Exchange Rate under the Deflationary Process

Exchange Rate under the Deflationary Process

Kunihiko Kaneko

Abstract

In this paper we examine the exchange rate movement of a small open economy under the deflationary process. To focus on the effects of price level expectation on the exchange rate, we assume that the expectations of price level and deflation rate are exogenously given. The time path of the future price level expected by the residents affects the current exchange rate in a simple model with perfect foresight. We show that the exchange rate continues to depreciate during the persistent deflationary process. We also show that the exchange rate, under the deflationary process with price level bottom, depreciates with an initial jump and then smoothly approaches the steady-state level with zero inflation.

Keywords: Exchange Rate; Deflationary Process; Price Level; Perfect Foresight
JEL Classification: F31; F41

1. Introduction

How does the exchange rate move when a small open economy is under the deflationary process? And what kind of behavior does the exchange rate show when the economy escapes from deflation? The purpose of this paper is to answer these questions in a simple model with perfect foresight.

Some authors already examine the macroeconomic performance under deflation in a fully micro-founded framework. (See, e.g., Obstfeld and Auerbach (2003).) In this paper, however, we take the opposite tack of the literature and use a reduced-form model which treats the issue as simply as possible.

Differing from the conventional approach, we assume that the time path of the expected price level formed by the residents of the economy is exogenously given. This treatment is crucially different from the standard approach where the expected price level is endogenously determined. And we also suppose the perfect foresight in our model. Therefore, the actual time path of price level is exogenously given in our model.

To focus on the effects of price level expectation on the exchange rate, we assume that the expectations of price level and deflation rate are exogenously given and other fundamental variables are set unchanged for simplicity. Further, our model seems like a monetary model of exchange rates. However, in our model, PPP is not imposed. That is, an arbitrage on goods and services is assumed to be incomplete.

The rest of the paper is organized as follows. In section 2 we present the model. It is shown that the current exchange rate is affected from the discounted sum of the future price level. The time path of price level is not specified there. The solution of exchange rate is applicable for a general time path of the price level. In section 3 we examine the exchange rate movement under the persistent deflationary process. On the other hand, in section 4, we investigate the exchange rate behavior when the economy escapes from the deflationary process. Finally, in section 5, concluding remarks are given.

2. The Model

In this section we present the deterministic, continuous-time model of a small open economy with flexible exchange rate and perfect capital mobility. The economy is formulated by the following equations (1), (2), and (3). All variables except for interest rates are in natural logarithm.

$$m-q=-\lambda i+\phi y \ , \tag{1}$$

$$i=i^{*}+\dot{s}^{e} \ , \tag{2}$$

$$q\equiv\theta(s+p^{*})+(1-\theta)p \ , \tag{3}$$

where $\lambda, \phi>0$ and $0<\theta<1$. Equation (1) is the domestic money market equilibrium condition. The left-hand side is the real money supply. On the other hand, the right-hand side expresses the real money demand. The real money supply is described by the nominal money supply m deflated by the general price index q defined in equation (3). The real money demand depends negatively on the domestic interest rate i and positively on real income y. Equation (2) is the uncovered interest parity condition. The domestic interest rate is equal to the sum of the world interest rate i^* and the expected rate of depreciation in nominal exchange rate $\dot{s}^e$. Equation (3) shows the general price index q. The domestic general price index is a weighted average of the world price level evaluated by the domestic currency $s+p^*$ and the domestic price level p. θ is a weight assigned to the world price level.

Differing from the monetary models of exchange rates, the purchasing power parity (PPP) is not assumed in the current analysis.[1] This means that an arbitrage on goods and services is incomplete, and the exchange rate and the price level move separately.

Substituting equations (2) and (3) into equation (1), we have

$$m-\theta(s+p^*)-(1-\theta)p \doteq -\lambda(i^*+\dot{s}^e)+\phi y \ . \tag{4}$$

For simplicity, we assume perfect foresight, i.e., rational expectations under certainty, on exchange rate and price level. On the exchange rate, we have

$$\dot{s}^e=\dot{s} \ . \tag{5}$$

Equation (5) means that the expected rate of depreciation is equivalent to the actual rate of depreciation. On the price level, we have

$$p^e=p \ . \tag{6}$$

Equation (6) implies that the expected time path of price level is also the actual time path of price level. (The specific time path of the price level is given in the next section.) Notice that the price level and the exchange rate are denoted in natural logarithm. Thus the time derivatives of them mean the rate of inflation (deflation) and the rate of depreciation, respectively.

