Effectiveness of Stiffened Plates on Rigidity under Lateral loads

Effectiveness of Stiffened Plates on Rigidity under Lateral Loads

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In this paper the effective breadth of stiffened plates subjected to lateral loads is calculated on the view point of the rigidity and numerical calculation is executed for some typical cases

1. Introduction

The conception of the effective breadth was introduced in structural engineering for the purpose of estimating the strength of structures consisting of plates and stiffners by the elementary beam theory. And by means of this effective breadth which has been treated by many researchers, the maximum stresses in structures subjected to bending loads can be estimated easily. While, there are some cases in practice where it is necessary to estimate the rigidity of the structures. For this purpose, the effective breadth men tioned above is inadequate.

In this paper, the effective breadth to estimate the maximum deflection of structures by the elementary beam theory is newly defined and numerical calculation is executed for some typical cases shown in Fig. 1, when both ends are simply supported.

2. Theory

Here, the plate is treated as a case of plane stress loaded only by shear stress imposed on it by the stiffners on the line of connection between plates and stiffners, and the stiffner is treated as a web which obeys the elementary beam theory. The coordinates are shown in Fig. 1.

For the plate, a stress function F which satisfies

$$\frac{\partial^4 F}{\partial x^4}+2\frac{\partial^4 F}{\partial x^2 \partial y^2}+\frac{\partial^4 F}{\partial y^4}=0 \qquad \cdots(1)$$

is employed. The stress components are given by

$$\sigma_x=\frac{\partial^2 F}{\partial y^2}, \quad \sigma_y=\frac{\partial^2 F}{\partial x^2}, \quad \tau=-\frac{\partial^2 F}{\partial x \partial y} \qquad \cdots(2)$$

For F, a harmonic form is assumed as follows;

$$F=\sum_n f_n \sin \omega_n x \qquad \cdots(3)$$

where $\omega_n=n\pi/L$ $\quad n$: integer

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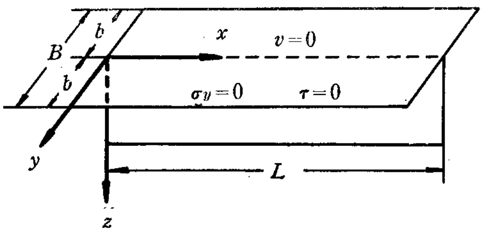
then, Eq. (1) requires that f_n takes are form as

$$f_n = (A_n + C_n\omega_n y)\cosh\omega_n y + (B_n + D_n\omega_n y)\sinh\omega_n y \qquad \cdots(4)$$

where A_n, B_n, C_n, D_n are arbitrary constants which are determined by the boundary conditions.

For typical cases, the relations between these constants determined from the physical boundary conditions are shown in Fig. 1.

Case I. Single web, flange with free sides



$$\frac{B_n}{C_n} = \frac{1-\nu}{1+\nu} \equiv j$$

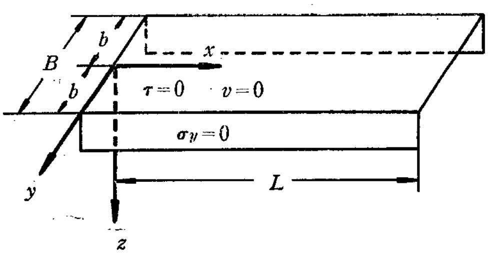
$$C_n = -\frac{A_n(\omega_n b + \sinh\omega_n b\cosh\omega_n b)}{\omega_n^2 b^2 + j\sinh^2\omega_n b}$$

$$D_n = \frac{A_n(j + \cosh^2\omega_n b)}{\omega_n^2 b^2 + j\sinh^2\omega_n b}$$

$$\frac{\lambda_n}{b} = \frac{4}{\alpha_n} \times \frac{\sinh\alpha_n + \alpha_n}{(3-\nu)(1+\nu)\cosh\alpha_n + (1+\nu)^2 \times \alpha_n^2/2 + (5-2\nu+\nu^2)}$$

where $\alpha_n = n\pi B/L$

Case II. Double web, flange bounded by webs



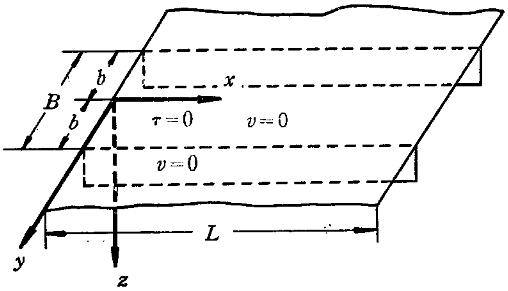
$$\frac{B_n}{C_n} = j$$

$$C_n = 0$$

$$D_n = \frac{-A_n}{\omega_n b \tanh\omega_n b}$$

$$\frac{\lambda_n}{b} = \frac{1}{\alpha_n} \times \frac{\sinh\alpha_n + \alpha_n}{\cosh\alpha_n + 1}$$

