Characteristics Calculation on Three Phase Induction Motor by Single Phase Rated Frequency Test

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Generally, the characteristics of three phase induction motor is calculated by circle diagram method and its circle diagram elements are obtained under the test of three phase rated balancing voltage and frequency source. But this ideal source is not easily obtainable everywhere. Then to get circle diagaam elements by single phase rated voltage and frequency test, we tried to draw out the converting equation. In this paper, about the converting equations are discribed and many experimental results are led from them. They are able to put practical use, as expected first.

1. Introduction

The characteristics of three phase induction motor is usually calculated by circle diagram method, in which the following tests are indispensable.

1. Measurement of the resistance between two terminals.
2. Three phase no load test at rated voltage and frequency.
3. Three phase short circuit test at nearly full load current and rated frequency.

However, generally, the three phase balancing voltage source can hardly be obtained from distribution lines, and no accurate results may be expected from these tests in such factories as are not equipped with a three phase a.c. generator. But if these accurate results can be obtained from single phase no load and short circuit tests and converting its results to equivalent three phase values, the unbalance of the three phase voltages becomes out of the question. Besides the meters and voltage-regulators for the test can be much simplified.

Therefore, in this paper, we derived converting equations for above mentioned problem, and compared the results converted by them with the directly measured values on many three phase induction motors. Consequently, the difference between both values remained quite small, as we had expected.

2. Conversion of Single Phase Tests Results to Three Phase Test Ones.

In order to draw a circle-diagram, the following elemental values are required.

$\dot{I}_0$=no load current$=i_{01}-ji_{02}$

i_{01}=active component of no load current

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i_{02}=reactive component of no load current

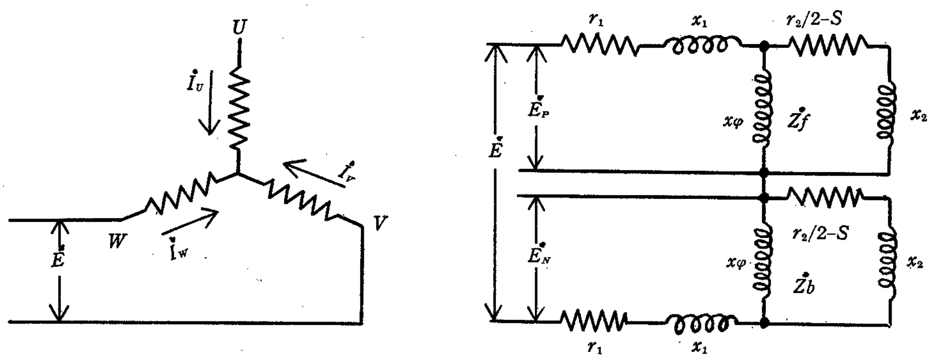
$\dot{I}_s$=short circuit current at rated voltage and frequency=$i_{s1}-ji_{s2}$

i_{s1}=active component of short circuit current

i_{s2}=reactive component of short circuit current

Therefore, it is necessary to calculate these elemental values i_{01}, i_{02}, i_{s1} and i_{s2} at rated voltage and frequency, from the values measured on the single phase rated frequency source.

While a three phase induction motor is running on three phase source, one of the supply lines is disconnected so as to leave a single phase source connected to the stator winding, the motor will continue to run, and its input current will be shown as follows,



$$\dot{Z}_1=r_1+jx_1$$

$$\dot{Z}_f=R_f+jX_f=\cfrac{1}{\cfrac{1}{jx_\varphi}+\cfrac{1}{r_2/S+jx_2}}$$

$$\dot{Z}_b=R_b+jX_b=\cfrac{1}{\cfrac{1}{jx_\varphi}+\cfrac{1}{r_2/(2-S)+jx_2}}$$

$$\dot{Z}_P=\dot{Z}_1+\dot{Z}_f$$

$$\dot{Z}_N=\dot{Z}_1+\dot{Z}_b$$

Fig. 1 The circuit when a three phase induction motor running at single phase source and its equivalent circuit.

