<ARTICLE>On Negative Consumption Tax

On Negative Consumption Tax*

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1. Introduction

To maintain social security system such as pension system [See for example Watanabe and Kambashi (2001) for analysis of the pension], the policy of raising the tax rate of consumption tax has been discussed. A purpose of this paper is, however, to examine negative consumption tax where the consumption tax rate is negative, instead of ordinal consumption tax where the consumption tax rate is positive. However, the negative consumption tax has not been examined except Watanabe (2000). In Watanabe (2000) the negative consumption tax has already been analyzed considering the tax evasion [See for example Allingham and Sandmo (1972), Peacock and Show (1982), Kreutzer and Lee (1986) and Watanabe (1986,1987,1988,2001) for the analysis of tax evasion]. From the analysis of Watanabe (2000) following main results have been derived. (I) A policy of setting the absolute value of negative consumption tax rate equal to the profit tax rate will eliminate tax evasion by means of understating proceeds. (II) Raising the absolute value of the negative consumption tax rate which is kept equal to the profit tax rate will increase the amount of employed labor and wage income. (III) Even if the consumption tax rate is negative, tax revenue can be positive so long as the tax on wage income is also considered.

However, in Watanabe (2000) only the tax evasion by means of the understating proceeds was examined. In this paper not only the tax evasion by means of proceeds understating but also the tax evasion by means of cost overstating will be considered in the generalized model. The policy of setting the absolute value of negative consumption tax rate equal to the profit tax rate can not eliminate the tax evasion by means of the overstating cost. Additional policy tool such as the penalty rate of tax evasion will be required in this

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generalized model to eliminate the tax evasion.

In the next section a generalized model will be presented. Effect of raising the penalty rate of tax evasion and that of raising the absolute value of the negative consumption tax rate on the amount of employed labor and on the expected tax revenue will be analyzed. And the policy mix of them will also be analyzed in section 3. In the last section concluding remarks will be given.

2. A Generalized Model

If both the cost overstatement and the proceeds understatement are not detected by tax authority, the profit π_1 is denoted by

$$\pi_1=(1-t_1)\frac{a-bx(m)}{1-t_1}x(m)-wm$$
$$-t_2\left\{(1-\varepsilon)\frac{a-bx(m)}{1-t_1}x(m)-(1+\delta)wm\right\}$$
$$+t_1(1-\varepsilon)\frac{a-bx(m)}{1-t_1}x(m), \tag{1}$$

where t_1 is absolute value of the negative consumption tax rate, t_2 is the profit tax rate, $p=a-bx(m)$, where p is the price level, x is output level which is an increasing function of employed labor m, ε is the rate of understatement with respect to the proceeds, δ is the rate of cost overstatement and w is wage rate respectively.

On the other hand, if only the proceeds understatement is detected, the profit π_2 is denoted by

$$\pi_2=\pi_1-Ft_2\varepsilon\frac{a-bx(m)}{1-t_1}x(m)+s_1t_1\varepsilon\frac{a-bx(m)}{1-t_1}x(m). \tag{2}$$

If only the cost overstatement is detected, the profit π_3 is denoted by

$$\pi_3=\pi_1-Ft_2\delta wm. \tag{3}$$

If both the cost overstatement and the proceeds understatement are detected by tax authority, the profit π_4 is denoted by

$$\pi_4 = \pi_2 - F t_2 \delta wm. \tag{4}$$

From (1), (2),(3) and (4) the expected profit $E\pi$ is denoted by

$$E\pi = \left\{1 - t_1 + (1-\varepsilon)(t_1 - t_2) - q(\varepsilon)\varepsilon(Ft_2 - s_1 t_1)\right\} \times \frac{a - bx(m)}{1-t_1} x(m) - \left\{1 - (1 + \delta - r(\delta)F\delta)t_2\right\} wm, \tag{5}$$

where $q(\varepsilon)$ is the probability of detection with respect to the understatement of proceeds and $r(\delta)$ is that of detection with respect to the cost overstatement.