Case III. Multiple webs



$$\frac{B_n}{C_n} = j$$

$$C_n = 0$$

$$D_n = \frac{-A_n\tanh\omega_n b}{\omega_n b - j\tanh\omega_n b}$$

$$\frac{\lambda_n}{b} = \frac{4}{\alpha_n} \times \frac{\cosh\alpha_n - 1}{(3-\nu)(1+\nu)\sinh\alpha_n - (1+\nu)^2\alpha_n}$$

Fig. 1. Boundary condition, the value of B_n, C_n, D_n and λ_n for typical cases.

Then, B_n, C_n, D_n are represented by A_n and f_n can be written as

$$f_n = A_n\varphi_n \qquad \cdots(5)$$

where φ_n is a function of y only and implies no indeterminate factor for each physical boundary condition.

And A_n can be determined by considering the equilibrium of a section and the continuity of strains along the line of a connection between a plate and a web, when a external bending moment (sagging) M is given.

The equilibrium condition of a section is

$$M=M_w-eX \qquad \cdots(6)$$

M_w : bending moment (sagging) to which the web is subjected,

$X=2t\left[\partial F/\partial y\right]_0^b=2t\sum_n A_n\left[\partial\varphi_n/\partial y\right]_0^b \sin\omega_n x$: total tension which acts on the plate,

e : distance between the plate and the centroid of the web,

t : thickness of the plate.

The continuity conditions is

$$-\left[\frac{\partial^2 F}{\partial y^2}-\nu\frac{\partial^2 F}{\partial x^2}\right]_{y=0 \text{ or } b}=\frac{eM_w}{I_w}+\frac{X}{A_w} \qquad \cdots(7)$$

I_w : moment of inertia of the web about its own neutral axis,

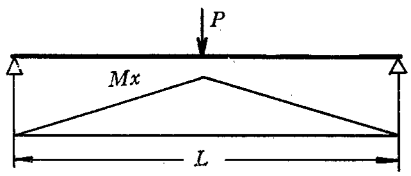
A_w: area of the web.

Now we expand M in a harmonic form as follows;

$$M=\sum_n M_n \sin\omega_n x \qquad \cdots(8)$$

the values of M_n for typical states of loading are given in Fig. 2.

(1) Concentrated load at centre (both ends supported)

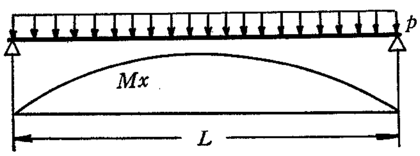


$$M_x=\sum_n M_n \sin\omega_n x$$

$$M_n=\frac{2PL}{\pi^2}\times(-1)^{(n-1)/2}\times\frac{1}{n^2}$$

$[n=1, 3, 5, \cdots]$

(2) Uniform load. (both ends supported)



$$M_x=\sum_n M_n \sin\omega_n x$$

$$M_n=\frac{4pL^2}{\pi^3}\times\frac{1}{n^3}$$

$[n=1, 3, 5, \cdots]$

Fig. 2. The value of M_n for some loading conditions.

Eliminating M_w from Eq. (6) and (7), and considering Eq. (8)

$$-A_n=\frac{M_n}{(I_w/e)\left[-\frac{\partial^2\varphi_n}{\partial y^2}+\nu\omega_n^2\varphi_n\right]_{y=0 \text{ or } b}+(2t/eA_w)(I_w+e^2A_w)\left[\partial\varphi_n/\partial y\right]_0^b} \qquad \cdots(9)$$

Then, the arbitrary constants in Eq. (4) are determined completely, and it results that we can find the stress distribution.

To determine the effective breadth based on the rigidity we will calculate the deflection of the web z. z must satisfy

$$-\frac{d^2z}{dx^2}=\frac{M_w}{EI_w}=\frac{M+eX}{EI_w} \qquad \cdots(10)$$

substituting M and X of the harmonic form in Eq. (10), and considering Eq. (9)

$$-\frac{d^2z}{dx^2}=\frac{1}{EI_w}\sum_n\frac{(I_w/e)\left[\frac{\partial^2\varphi_n}{\partial y^2}+\nu\omega_n^2\varphi_n\right]_{y=0\text{ or }b}+(2t/eA_w)I_w(\partial\varphi_n/\partial y)]_0^b}{(I_w/e)\left[\frac{\partial^2\varphi_n}{\partial y^2}+\nu\omega_n^2\varphi_n\right]_{y=0\text{ or }b}+(2t/eA_w)(I_w+e^2A_w)[\partial\varphi_n/\partial y]_0^b}$$
$$\times M_n\sin\omega_n x \qquad \cdots(11)$$

