



Simplified Machine Diagnosis Techniques by Impact Vibration using 6th Normalized Moment

メタデータ	言語: eng 出版者: 公開日: 2009-08-25 キーワード (Ja): キーワード (En): 作成者: Takeyasu, Kazuhiro メールアドレス: 所属:
URL	https://doi.org/10.24729/00000894

Simplified Machine Diagnosis Techniques by Impact Vibration using 6th Normalized Moment

Kazuhiro Takeyasu

Professor of College of Economics, Osaka Prefecture University

Among many dimensional and dimensionless amplitude parameters, kurtosis (4th normalized moment of probability density function) is said to be a sensitive good parameter for machine diagnosis. In this paper, 6th normalized moment of probability density function is introduced so as to intensify the sensitivity of diagnosis. Furthermore, simplified calculation method for this new parameter by impact vibration is introduced. Compared with the past papers' results, the proposed new method shows good result.

Keywords: impact vibration, probability density function, kurtosis, 6th moment

1. INTRODUCTION

In mass production firms such as steel making that have big equipments, sudden stops of production processes by machine failure cause great damages such as shortage of materials to the later processes, delays to the due date and the increasing idling time.

To prevent these troubles, machine diagnosis techniques play important roles. So far, Time Based Maintenance (TBM) technique has constituted the main stream of the machine maintenance, which makes checks for maintenance at previously fixed time. But it has a weak point that it makes checks at scheduled time without taking into account whether the parts still keeping good conditions or not. On the other hand, Condition Based Maintenance (CBM) makes maintenance checks by watching the condition of machines. Therefore, if the parts are still keeping good condition beyond its supposed life, the cost of maintenance may be saved because machines can be used longer than planned. Therefore the use of CBM has become dominant. The latter one needs less cost of parts, less cost of maintenance and leads to

lower failure ratio.

However, it is mandatory to catch a symptom of the failure as soon as possible of a transition from TBM to CBM is to be made. Many methods are developed and examined focusing on this subject. In this paper, a method for the early detection of the failure on rotating machines is proposed which is the most common theme in machine failure detection field.

So far, many signal processing methods for machine diagnosis have been proposed (Bolleter, 1998). As for sensitive parameters, Kurtosis, Bicoherence, Impact Deterioration Factor (ID Factor) were examined (Yamazaki, 1977; Maekawa et al. 1997; Shao et al. 2001; Song et al. 1998; Takeyasu, 1989). In this paper, the index parameters of vibration are focused.

Kurtosis is one of the sophisticated inspection parameters which calculates normalized 4th moment of Probability Density Function (PDF). In the industry, there are cases where quick reactions are required on watching the waveform at the machine site.

In this paper, the case such that the impact vibration occurs on the gear when the failure arises is considered. Higher moment would be more sensitive compared with 4th moment. In this paper, 6th normalized moment of probability density function is introduced so as to intensify the sensitivity of diagnosis. Furthermore, simplified calculation method for this new parameter by impact vibration is introduced. Compared with the past papers' results, the proposed new method shows good result. Indices of deterioration are surveyed in section 2. Simplified absolute index of 6th moment is proposed in section 3. In section 4, numerical examples are presented which are followed by remarks of section 5. Section 6 is a summary.

2. SIMPLIFIED CALCULATION METHOD OF KURTOSIS

2.1 About Kurtosis

In cyclic movements such as those of bearings and gears, the vibration grows larger whenever the deterioration becomes bigger. Also, it is well known that the vibration grows large when the setting equipment to the ground is unsuitable (Yamazaki, 1977). Let the vibration signal be presented by the function of time $x(t)$. And also assume that it is a stationary time

series with mean \bar{x} . Denote the probability density function of these time series as $p(x)$.