$$\dot{I}_V=-\dot{I}_W=\dot{E}/(\dot{Z}_P+\dot{Z}_N) \qquad (1)$$

where, $\dot{E}$=source voltage

$\dot{Z}_P$=positive sequence impedance per phase

$\dot{Z}_N$=negative sequence impedance per phase

$\dot{I}_V$, $\dot{I}_W$=line current

Multiplying numerator and denominator of Eq. (1) by $\dot{E}/\dot{Z}_N$, it becomes

$$\dot{I}_V = -\dot{I}_W = \frac{\dot{E}}{\dot{Z}_P} \frac{\dot{E}/\dot{Z}_N}{\dot{E}/\dot{Z}_N + \dot{E}/\dot{Z}_P} \tag{2}$$

2.1. Conversion of single phase short circuit test values. The single phase short circuit test is easier than the three phase test, because there is no need to lock the stator. At standstill (i.e. slip $s=1$)

$$(\dot{Z}_P)_{s=1} = (\dot{Z}_N)_{s=1} \tag{3}$$

where $(\dot{Z}_P)_{s=1}$=positive sequence impedance at $s=1$

$(\dot{Z}_N)_{s=1}$=negative sequence impedance at $s=1$

Hence, from Eq. (1) and Eq. (3) the input current at single phase short circuit test is shown as

$$\dot{A}_S = \dot{E}/2(\dot{Z}_P)_{s=1} \tag{4}$$

Otherwise, the input current at three phase short circuit test is represented by

$$\dot{I}_S = \dot{E}/\sqrt{3}(\dot{Z}_P)_{s=1} \tag{5}$$

Substituting Eq. (4) into Eq. (5), obtain the converting equation

$$\dot{I}_S = 2/\sqrt{3} \cdot \dot{A}_S \tag{6}$$

It must be noted that, because the frequencies at both single phase and three phase tests are kept identical, both of the short circuit power factors are also identical and constant. Therefore, if the following results are obtained from the single phase short circuit test,

$\dot{A}_s'$ =short circuit current (nearly rated current)

$\dot{E}_s'$ =impressed voltage (for $\dot{A}_s'$)

P_s' =short circuit loss (for $\dot{A}_s'$)

it follows that

$$\left.\begin{aligned} &\dot{A}_s = \dot{A}_s' E/E_s' = \text{short circuit current at rated voltage} \\ &\dot{A}_s = a_{s1} - ja_{s2} \\ &a_{s1} = P_s' E/E_s'^2 \\ &a_{s2} = \sqrt{A_s^2 - a_{s1}^2} \end{aligned}\right\} \tag{7}$$

Converting these values into corresponding three phase values, which are necessary for circuit diagram construction

$$\left.\begin{aligned} &I_s = 2/\sqrt{3} \cdot A_s \\ &\dot{I}_s = i_{s1} - ji_{s2} \\ &i_{s1} = 2/\sqrt{3} \cdot a_{s1} \\ &i_{s2} = 2/\sqrt{3} \cdot a_{s2} \end{aligned}\right\} \tag{8}$$

2.2. Conversion of single phase no load values. Where a three phase source is available (unbalance of voltages may be left out of consideration) we can make a single phase no load test in which start the motor with this source and then cut off one of the supply lines. But, special starting methods most be adopted where only a single phase source is available. For instance, one of the methods is parallel capacitos must be connected for starting, as shown in Fig. 2.

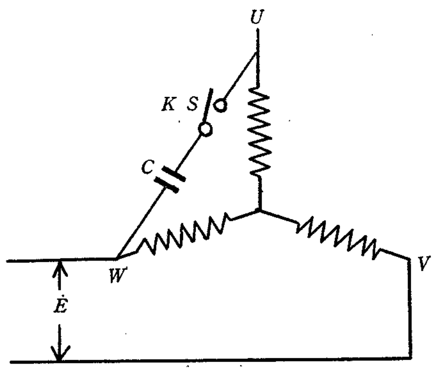


Fig. 2. A starting capacitor connecting circuit.