Maximizing (5) with respect to m , ε and δ yields

$$\frac{\partial E\pi}{\partial m} = \left\{1 - t_1 + (1-\varepsilon)(t_1 - t_2) - \varepsilon^2 (Ft_2 - s_1 t_1)\right\} \times \frac{1}{1-t_1}(a\kappa - 2b\kappa^2 m) - \left\{1 - (1 + \delta - \delta^2 F)t_2\right\} w = 0, \tag{6}$$

where $q(\varepsilon)$, $r(\delta)$ and $x(m)$ are specified such that $q(\varepsilon) = \varepsilon$, $r(\delta) = \delta$ and $x(m) = km$ for simplification.

$$\frac{\partial E\pi}{\partial \varepsilon} = \left\{t_2 - t_1 - 2(Ft_2 - s_1 t_1)\varepsilon\right\} \frac{a - bx(m)}{1-t_1} x(m) = 0, \tag{7}$$

$$\frac{\partial E\pi}{\partial \delta} = (1 - 2\delta F)t_2 wm = 0. \tag{8}$$

Second order conditions are satisfied;

$$\frac{\partial^2 E\pi}{\partial m^2} = \frac{-2b\kappa^2}{1-t_1}\left\{1-t_1+(1-\varepsilon)(t_1-t_2)-\varepsilon^2(Ft_2-s_1t_1)\right\}$$
$$< 0, \qquad (9)$$

where $1-t_1+(1-\varepsilon)(t_1-t_2)-\varepsilon^2(Ft_2-s_1t_1)$

$$= 1-t_2+\frac{(t_2-t_1)^2}{4(Ft_2-s_1t_1)} > 0$$

since $\varepsilon = \dfrac{t_2-t_1}{2(Ft_2-s_1t_1)}$ from (7).

$$\begin{vmatrix} \dfrac{\partial^2 E\pi}{\partial m^2} & \dfrac{\partial^2 E\pi}{\partial m\,\partial\varepsilon} \\ \dfrac{\partial^2 E\pi}{\partial\varepsilon\,\partial m} & \dfrac{\partial^2 E\pi}{\partial\varepsilon^2} \end{vmatrix} > 0, \qquad (10)$$

where $\dfrac{\partial^2 E\pi}{\partial m^2} < 0$ from (9),

$$\frac{\partial^2 E\pi}{\partial\varepsilon^2} = -2(Ft_2-s_1t_1)\frac{a-bx(m)}{1-t_1}x(m) < 0,$$

and $$\frac{\partial^2 E\pi}{\partial\varepsilon\,\partial m} = \left\{t_2-t_1-2(Ft_2-s_1t_1)\varepsilon\right\}MR\frac{dx}{dm}$$
$$= 0$$

since $t_2-t_1-2(Ft_2-s_1t_1)\varepsilon = 0$ from (7).

$$\begin{vmatrix} \dfrac{\partial^2 E\pi}{\partial m^2} & \dfrac{\partial^2 E\pi}{\partial m \partial \varepsilon} & \dfrac{\partial^2 E\pi}{\partial m \partial \delta} \\ \dfrac{\partial^2 E\pi}{\partial \varepsilon \partial m} & \dfrac{\partial^2 E\pi}{\partial \varepsilon^2} & \dfrac{\partial^2 E\pi}{\partial \varepsilon \partial \delta} \\ \dfrac{\partial^2 E\pi}{\partial \delta \partial m} & \dfrac{\partial^2 E\pi}{\partial \delta \partial \varepsilon} & \dfrac{\partial^2 E\pi}{\partial \delta^2} \end{vmatrix} < 0,$$

where $\dfrac{\partial^2 E\pi}{\partial m^2} < 0, \qquad \dfrac{\partial^2 E\pi}{\partial \varepsilon^2} < 0,$

$$\frac{\partial^2 E\pi}{\partial \delta^2} = -2Ft_2 wm < 0, \qquad \text{and}$$

$$\frac{\partial^2 E\pi}{\partial m \partial \varepsilon} = \frac{\partial^2 E\pi}{\partial m \partial \delta} = \frac{\partial^2 E\pi}{\partial \varepsilon \partial \delta} = 0.$$

m^*, ε^* and δ^* which maximize the expected profit are derived from (6), (7) and (8) in the following ;

$$m^* = \frac{a}{2b\kappa} + \frac{(1-t_1)w\left\{(1+\frac{1}{4F})t_2 - 1\right\}}{2b\left\{1 - t_2 + \frac{(t_2-t_1)^2}{4(Ft_2 - s_1t_1)}\right\}\kappa^2}, \tag{12}$$

which is assumed to be positive,

$$\varepsilon^* = \frac{t_2 - t_1}{2(Ft_2 - s_1t_1)}, \tag{13}$$

where $t_2 \geqq t_1$ is assumed

and $\delta^* = \frac{1}{2F}$. (14)