Mean value \bar{x} of $x(t)$ is calculated as

$$\bar{x} = \int_{-\infty}^{\infty} xp(x) dx \quad (1)$$

Discrete time series are stated as follows.

$$x_k = x(k\Delta t) \quad (k=1, 2, \dots) \quad (2)$$

Where Δt is a sampling time interval. \bar{x} is stated as follows under discrete time series.

$$\bar{x} = \lim_{M \rightarrow \infty} \frac{1}{M} \sum_{i=1}^M x_i \quad (3)$$

Under the following Gaussian distribution

$$\phi(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2} \quad (4)$$

its moment is described as follows which is well known (Hino, 1977).

$$\overline{x^{(2n-1)}} = 0 \quad (5)$$

$$\overline{x^{(2n)}} = \prod_{k=1}^n (2k-1) \sigma^{2n} \quad (6)$$

Normalized one is the one divided by σ^{2n} in Eq(6). Kurtosis (KT) (normalized 4-th moment) is stated as follows.

$$KT = \frac{\int_{-\infty}^{\infty} (x-\bar{x})^4 p(x) dx}{\left[\int_{-\infty}^{\infty} (x-\bar{x})^2 p(x) dx \right]^2} \quad (7)$$

In discrete time system, it is described as

$$KT = \lim_{M \rightarrow \infty} \frac{\frac{1}{M} \sum_{i=1}^M (x_i - \bar{x})^4}{\left\{ \frac{1}{M} \sum_{i=1}^M (x_i - \bar{x})^2 \right\}^2} \quad (8)$$

We describe KT as KT_N if it is calculated by using N amount of data.

$$KT_N = \frac{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^4}{\left\{ \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 \right\}^2} \quad (9)$$

2.2 Simplified Calculation Method of Kurtosis

When the number of failures on bearings or gears arise, the peak value arise cyclically. In the early stage of the defect, this peak signal usually appears clearly. Generally, defects will injure other bearings or gears by contacting the inner covering surface as time passes.

Assume that we get N amount of data and then newly get l amount of data. Assume that mean, variance and moment are same with $1 \sim N$ data and $N+1 \sim N+l$ data except for the case where a special peak signal arises.

Let mean, variance and 4th moment of $1 \sim N$ data state as

$$\bar{x}_N, \sigma_N^2, M_N$$

And as for $N+1 \sim N+l$, let them state as

$$\bar{x}_{N/l}, \sigma_{N/l}^2, M_{N/l}$$

Where

$$M_N = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^4 \quad (10)$$

$$M_{N/l} = \frac{1}{l} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^4 \quad (11)$$

Therefore, Eq(9) is stated as

$$KT_N = \frac{M_N}{\sigma_N^4} \quad (12)$$

Assume that the peak signal which has S times impact from normal signals arises in each m times samplings. As for determining the sampling interval, the sampling theorem which is well known can be used (Tokumaru et al. 1982). But in this paper, we do not pay much attention on this point in order

to focus on our proposal theme. Let $\sigma_{N/l}^2$ and $M_{N/l}$ of this case, of $N+1 \sim N+l$ be $\bar{\sigma}_{N/l}^2$, $\bar{M}_{N/l}$, then we get

$$\begin{aligned}\bar{\sigma}_{N/l}^2 &= \frac{1}{l} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^2 \\ &= \frac{l - \frac{l}{m}}{l} \sigma_N^2 + \frac{\frac{l}{m}}{l} S^2 \sigma_N^2 \\ &= \sigma_N^2 \left(1 + \frac{S^2 - 1}{m} \right)\end{aligned}\quad (13)$$

$$\begin{aligned}\bar{M}_{N/l} &= \frac{1}{l} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^4 \\ &= \frac{l - \frac{l}{m}}{l} M_{N/l} + \frac{\frac{l}{m}}{l} S^2 M_{N/l} \\ &= \left(1 + \frac{S^4 - 1}{m} \right) M_{N/l}\end{aligned}\quad (14)$$

From these equations, we obtain $\bar{K}T_{N+l}$ as KT_{N+l} of the above case

$$\begin{aligned}\bar{K}T_{N+l} &= \frac{\frac{N}{N+l} M_N + \frac{l}{N+l} \left(1 + \frac{S^4 - 1}{m} \right) M_N}{\left\{ \frac{N}{N+l} \sigma_N^2 + \frac{l}{N+l} \sigma_N^2 \left(1 + \frac{S^2 - 1}{m} \right) \right\}^2} \\ &= \frac{\left(1 + \frac{l}{N+l} \frac{S^4 - 1}{m} \right) M_N}{\left(1 + \frac{l}{N+l} \frac{S^2 - 1}{m} \right)^2 \sigma_N^4} \\ &= \frac{\left(1 + \frac{l}{N+l} \frac{S^4 - 1}{m} \right)}{\left(1 + \frac{l}{N+l} \frac{S^2 - 1}{m} \right)^2} KT_N\end{aligned}\quad (15)$$