When a motor is running at no load, the slip may be considered approximately $s=0$. Further it must be noted that

$$\left.\begin{aligned}(\dot{Z}_N)_{s=2}&=(\dot{Z}_P)_{s=2}\\(\dot{Z}_P)_{s=2}&\simeq(\dot{Z}_P)_{s=1}\end{aligned}\right\}\qquad(9)$$

So, the input current at single phase rated voltage no load running is given from Eq. (2) and Eq. (9) by the relation

$$\left.\begin{aligned}\dot{A}_0&=\frac{\dot{E}}{(\dot{Z}_P)_{s=0}}\;\frac{\dot{E}/(\dot{Z}_N)_{s=0}}{\dot{E}/(\dot{Z}_P)_{s=0}+\dot{E}/(\dot{Z}_N)_{s=0}}\\&\simeq\frac{\dot{E}}{(\dot{Z}_P)_{s=0}}\;\frac{\dot{E}/(\dot{Z}_P)_{s=1}}{\dot{E}/(\dot{Z}_P)+\dot{E}/(\dot{Z}_P)_{s=1}}\end{aligned}\right\}\qquad(10)$$

from which

$$\dot{E}/(\dot{Z}_P)_{s=0}\simeq\dot{A}_0\frac{\dot{E}/(\dot{Z}_P)_{s=1}}{\dot{E}/(\dot{Z}_P)_{s=1}-\dot{A}_0}\qquad(11)$$

On the other hand, the input current I at three phase rated voltage no load test is

$$\dot{I}_0=\dot{E}/\sqrt{3}(\dot{Z}_P)_{s=0}\qquad(12)$$

Therefore, substituting Eq. (11) into Eq. (12)

$$\dot{I}_0\simeq\frac{\dot{A}_0}{\sqrt{3}}\;\frac{\dot{E}/\sqrt{3}(\dot{Z}_P)_{s=1}}{\dot{E}/\sqrt{3}(\dot{Z}_P)_{s=1}-\dot{A}_0/\sqrt{3}}\qquad(13)$$

from Eq. (5)

$$\dot{E}/\sqrt{3}(\dot{Z}_P)_{s=1}=\dot{I}_s$$

then Eq. (13) reduces to

$$\dot{I}_0\simeq\dot{A}_N\cdot\dot{I}_S/(\dot{I}_S-\dot{A}_N)\qquad(14)$$

where $\dot{A}_N=\dot{A}_0/\sqrt{3}$, from which calculate the three phase no load input current $\dot{I}_0$ from the sidgle phas no load input current $\dot{A}_0$.

Therefore, from the single phase no load test, the results such as

$\dot{E}$ =rated terminal voltage

$\dot{A}_0$=single phase no load input current

$\dot{P}_0$=single phase no load loss

are obtained, it follows

$$\left.\begin{aligned}
&\dot{A}_0=a_{01}-ja_{02}\\
&a_{01}=P_0/E\\
&a_{02}=\sqrt{A_0^2-a_{01}^2}\\
&\dot{A}_N=\dot{A}_0/\sqrt{3}=a_{01}/\sqrt{3}-ja_{02}/\sqrt{3}=a_{N1}-ja_{N2}
\end{aligned}\right\} \quad (15)$$

Using these results and the results of short circuit test

$\dot{I}_S=i_{S1}-ji_{S2}$

the circuit diagram elements of three phase no load test are obtained as follows

$$\left.\begin{aligned}
&\dot{I}_s=i_{s1}-ji_{s2}=I_s\exp(-j\theta_S)\\
&\qquad\theta_s=\tan^{-1}i_{s2}/i_{s1}\,,\quad I_s=\sqrt{i_{s1}^2+i_{s2}^2}\\
&\dot{A}_N=a_{N1}-ja_{N2}=A_N\exp(-j\theta_N)\\
&\qquad\theta_N=\tan^{-1}a_{N2}/a_{N1}\,,\quad A_N=\sqrt{a_{N1}^2+a_{N2}^2}\\
&\dot{I}_s-\dot{A}_N=(i_{s1}-a_{N1})-j(i_{s2}-a_{N2})=F\exp(-j\theta_F)\\
&\qquad\theta_F=\tan^{-1}(i_{s2}-a_{N2})/(i_{s1}-a_{N1})\\
&\qquad F=\sqrt{(i_{s1}-a_{N1})^2+(i_{s2}-a_{N2})^2}\\
&\therefore\quad \dot{I}_0=A_NI_s/F\exp\{-j(\theta_N+\theta_s-\theta_F)\}=D\exp(-j\theta)\\
&\qquad\theta=\theta_N+\theta_s-\theta_F\,,\quad D=A_NI_s/F\\
&\dot{I}_0==i_{01}-ji_{02}=D\cos\theta-jD\sin\theta\\
&\qquad i_{01}=D\cos\theta\,,\quad i_{02}=D\sin\theta
\end{aligned}\right\} \quad (16)$$

These calculation can be made in a construction method more conveniently.