From (13) $\varepsilon^* = 0$ if $t_1 = t_2$.
Then m* reduces to

$$m^* = \frac{1}{2b\kappa^2}\left[a\kappa + \left\{ \left(1+\frac{1}{4F}\right)t - 1 \right\} w \right]. \tag{15}$$

Even if ,however, $t_1 = t_2$, the inequity due to the tax evasion by means of the overstating cost cannot be eliminated, though the inequity due to the tax evasion by means of understating proceeds can be eliminated.

3. Policy Mix of Negative Consumption Tax Rate and Penalty Rate

In order to reduce the inequity due to the tax evasion the penalty rate F must be raised from (14) , however, as shown in the following raising the penalty rate will decrease the amount of employed labor.

Differentiating (15) with respect to F yields

$$\frac{\partial m^*}{\partial F} = \frac{1}{2b\kappa^2} w \frac{t}{4} (-1) F^{-2} < 0. \tag{16}$$

Differentiating (15) with respect to t yields

$$\frac{\partial m^*}{\partial t} = \frac{1}{2b\kappa^2} w \left(1+\frac{1}{4F}\right) > 0. \tag{17}$$

Therefore raising the absolute value of the negative consumption tax rate will increase the amount of the employed labor.

Totally differentiating m* yields

$$dm^* = \frac{w}{8b\kappa^2 F}\left\{-\frac{t}{F}dF + (1+4F)dt\right\}.$$

Hence, following relation can be derived;

$$dm^* \gtreqless 0, \quad \text{according to} \quad \frac{F}{t}\frac{dt}{dF} \gtreqless \frac{1}{1+4F}. \tag{19}$$

On the other hand, expected value of tax revenue E.T.R. is denoted by

$$\begin{aligned} E.T.R. = & \left\{(1-\varepsilon)\frac{a-bx(m)}{1-t_1}x(m) - (1+\delta)wm\right\}t_2 \\ & -(1-\varepsilon)\frac{a-bx(m)}{1-t_1}x(m)\,t_1 \\ & + q(\varepsilon)\left\{\varepsilon\frac{a-bx(m)}{1-t_1}x(m)\,Ft_2 - \varepsilon\frac{a-bx(m)}{1-t_1}x(m)\,s_1t_1\right\} \\ & + r(\delta)\,\delta\, wmFt_2 + t_w\, wm, \end{aligned} \tag{20}$$

where t_w is the rate of tax on the wage income.

When $t_1 = t_2 = t$, $\varepsilon^* = 0$ is attained and using $\delta^* = \frac{1}{2F}$, E.T.R. is reduced to

$$E.T.R. = wm\left\{t_w - \left(1+\frac{1}{4F}\right)t\right\}. \tag{21}$$

Hence following relationship can be obtained;

$$E.T.R. > 0,$$

$$\text{so long as} \quad \frac{t_w}{t} > 1 + \frac{1}{4F}. \tag{22}$$

Therefore the expected value of the tax revenue can be positive even if the tax rate of consumption tax is negative so long as the condition in (22) is satisfied.

Differentiating (21) with respect to F yields

$$\frac{\partial E.T.R}{\partial F} = \frac{1}{4} \mathrm{wmt}F^{-2} > 0. \tag{23}$$

Hence, raising the penalty rate will increase the expected value of the tax revenue.

On the other hand, differentiating (21) with respect to t yields

$$\frac{\partial E.T.R}{\partial t} = -\mathrm{wm}\left(1+\frac{1}{4F}\right) < 0. \tag{24}$$

Therefore raising the tax rate will decrease the expected value of the tax revenue.