Here we introduce the following number. Each index is compared with the normal index as follows.

$$F_a = \frac{P_{abn}}{P_{nor}} \quad (16)$$

P_{nor} : Index at normal condition

P_{abn} : Index at abnormal condition

Then F_a as of (15) becomes as follows.

$$F_a(\bar{K}T_{N+l}) = \frac{1 + \frac{l}{N+l} \frac{S^4-1}{m}}{\left(1 + \frac{l}{N+l} \frac{S^2-1}{m}\right)^2} \quad (17)$$

We assume that time series are stationary as is stated before in 2. Therefore, even if sample pass may differ, mean and variance are naturally supposed to be the same when the signal is obtained from the same data occurrence point of the same machine.

We consider such case when the impact vibration occurs. Except for the impact vibration, other signals are assumed to be stationary and have the same means and variances. Under this assumption, we can derive the simplified calculation method for machine diagnosis which is a very practical one.

3. SIMPLIFIED CALCULATION METHOD OF 6TH NORMALIZED MOMENT

3.1 About 6th Normalized Moment

6th Normalized Moment is stated as follows

$$Q = \frac{\int_{-\infty}^{\infty} (x - \bar{x})^6 p(x) dx}{\left[\int_{-\infty}^{\infty} (x - \bar{x})^2 p(x) dx \right]^3} \quad (18)$$

In discrete time system, it is described as

$$Q = \lim_{M \rightarrow \infty} \frac{\frac{1}{M} \sum_{i=1}^M (x_i - \bar{x})^6}{\left\{ \frac{1}{M} \sum_{i=1}^M (x_i - \bar{x})^2 \right\}^3} \quad (19)$$

We describe Q as Q_N if it is calculated by using N amount of data. Let mean, variance, 6th moment and 6th normalized moment calculated by using $1 \sim N$ data state as

$$\bar{x}_N, \sigma_N^2, M_N, Q_N$$

And as for $N+1 \sim N+l$ let them state as

$$\bar{x}_{N/l}, \sigma_{N/l}^2, M_{N/l}, Q_{N/l}$$

Where

$$M_N = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^6 \quad (20)$$

$$M_{N/l} = \frac{1}{l} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^6 \quad (21)$$

3.2 Simplified Calculation Method of 6th Normalized Moment

Assume that we get N amount of data and then newly get l amount of data. Assume that mean, variance and moment are same with $1 \sim N$ data and $N+1 \sim N+l$ data except for the case special peak signals arises. Assume that the peak signal which has S times impact from normal signals arises in each m time gamplings. Let $\sigma_{N/l}^2$ and $M_{N/l}$ of this case, of $N+1 \sim N+l$ be $\bar{\sigma}_{N/l}^2$, $\bar{M}_{N/l}$, them we get

$$\begin{aligned} \bar{\sigma}_{N/l}^2 &= \frac{1}{l} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^2 \\ &= \frac{l - \frac{l}{m}}{l} \sigma_N^2 + \frac{\frac{l}{m}}{l} S^2 \sigma_N^2 \\ &= \sigma_N^2 \left(1 + \frac{S^2 - 1}{m} \right) \end{aligned} \quad (22)$$

$$\begin{aligned} \bar{M}_{N/l} &= \frac{1}{l} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^6 \\ &= \frac{l - \frac{l}{m}}{l} M_{N/l} + \frac{\frac{l}{m}}{l} S^6 M_{N/l} \\ &= \left(1 + \frac{S^6 - 1}{m} \right) M_{N/l} \end{aligned} \quad (23)$$

From these equations, we obtain \bar{Q}_{N+l} as Q_{N+l} of the above case

$$\begin{aligned}\bar{Q}_{N+l} &= \frac{\frac{N}{N+l} M_N + \frac{l}{N+l} \left(1 + \frac{S^6 - 1}{m}\right) M_N}{\left\{ \frac{N}{N+l} \sigma_N^2 + \frac{l}{N+l} \sigma_N^2 \left(1 + \frac{S^2 - 1}{m}\right) \right\}^3} \\ &= Q_N \frac{\left(1 + \frac{l}{N+l} \cdot \frac{S^6 - 1}{m}\right)}{\left(1 + \frac{l}{N+l} \cdot \frac{S^2 - 1}{m}\right)^3}\end{aligned}\quad (24)$$