Taking into account equations (5) and (6) in equation (4), we obtain the following first-order differential equation describing the exchange rate movement.

$$\dot{s}=\frac{\theta}{\lambda}s+\frac{1-\theta}{\lambda}p-\frac{1}{\lambda}z \ , \tag{7}$$

where $z\equiv -m+\theta p^*-\lambda i^*+\phi y$. As we concern solely the role of expectation of the future price level on the exchange rate, we can assume z equal to constant in equation (7) for simplicity and without loss of generality. Setting a fundamental factor $z=\bar{z}=$const., we have the following bubble-free, forward-looking solution of the exchange rate.

$$s_t=\bar{z}/\theta-\left(\frac{1-\theta}{\lambda}\right)\int_t^{\infty}p_{\tau}\exp\left[-\frac{\theta}{\lambda}(\tau-t)\right]d\tau\ . \tag{8}$$

Equation (8) implies that the exchange rate at time t is influenced by the discounted sum of the future price level. Under the perfect foresight assumption, this means that the expectation of the future price level formed by the residents affects the current level of the exchange rate. As the time path of the price level is not yet specified, equation (8) is a general forward-looking solution applicable for any time path of the price level.

To investigate the steady state of the economy with zero inflation as a benchmark case, let us assume $p_{\tau}=p_0=$const. in equation (8). Then, we have

$$s_0=\bar{z}/\theta-\left(\frac{1-\theta}{\theta}\right)p_0\ . \tag{9}$$

Equation (9) means that (i) the exchange rate is constant forever, and (ii) the lower the price level is, the higher the exchange rate is. The time paths of the price level and the exchange rate at the steady state with zero inflation are described by dotted horizontal lines in Figure 1.

3. Under the Persistent Deflationary Process

In this section we consider a small open economy under the persistent deflationary process. Facing a deflation, the residents of the economy may anticipate the deflationary process to continue persistently. When the residents of the economy believe that a deflation persists for a long time, how does the exchange rate move?

To obtain a closed-form solution for the exchange rate, we specify the time path of the expected price level formed by the residents. Under the perfect foresight assumption we impose below, this expected time path of price level coincides with the actual one that the economy proceeds.

The residents are assumed to expect the economy to be under the persistent deflationary process. For concreteness, we assume that a deflation continues at the constant rate $-\pi$ where $\pi > 0$. If the initial value of the price level $p_0 > 0$ is exogenously given, then the entire path of the price level p_t is expressed as follows.

$$p_t = p_0 - \pi t \, , \tag{10}$$

for $0 \leq t < \infty$. Under the assumption of perfect foresight, the above formulation of the actual path reflects the expected path of the price level formed by the residents of the economy.

Substituting equation (10) into equation (8), we have the following equation (11) describing the entire path of the exchange rate under the persistent deflationary process.[2)]

$$s_t = \bar{z}/\theta - \left(\frac{1-\theta}{\theta}\right)\left(p_t - \frac{\lambda}{\theta}\pi\right) . \tag{11}$$

Equation (11) means that the current exchange rate depends negatively on the current price level and the rate of deflation.

Taking a time derivative in equation (11), we obtain the expected and actual rate of depreciation of the exchange rate.

$$\dot{s}_t = -\left(\frac{1-\theta}{\theta}\right)\dot{p}_t = \left(\frac{1-\theta}{\theta}\right)\pi \, . \tag{12}$$

Equation (12) means that the rate of depreciation of the exchange rate is positive, and depends negatively on the deflationary rate of the price level. This suggests that the exchange rate under the persistent deflationary process continues to depreciate forever. If the deflationary rate is higher, then the corresponding rate of depreciation of exchange rate is also higher. The time paths of the price level and the exchange rate under the persistent deflationary process are described by solid lines in Figure 1.