Now, we employ the function;

$$\lambda_n=\frac{[\partial\varphi_n/\partial y]_0^b}{[(\partial^2\varphi_n/\partial y^2)+\nu\omega_n^2\varphi_n]_{y=0\text{ or }b}} \qquad \cdots(12)$$

which is equivalent to the effective breadth of the flange, when $M=M_n\sin\omega_n x$. This term was defined and numerically calculated by A. Schade[1)2)] to estimate the effective breadth based on the strength. (the equation of λ_n is given in Fig. 1 for each case.)

Substituting Eq. (12) in Eq. (11)

$$-\frac{d^2z}{dx^2}=\frac{1}{E}\sum_n\frac{M_n}{I_n}\sin\omega_n x \qquad \cdots(13)$$

$I_n=\dfrac{I_wA_w+2\lambda_nt(I_w+e^2A_w)}{2\lambda_nt+A_w}$: moment of inertia of the section, the flange breadth of which is $2\lambda_n$.

As both ends are simply supported,

$$z=\frac{1}{E}\sum_n\frac{M_n}{\omega_n^2I_n}\sin\omega_n x \qquad \cdots(14)$$

The deflection of the mid-point of the span δ is given as follows;

$$\left.\begin{array}{lll}\text{for concentrated load,} & \delta=\sum_n\frac{I}{EI_n}\times\frac{2PL^3}{n^4\pi^4} & [n=1,3,5\cdots]\\ \text{for uniform load,} & \delta=\sum_n\frac{(-1)^{(n-1)/2}}{EI_n}\times\frac{4pL^4}{n^5\pi^5} & [n=1,3,5\cdots]\end{array}\right\} \qquad \cdots(15)$$

While, the corresponding deflection of equivalent beam is

$$\left.\begin{array}{ll}\text{for concentrated load,} & \delta=\frac{PL^3}{48EI}\\ \text{for uniform load,} & \delta=\frac{5pL^4}{384EI}\end{array}\right\} \qquad \cdots(16)$$

Equating Eq. (15) and (16), the 'effective' moment inertia of the equivalent beam I is determined. And the effectiveness of the plate λ/b based on the rigidity is gained from the equation;

$$\frac{\lambda}{b}=\frac{I-I_w}{I_w+e^2A_w-I}\times\frac{A_w}{A} \qquad \cdots(17)$$
$$A=2bt$$

3. Numerical calculation

For the numerical calculation, we consider the cases, where the structures are shown in Fig. 1 and the load conditions are shown in Fig. 2. And when the web is flat bar, Eq. (17) is

$$\left.\begin{aligned}
&\text{for concentrated load,} \quad \frac{\lambda}{b}=\frac{\pi^4-96\sum\limits_n(\psi_n/n^4)}{384\sum\limits_n(\psi_n/n^4)-\pi^4}\times\frac{A_w}{A} \\
&\qquad\qquad [n=1,3,5\cdots] \\
&\text{for uniform load,} \quad \frac{\lambda}{b}=\frac{5\pi^5-1536\sum\limits_n(-1)^{(n-1)/2}(\psi_n/n^5)}{6144\sum\limits_n(-1)^{(n-1)/2}(\psi_n/n^5)-5\pi^5}\times\frac{A_w}{A} \\
&\qquad\qquad [n=1,3,5\cdots]
\end{aligned}\right\}\cdots(18)$$

where $\qquad \psi_n=\dfrac{(\lambda_n/b)(A/A_w)+1}{4(\lambda_n/b)(A/A_w)+1}$

This Eq. (18) contains A/A_w for a parameter, and we take 1, 5 and 10 for its values

