values of currents at low and high voltages should be extremely accurate. To determine the constants of this function is simple compared with that of Eq. (13). This function, however, does not always satisfy all the conditions of Eqs. (2).

The approximate function of Eq. (16) satisfies all the conditions of Eqs. (2), and the error of this function, as expressed by Eq. (17), has the same degree with that of Eq. (13). However, the errors of these functions in the range of $V_v<V<V_s$ have different signs; that is, the error of Eq. (16) is negative and that of Eq. (13) is positive in this range.

The approximate function of Eq. (13), as derived in this paper, differs from the approximate functions from Eq. (14) to Eq. (16) in point of the fact that this function is expressed by one term over all the ranges of working voltage. Further, it is noticable that this function has only one exponential factor, while the functions of Eqs. (15) and (16) have two exponential factors.

Well, the approximate function of Eq. (13) has six constants, and $m$ is determined so that the value of $\gamma$ has a maximum value, regardless of the conditions of Eqs. (2). It is a knotty problem to determine the approximate function of some tunnel diode, because the approximate function includes such unknown constants mentioned above. But if the actual diode characteristics do not differ so much from those which were given at the beginning of Chapter 5, the value of $m$ will not change so much. Further, as shown in Fig. 3, if the deviation of the value of $m$ from correct value is small, the deviation of the approximate function also is small. Hence, it is clear that $m$ is warranted in taking the value in Eq. (13), and that the approximate function of the actual diode characteristics also may be expressed as follows:

$$I=(aV^2+bV+c)^{25}V^r\exp(kV). \tag{18}$$

It goes without saying that five constants of Eq. (18) can be determined by the conditions of Eqs. (2).

In the approximate function of Eq. (13), the value of $dV/dI$ at $V=0$ becomes zero. This contradicts to the fact that the value of $dV/dI$ of the actual diode characteristics at $V=0$ does not become zero. This, however, is out of the question, because the tunnel diode is used under a proper d-c bias voltage in general.

The actual diode characteristics is influenced by temperature, and this influence is very large in the range of $V>V_v$. Therefore, it is desirable that Eq. (3) includes a factor of temperature, to emphasize the physical meaning of the approximate function. In this case, from a close examination of the deviation of the actual diode characteristics due to temperature, it seems that such a function will be realized without particular difficulty. Further, in this case, it is desirable that a physical consideration, as shown in reference (8), is introduced into Eq. (3).

The actual diode characteristic, as shown in Fig. 4, was recorded by X-Y recorder. In this paper, the authors took no notice on an accuracy of the recorder, practically, however, the recorder has some errors due to recording. The error due to this sort, as shown in Fig. 4, is large in the part of very low voltage. By this reason, the error which is expressed by Eq. (17) increases rapidly when the voltage approaches to zero, as shown in Fig. 5.

## 8. Conclusions

As described above, the authors derived an approximate function of the actual diode characteristics which can be expressed by one analytic function over all the ranges of working voltage.

It is necessary that the approximate functions should be simple and close to the curve expressing the actual diode characteristics as much as possible. On the other hand, the more complicated the approximate functions are, the better the degree of approximation becomes. This fact is understood by comparing Eq. (15) with Eq. (16).

Deriving the approximate function obtained in this paper, the authors considered the above circumstances wholly. But the degree of approximation of this function is still not so good in the part of negative resistance. This fault will hereafter be removed by a detailed examination of the actual diode characteristics.

As mentioned above, there are some faults in the approximate function derived by the authors. However, it is expected that this function is possible to be utilized in an analysis of circuits using tunnel diodes.

## Acknowledgement

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## References

1) T. Kasai and W. Sasaki: Lecture 1532 in Annual Joint Meeting of Four Institutes related with Electrical Engineers of Japan, (April, 1963).
2) for example, E. Iwahashi and H. Saito: Lecture S9-13 in Annual Meeting of I.E.C.E. of Japan, (Nov. 1961).
3) T. Kasai: Lecture 1131 in Annual Joint Meeting of Four Institutes related with Electrical Engineers of Japan, (April, 1962).
4) A. A. Clark and W. H. Ko: Electronics, **34**, No. 12, 36, (1961).
5) K. Tarnay: Proc. IRE, **50**, 202, (1962).
6) A. Ferendeci and W. H. Ko: Proc. IRE, **50**, 1852, (1962).
7) H. R. Kaupp and D. R. Crosby: Proc. IRE, **49**, 146, (1961).
8) M. I. Nathan: J.A.P., **33**, 1460, (1962).