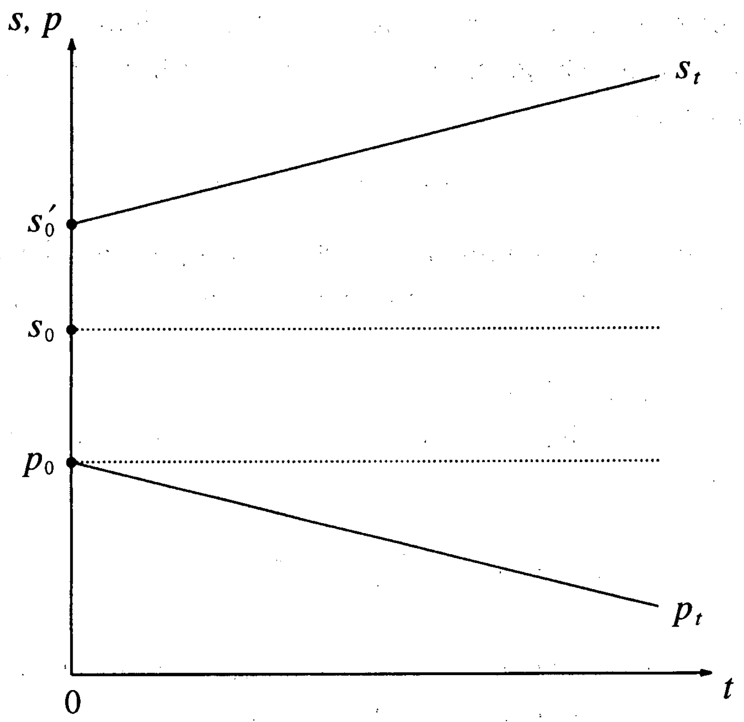

Figure 1. Exchange Rate under the Deflationary Process

4. Under the Deflationary Process with Price Level Bottom

In this section we consider the case where the residents of a small open economy change their expectation of the time path of the future price level. We assume that the residents of the economy expect an escape from the deflationary process in the future. Let us denote such a timing t^*. Under perfect foresight, the timing t^* is correctly anticipated and actually holds. For example, if a government commits to recover from a deflation and the residents of the economy believe that such a policy is credible, then the expected path of the price level will be modified by the residents.

For simplicity, we assume that the deflationary rate becomes zero at time t^* and the economy is at the steady state with zero inflation after that. Then the time path of the future price level is given as follows.

$$p_t = \begin{cases} p_0 - \pi t & \text{for } \ 0 \leq t < t^* , \\ \underline{p} & \text{for } \ t^* \leq t < \infty . \end{cases} \tag{13}$$

Equation (13) means that the economy is under the deflationary process for $0 \leq t < t^*$. On the other hand, the economy is at the steady state with zero inflation for $t^* \leq t < \infty$. t^* is the turning point from the deflationary process to the steady state with zero inflation, and we denote the corresponding price level as $\underline{p}$ and call it the price level bottom. Needless to say, $\underline{p}$ satisfies $\underline{p} = p_0 - \pi t^*$. Graphically speaking, the new time path of the price level is described by the kinked solid line with the price level bottom $\underline{p}$ in Figure 2.

We next investigate the movement of exchange rate when the economy escapes from the deflationary process in the future. When the entire path of the price level is given by equation (13), the current exchange rate can be obtained by equation (8). After some cumbersome calculation, we have the closed-form solution on the exchange rate under the deflationary process with price level bottom as follows.[3)]

For $0 \leq t < t^*$,

$$s_t = \bar{z}/\theta - \left(\frac{1-\theta}{\theta}\right)\left[p_t + \frac{\lambda}{\theta}\pi\left\{\exp\left[-\frac{\theta}{\lambda}(t^*-t)\right]-1\right\}\right] , \tag{14}$$

and for $t^* \leq t < \infty$,

$$s_t = \bar{z}/\theta - \left(\frac{1-\theta}{\theta}\right)\underline{p} . \tag{15}$$

The corresponding time path of the exchange rate is the solid curve passing through point s_0'' in Figure 2. The rate of depreciation of the exchange rate, corresponding to equations (14) and (15), becomes as follows.