F_a for \bar{Q}_{N+l} becomes as follows.

$$F_a(\bar{Q}_{N+l}) = \frac{\left(1 + \frac{l}{N+l} \cdot \frac{S^6 - 1}{m}\right)}{\left(1 + \frac{l}{N+l} \cdot \frac{S^2 - 1}{m}\right)^3}\quad (25)$$

4. Numerical Example

If the system is under normal condition, we may suppose $p(x)$ becomes a normal distribution function. Under this condition, Q is always

$$Q = 15.0$$

Under the assumption of 3.2, let $m = 12$. Considering the case $S = 2, 3, \dots, 6$ for 3.2, and setting $l = N/10$, we obtain Table 1 from the calculation of (17) and (25).

If the system is under the normal condition, $Q_N = 15.0$ for any N . So, we set N as ϵ ($\epsilon \rightarrow 0$) and the newly added l to the former value of N . Then we get Table 2.

5. REMARKS

5.1 Remarks on the result of Numerical Examples

Here, we introduce a simplified calculation method for Kurtosis which is one of the sensitive index for the failure detection. When we get newly coming l amount of data after normal N amount of data, we can easily calculate KT

Table 1: $F_a(\bar{K}T_{N+l}), F_a(\bar{Q}_{N+l})$ in the case of $m=12, l=N/10$

S	$\bar{K}T_{N+l}$	\bar{Q}_{N+l}	$F_a(\bar{K}T_{N+l})$	$F_a(\bar{Q}_{N+l})$
2	3.19	20.72	1.06	1.38
3	4.28	81.92	1.43	5.46
4	7.09	347.79	2.36	23.19
5	12.30	1084.71	4.10	72.31
6	20.30	2625.51	6.77	175.03

Table 2: $F_a(\bar{K}T_{N+l}), F_a(\bar{Q}_{N+l})$ in the case of $m=12, N=\epsilon (\epsilon \rightarrow 0), l=N$

S	$\bar{K}T_{N+l}$	\bar{Q}_{N+l}	$F_a(\bar{K}T_{N+l})$	$F_a(\bar{Q}_{N+l})$
2	4.32	48.65	1.44	3.24
3	8.28	199.80	2.76	13.32
4	13.20	450.71	4.40	30.05
5	17.70	723.89	5.90	48.26
6	21.30	970.89	7.10	64.73

in a simple way. The result of this simplified calculation method is a reasonable one compared with the result obtained so. In this paper, we introduced simplified calculation method to the 6th normalized moment which would be the more sensitive parameter than Kurtosis.

The results show that \bar{Q} is much more sensitive than $\bar{K}T$ and is easy to make failure detection. This method is properly considered to be effective for early stage failure detection especially.

The steps for the failure detection by this method are as follows.

1. Prepare standard $F_a(\bar{Q})$ Table for each normal or abnormal level
2. Measure peak values by signal data and compare the peak ratio to the normal data
3. Calculate $F_a(\bar{Q})$ by Eq(25)
4. Judge the failure level by the score of $F_a(\bar{Q})$

Preparing standard Table of $F_a(\bar{Q})$ for each normal and abnormal level, we can easily judge the failure level only by taking ratio of the peak value to the normal level and calculating $F_a(\bar{Q})$ by (25). This calculation method is simple enough to execute even on pocketsize calculator and is very practical

at the factory of maintenance site. This can be installed in microcomputer chips and utilized as the tool for early stage detection of the failure.