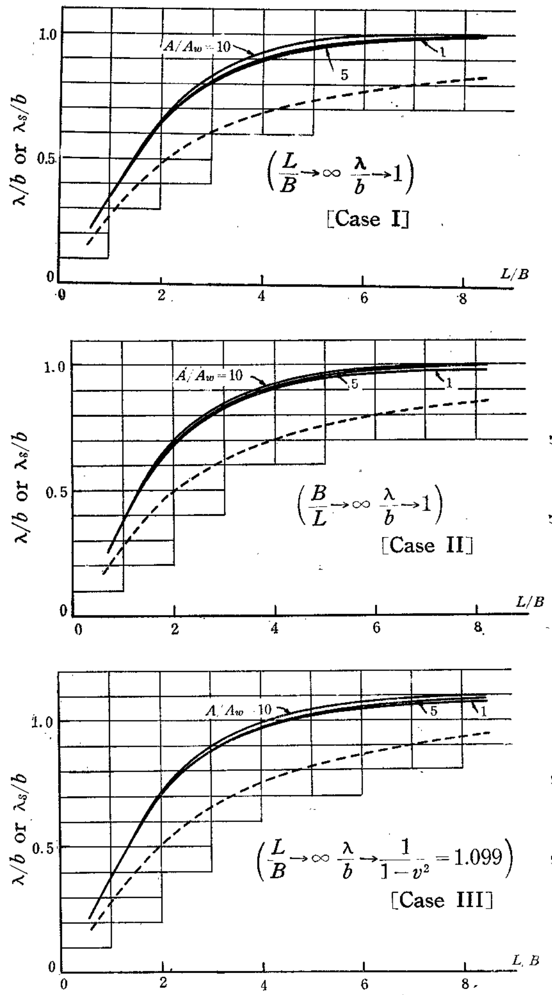


Fig. 3. λ/b for the concentrated load. (Broken line shows the value λ_s/b)

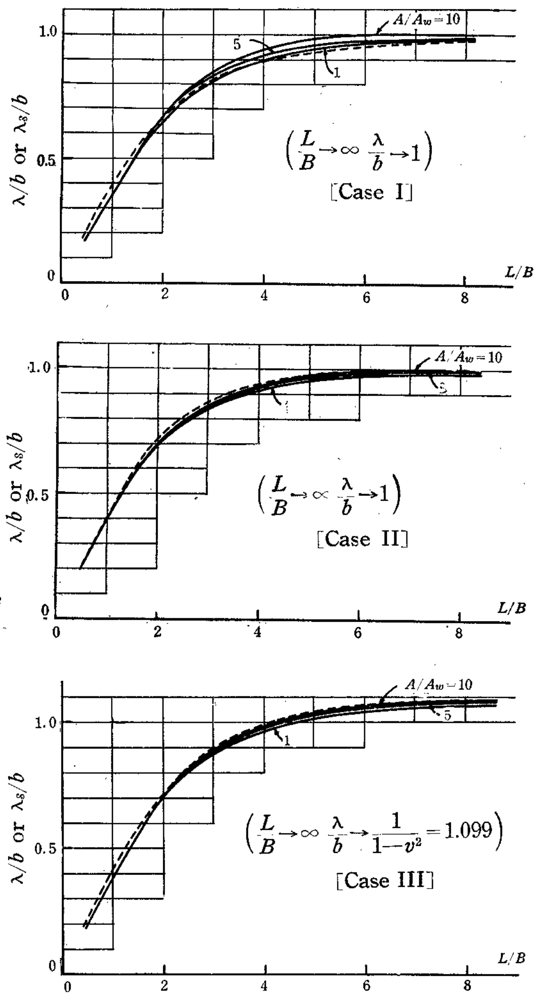


Fig. 4. λ/b for the uniform load. (Broken line shows the value λ_s/b)

here. As the convergency of the series in Eq. (18) is rapid, it is sufficient to take account of the first two terms, and the values of λ/b are plotted against L/B in Fig. 3 and Fig. 4.

From these, it is found that λ/b are affected little by the loading condition and A/A_w.

In this connection, we will compare λ/b with λ_s/b (effectiveness considering the strength) which was calculated by A. Schade. The values of λ_s/b are given for a parameter β, the relation of which to A/A_w is

$$\beta=\frac{1}{4}\times\frac{A_w}{A}\left[\frac{4(A/A_w)+1}{3(A/A_w)+1}\right] \qquad \cdots(19)$$

when the web is a flat bar. Schade's broken lines in Fig. 3 and Fig. 4 are curves of λ_s/b for $\beta=1/6$ ($A/A_w \doteqdot 2$).

These lines are affected by the loading conditions, contrary to λ/b. And it is found that λ/b is larger than λ_s/b for a concentrated load while λ/b agrees nearly with λ_s/b for a uniform load.

4. Conclusion

There are cases where it is appropriate to design the structures on the view point of the rigidity, but it seems that there is no simple method to calculate the deflection of the stiffened plate under bending loads. So, we define the effective breadth based on the rigidity and calculate it for some practical cases and show the results in Fig. 3 and Fig. 4. These results are applicable to calculate the maximum deflections of the structures by the elementary beam theory.

References

1) A. Schade, SNAME, vol. 59, (1951).
2) A. Schade, SNAME, vol. 61, (1953).