For $0 \leq t < t^*$,

$$\dot{s}_t = \left(\frac{1-\theta}{\theta}\right)\pi\left\{1-\exp\left[-\frac{\theta}{\lambda}(t^*-t)\right]\right\} , \tag{16}$$

and for $t^* \leq t < \infty$,

$$\dot{s}_t = 0 . \tag{17}$$

As t approaches t^* from the left, $\dot{s}_t$ tends to 0. This means that the exchange rate approaches its ceiling level $\bar{s}$ *smoothly* even if the expected rate of deflation by the residents discontinuously changes from $-\pi$ to zero.

Consider the relationship between new and old time paths of the exchange rate. Let us denote the initial values of exchange rate and rate of depreciation, corresponding to the new time path of the price level, as s_0'' and $\dot{s}_0''$. At $t=0$, we have

$$s_0'' = \bar{z}/\theta - \left(\frac{1-\theta}{\theta}\right)\left\{p_0 + \frac{\lambda}{\theta}\pi\left[\exp\left(-\frac{\theta}{\lambda}t^*\right)-1\right]\right\} \tag{18}$$

and

$$\dot{s}_0'' = \left(\frac{1-\theta}{\theta}\right)\pi\left[1-\exp\left(-\frac{\theta}{\lambda}t^*\right)\right]. \tag{19}$$

We have $s_0'' < s_0'$ and $\dot{s}_0'' < \dot{s}_0' = \left(\frac{1-\theta}{\theta}\right)\pi$. When the residents realize that the economy escapes from the deflation process at time t^* in the future, the exchange rate jumps down immediately from s_0' to s_0'' and the rate of depreciation falls discontinuously from $\dot{s}_0'$ to $\dot{s}_0''$. And then the exchange rate approaches *smoothly* its ceiling level $\bar{s}$. The adjustment process of the exchange rate is described in Figure 2.

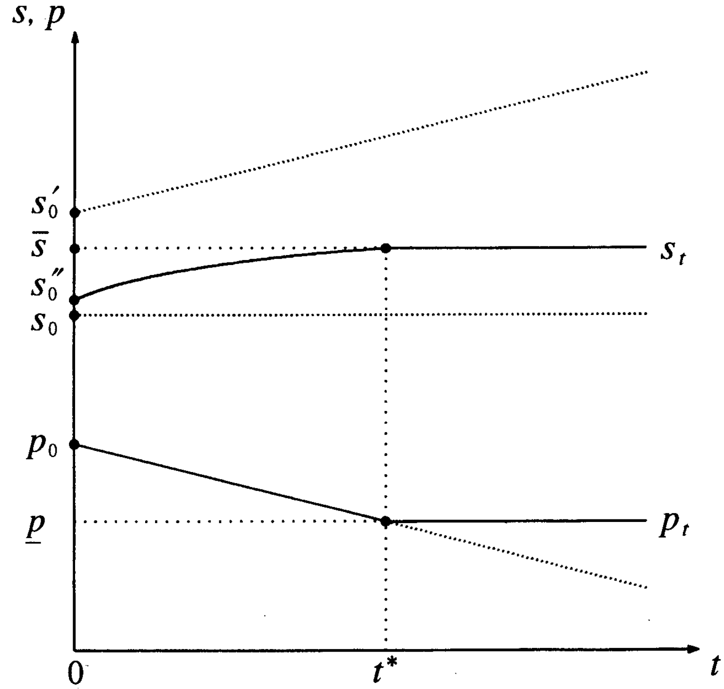

Figure 2. Exchange Rate under the Deflationary Process with Price Level Bottom

5. Concluding Remarks

In this paper we examine the exchange rate movement under the deflationary process in a simple model with perfect foresight. We show that the exchange rate continues to depreciate under the persistent deflationary process. Also we show that when there is a expectation switching from the deflationary process to the zero inflation, the exchange rate, with an initial jump, continues to depreciate and approaches smoothly its ceiling level.