5.2 Estimation of the peak level by $F_a(\bar{Q})$

When $F_a(\bar{Q})$ is calculated, we can get the peak level by (25) conversely. It is considered that there is a case machine is watched and $F_a(\bar{Q})$ is calculated and sent to the central operating room. On such a case, a converse situation may occur stated above. Assume that the machine is in irregular condition. Set $N = \epsilon$ ($\epsilon \rightarrow 0$), $l = N$. From (24), we get

$$\bar{Q}_{\epsilon+N} \approx Q_{\epsilon} \frac{\left(1 + \frac{S^6 - 1}{m}\right)}{\left(1 + \frac{S^2 - 1}{m}\right)^3} \quad (26)$$

Set

$$\alpha = \frac{Q_{\epsilon}}{\bar{Q}_{\epsilon+N}} \quad (27)$$

Then $\alpha = \frac{1}{F_a(\bar{Q}_{\epsilon+N})}$ and from (26)

$$\begin{aligned} (m^2 \alpha - 1) S^6 - 3(m-1) S^4 - 3(m-1)^2 S^2 \\ + (m-1) \{m^2 (\alpha - 1) + 2m - 1\} \approx 0 \end{aligned} \quad (28)$$

Set $S^2 = x$, then (28) becomes 3rd order equation as for x and we can obtain S using the formula of solving 3rd order equation. For example, examine the case $S=4$ in Table 2. As $m=12$ and $\alpha=1/30.05$, putting these into (28), we obtain $S=3.85$ which is similar to $S=4$.

5.3 Remarks on the progress of the deterioration

In 2.2, 3.2 we assume that S times peak signal arise during m time measurement of sampling in order to focus on simplified calculation. But in reality, as deterioration progresses, n th harmonics may arise and defects may transfer. Therefore much more peaks may arise in the same interval. This implies that when we estimate m , much smaller one would be estimated if

deteriorations are progressed. We examine this by the calculation hereafter. Equation (24) may be much more simply approximated as

$$\begin{aligned}
 & \bar{Q}_{N+l} \\
 & \simeq Q_N \left(1 + \frac{l}{N+l} \cdot \frac{S^6-1}{m} \right) \left(1 - 3 \frac{l}{N+l} \cdot \frac{S^2-1}{m} \right) \\
 & \simeq Q_N \left(1 + \frac{l}{N+l} \cdot \frac{S^6-1}{m} - 3 \frac{l}{N+l} \cdot \frac{S^2-1}{m} \right) \\
 & = Q_N \left\{ 1 + \frac{l}{N+l} \cdot \frac{(S^2-1)^2(S^2+2)}{m} \right\} \tag{29}
 \end{aligned}$$

Set the increment of \bar{Q}_{N+l} as $\Delta \bar{Q}_{N+l}$, then we get

$$\frac{\Delta \bar{Q}_{N+l}}{Q_N} \simeq \frac{l}{N+l} \cdot \frac{(S^2-1)^2(S^2+2)}{m} \tag{30}$$

As for variance, we can calculate as

$$\begin{aligned}
 \bar{\sigma}_{N+l}^2 &= \frac{N}{N+l} \sigma_N^2 + \frac{N}{N+l} \sigma_N^2 \left(1 + \frac{S^2-1}{m} \right) \\
 &= \sigma_N^2 + \frac{l}{N+l} \sigma_N^2 \cdot \frac{S^2-1}{m} \tag{31}
 \end{aligned}$$

Therefore we can get the ratio of the increment as

$$\frac{\Delta \bar{\sigma}_{N+l}^2}{\bar{\sigma}_N^2} \simeq \frac{l}{N+l} \cdot \frac{S^2-1}{m} \tag{32}$$

Then we get

$$\begin{aligned}
 \Delta \xi &= \frac{\frac{\Delta \bar{Q}_{N+l}}{Q_{N+l}}}{\left(\frac{\Delta \bar{\sigma}_{N+l}^2}{\sigma_N^2} \right)^3} \\
 &= \left(\frac{N+l}{l} \right) m^2 \cdot \frac{S^2+2}{S^2-1} \tag{33}
 \end{aligned}$$

$\Delta \xi$ would be approximated as

$$\begin{aligned}
 \Delta\xi &= m^2 \cdot \frac{1 + \frac{2}{S^2}}{1 - \frac{1}{S^2}} \\
 &\approx m^2 \left(1 + \frac{2}{S^2}\right) \left(1 + \frac{1}{S^2}\right) \\
 &\approx m^2 \left(1 + \frac{3}{S^2}\right)
 \end{aligned} \tag{34}$$

When S becomes more large, $\Delta\xi$ becomes the function of m as follows.

$$\Delta\xi \approx m^2 \tag{35}$$

We examine following two cases.

case a: $N \leftarrow \varepsilon$ ($\varepsilon \rightarrow 0$), $l \leftarrow N$, the peak arises every m times measurement of sampling

case b: $N \leftarrow \varepsilon$ ($\varepsilon \rightarrow 0$), $l \leftarrow N$, the peak arises every $m/2$ times measurement of sampling

Case b is the case deterioration proceeds much more than case a.