2.3. Results of experiment. The authors made single phase and three phase tests on testing motors of various rating. The table shows the comparison of the results converted from single phase test values with that directly measured in the three phase tests.

3. Conclusion

When three phase source is not available, by using above mentioned converting equation, the three phase induction motor test which is indispensable to calculate the

Table. Results of Experiment

		Item	Symbol																
Ratings		Capacity	P(kw)	1.5		1.5		2.2		2.2		2.2		3.7		3.7		3.7	
		Voltage	E(V)	200		200		220		200		200		220		220		200	
		Current	I_1(A)	6		6		8		9		9		13		15		15	
		Frequency	f(c/s)	60		50		60		60		50		60		60		50	
		No of Poles	p	4		4		4		4		4		4		4		4	
Three Phase Test	No Load Test	Voltage	E(V)	200		200		220		200		200		220		200		200	
		Current	I_0(A)	2.03		2.74		3.24		2.84		3.46		5.28		3.61		4.55	
		In Put	W_0(W)	150		130		294		175		170		190		280		280	
	Lock Test	Current	I_S (A)	6		6		8		9		9		13		15		15	
		Voltage	E_S'(V)	41		36		43.8		42.3		37.5		42.6		40.5		35	
		In Put	W'_S(V)	210		200		452		390		380		461		525		515	
Single Phase Test	No Load Test	Voltage	E(V)	200		200		220		200		200		220		200		200	
		Current	A_0(A)	3.15		4.25		5.21		4.4		5.6		8.1		5.9		7.4	
		In put	P_0(W)	176		168		344		192		208		230		290		312	
	Lock Test	Current	A'_S(A)	6		6		8		9		9		13		15		15	
		Voltage	E'_S(V)	48		42		50.5		48		42		50.2		46		39	
		In put	P'_S(W)	140		136		302		250		255		319		330		340	
				*A	*B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
Circle Diagram	Elements	No Load Current	I_0	2.03	1.94	2.74	2.64	3.24	3.21	2.84	2.68	3.46	3.41	5.28	5.02	3.61	3.55	4.55	4.39
		〃 active component	i_{01}	0.43	0.51	0.38	0.42	0.77	0.80	0.51	0.50	0.49	0.52	0.50	0.50	0.81	0.80	0.81	0.82
		〃 reactive component	i_{02}	1.98	1.88	2.72	2.61	3.15	3.15	2.80	2.63	3.43	3.38	5.25	5.00	3.52	3.50	4.48	4.30
		Short Circuit Current	I_S	29.3	28.9	33.3	33.0	40.2	40.2	42.6	43.2	48.0	49.5	66.5	66.0	74.1	75.4	85.8	88.9
		〃 active component	i_{S1}	14.45	14.10	17.8	17.8	29.9	30.0	25.1	25.2	31.2	33.5	32.3	32.4	37.0	36.0	48.5	51.8
		〃 reative component	i_{S2}	25.45	25.2	28.2	27.8	27.2	27.0	34.5	35.2	36.6	36.5	58.2	57.6	64.3	66.2	70.6	73.7
		Resistance between terminals	$R(\Omega)$	1.9		1.9		1.6		1.52		1.52		0.915		1.52		1.52	
		Temperature	t°C	29		29		28		29.2		29.2		25		29.2		29.2	

*A: Three phase test results.

*B: Converted results, from the single phase test to the three pase values.

characeristic, is to replace the test at single phase rated frequency source. This method causes only tolerable errors and can be put to practical use.

The merit of this method is as follows.

(1). Three phase source is not necessary.

(2). It requires only simple voltage regulator.

(3). It requires only one ammeter and wattmeter.

(4). The rotor need not be looked at short circuit test.

On the other hand, the following inconvenience attaches to it.

(1). The conversion of no load current is a little complicated. (but the use of constructing method makes it pretty simple).

(2). Special starting device such as starting capacitors are necessary, because the motor per se had no starting torque at single phase source.

But it is a great advantage of this method, as it is possible to obtain the characteristics of three phase induction motors without three phase balancing source.

As another case, because two separated distribution areas exist in the nation, it often becomes necessary to make the characteristic test of a 50 c/s three phase induction motor within the 60 c/s distribution. For the problem we will discuss in another paper.