In our deterministic setting with perfect foresight, the exact timing of escaping from deflation is predictable and a priori given. This is a crucial deficiency and a limitation of our results. We need to analyze the current issue in a stochastic framework.

Notes

1) When $\theta=1$ in equation (3), we have $q\equiv s+p^*$. On the other hand, we have $s+p^*=p$ under PPP. Thus assuming $\theta=1$ in the current model is equivalent to supposing PPP all the time in the model where money supply m is deflated only by domestic price p.

2) To obtain equation (11), we apply integration by parts. When $p_t=p_0-\pi t$ for $0\leq t<\infty$, we have

$$\int_t^{\infty} p_\tau \exp\left[-\frac{\theta}{\lambda}(\tau-t)\right]d\tau = -\left(\frac{\lambda}{\theta}\right)p_\tau \exp\left[-\frac{\theta}{\lambda}(\tau-t)\right]\Bigg|_t^{\infty} + \left(\frac{\lambda}{\theta}\right)\int_t^{\infty}\dot{p}_\tau \exp\left[-\frac{\theta}{\lambda}(\tau-t)\right]d\tau$$
$$= \left(\frac{\lambda}{\theta}\right)p_t - \left(\frac{\lambda}{\theta}\right)^2\pi \ .$$

Using the above result in equation (8), we get equation (11).

3) Applying integration by parts, we get the following results:

$$\int_t^{t^*} p_\tau \exp\left[-\frac{\theta}{\lambda}(\tau-t)\right]d\tau = -\left(\frac{\lambda}{\theta}\right)\underline{p}\exp\left[-\frac{\theta}{\lambda}(t^*-t)\right] + \left(\frac{\lambda}{\theta}\right)p_t + \left(\frac{\lambda}{\theta}\right)^2\pi\left\{\exp\left[-\frac{\theta}{\lambda}(t^*-t)\right]-1\right\} ,$$

and

$$\int_{t^*}^{\infty} p_\tau \exp\left[-\frac{\theta}{\lambda}(\tau-t)\right]d\tau = \left(\frac{\lambda}{\theta}\right)\underline{p}\exp\left[-\frac{\theta}{\lambda}(t^*-t)\right].$$

As

$$\int_{t}^{\infty} p_\tau \exp\left[-\frac{\theta}{\lambda}(\tau-t)\right]d\tau = \int_{t}^{t^*} p_\tau \exp\left[-\frac{\theta}{\lambda}(\tau-t)\right]d\tau + \int_{t^*}^{\infty} p_\tau \exp\left[-\frac{\theta}{\lambda}(\tau-t)\right]d\tau,$$

substituting the above results into equation (8), we obtain equation (14).

References

Auerbach, A. J., and M. Obstfeld (2003) "The Case for Open-Market Purchases in a Liquidity Trap", NBER Working Paper Series No. 9814.

Bertola, G. (1994) "Continuous-Time Models of Exchange Rates and Intervention", in F. van der Ploeg (ed.), *The Handbook of International Macroeconomics*, Ch. 9, pp. 251-298, Blackwell.

Calvo, G. A. (1977) "The Stability of Models of Money and Growth with Perfect Foresight: A Comment", *Econometrica*, Vol. 45, No. 5, pp. 1737-1739.

Kawai, M. (1994) *International Finance* (in Japanese), University of Tokyo Press.

Mark, N. C. (2002) *International Macroeconomics and Finance: Theory and Econometric Methods*, Blackwell Publishers.

Obstfeld, M., and K. Rogoff (1996) *Foundations of International Macroeconomics*, The MIT Press.

Sarno, L., and M. P. Taylor (2002) *The Economics of Exchange Rates*, Cambridge University Press.

Sargent, T. J., and N. Wallace (1973) "The Stability of Models of Money and Growth with Perfect Foresight", *Econometrica*, Vol. 41, No. 6, pp. 1043-1048.

Svensson, L. E. O. (2001) "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap", *Monetary and Economic Studies (Special Edition)*, Vol. 19 (S-1), pp. 277-312, Institute for Monetary and Economic Studies, Bank of Japan.