In the case of $S=2$, $\Delta\xi$ is calculated in Table 3. As for case b, set $S=3$, and then calcute $\Delta\xi$ (Table 4). In Table 3, $\Delta\xi$ of case b is $1/4$ compared with those of case a and in Table 4, $\Delta\xi$ of case b is about $1/6$ compared with those of case a. $\Delta\xi$ is the function of m and when S becomes large, $\Delta\xi$ becomes

Table 3: $\Delta\xi$ under $S=2$, for each case a and case b

	S	(33)	(34)
Case a	2	$2m^2$	$\frac{7}{4}m^2$
Case b	2	$\frac{m^2}{2}$	$\frac{7}{16}m^2$

Table 4: $\Delta\xi$ under $S=2$, for case a and $S=3$ for case b

	S	(33)	(34)
Case a	2	$2m^2$	$\frac{7}{4}m^2$
Case b	3	$\frac{11}{32}m^2$	$\frac{m^2}{3}$

small. It implies that m would be estimated smaller than the initially supposed value.

Furthermore, we can estimate the state whether it is the case that added l amount of data have large value everywhere in the interval (case α) or it is the case that m times measurement data increase \bar{Q} (case β). Even if the variance and \bar{Q} increase on the added l amount of data, we can assume that it is the case α when calculation result of (33) is not the function of m .

If it is possible to keep watching the power spectrum, we may easily identify these situations. However, a spectrum analyzer is not always close at hand. Even in such cases, our method provides a simple way to detect the condition.

6. CONCLUSIONS

We proposed a simplified calculation method for 6th normalized moment. We previously proposed simplified calculation method of Kurtosis which is said to be the good index for failure detection of rotating machine.

Compared with the results of simplified calculation of Kurtosis, proposed simplified calculation method of 6th normalized moment found to be much more sensitive.

Judging from these results, our method is properly considered to be effective for early stage failure detection especially. This calculation method is simple enough to execute even on a pocket-size calculator and is very practical at the factory of maintenance site. This can be installed in microcomputer chips and utilized as a tool for early stage detection of the failure. The effectiveness of this method should be examined in various cases.

Reference

- [1] Bolleter, U. (1988). Blade Passage Tones of Centrifugal Pumps. *Vibration* 4 (3), 8-13
- [2] Maekawa, K., S. Nakajima, and T. Toyoda (1997). New Severity Index for Failures of Machine Elements by Impact Vibration (in Japanese). *J. SOPE Japan* 9 (3), 163-168. Mathematical Society of Japan (1985). Iwanami Mathematical Dictionary (In Japanese). Iwanami Publishing.

- [3] Noda (1987). Diagnosis Method for a Bearing (in Japanese). *NSK Tec. J.* (647), 33-38.
- [4] Shao, Y., K. Nezu, T. Matsuura, Y. Hasegawa, and N. Kansawa (2001). Bearing Fault Diagnosis Using an Adaptive Filter (in Japanese). *J.SOPE Japan* 12 (3), 71-77.
- [5] Song, J. W., H. Tin, and T. Toyoda (1998). Diagnosis Method for a Gear Equipment by Sequential Fuggy Neural Network (in Japanese). *J.SOPE Japan* 10 (1), 15-20.
- [6] Takeyasu, K. (1989). Watching Method of Circulating Moving Object (in Japanese). Certified Patent by Japanese Patent Agency.
- [7] Takeyasu, K., T. Amemiya, K. Ino, and S. Masuda (2003). Machin Diagnosis Techniques by Simplified Calculation Method. *IEMS* 2 (1), 1-8
- [8] Hino, M. (1977). Spectrum Analysis (in Japanese). Asakura shoten Publishing.
- [9] Tokumaru, H., T. Soeda, T. Nakamizo, and K. Akizuki (1982). Measurement and Calculation (in Japanese). Baifukan Publishing.
- [10] Yamazaki, H. (1977). Failure Detection and Prediction (In Japanese). Kogyo Chosakai Publishing.