Totally differentiating E.T.R. yields

$$\mathrm{d\,E.T.R.} = \frac{(1+4F)\ \mathrm{wmt}}{4F^2}\left(\frac{1}{1+4F} - \frac{F}{\mathrm{t}}\frac{\mathrm{dt}}{\mathrm{d}F}\right)\mathrm{d}F \tag{25}$$

Hence the following relationship can be derived;

$$\mathrm{d\,E.T.R.} \gtreqless 0$$

according to $$\frac{1}{1+4F} \gtreqless \frac{F}{\mathrm{t}}\frac{\mathrm{dt}}{\mathrm{d}F} \tag{26}$$

3. Concluding Remarks

From the analysis of this paper following main results have been derived.

(I) A policy which keeps the absolute value of negative consumption tax rate equal to the profit tax rate can eliminate the inequity due to tax evasion by means of proceeds understatement but the inequity due to tax evasion by means of cost overstatement can not be eliminated.

(II)In order to reduce the inequity due to tax evasion by means of cost overstatement additional policy will be required with respect to the penalty rate of tax evasion. Raising the penalty rate will reduce the inequity due to tax evasion by means of cost overstatement.

(III) However, raising the penalty rate will decrease the amount of employed labor.

(IV) On the other hand, raising the absolute value of negative consumption tax rate which is kept equal to the profit tax rate will increase the amount of employed labor.

(V) The expected value of the tax revenue can be positive even if the tax rate of consumption tax is negative so long as the condition in (22) is satisfied. The higher the penalty rate or the higher the tax rate on wage in come, the higher the probability that the condition in (22) is satisfied.

(VI) Raising the penalty rate of the tax evasion will increase the expected value of the tax revenue, though raising the absolute value of negative consumption tax rate will decrease the expected value of the tax revenue.

(VII) If the policy mix is implemented such that the absolute value of negative consumption tax rate is raised with the increase in the penalty rate, the expected value of the tax revenue will be increased with reducing cost overstatement rate though the amount of employed labor will be decreased if the elasticity of the absolute value of negative consumption tax rate with respect to the penalty rate is lower than $\frac{1}{1+4F}$.

(VIII)On the other hand, if the elasticity of the absolute value of negative consumption tax rate with respect to the penalty rate is equal to $\frac{1}{1+4F}$, both the expected value of the tax revenue and the amount of employed labor will not be affected by the policy mix, even if the inequity due to tax evasion by means of cost overstatement can be reduced by the policy mix. If the elasticity of the absolute value of negative consumption tax rate with respect to the penalty rate is higher than $\frac{1}{1+4F}$, the expected value of the tax revenue will be decreased though the amount of employed labor will be increased by the policy mix.

In Watanabe (2000) not only the relationship between the negative consumption tax and the tax evasion by means understating proceeds but also the relationship between the tax based on total wage payment and tax evasion was analyzed. From the analysis of latter relationship following result was derived. Under the tax policy which keeps the profit tax rate equal to the tax rate with respect to total wage payment, neither the understatement for evading tax based on total wage payment nor overstatement of total wage payment for evading profit tax will be made.

If the tax based on total wage payment is also taken into consideration in the model analyzed in this paper, the expected profit (5) will be rewritten as

$$E\pi = \left\{1 - t_1 + (1-\varepsilon)(t_1 - t_2) + (s_1 t_1 - Ft_2)\varepsilon^2\right\} \frac{a - bx(m)}{1 - t_1} x(m)$$

$$- \left\{1 + (1+\delta)(t_3 - t_2) + (Ft_2 - s_3 t_3)\delta^2\right\} wm,$$

where t_3 is the tax rate of total wage payment and s_3 is the rate of tax refund as to the profit tax when real wage payment is detected to be larger than the reported one.

Maximizing the above $E\pi$ with respect to m, ε and δ will yield the result that both ε^* and δ^* will become zero under the policy such that $t_1 = t_2 = t_3$, since

$$\varepsilon^* = \frac{t_2 - t_1}{2\,(Ft_2 - s_1 t_1)}$$

and

$$\delta^* = \frac{t_2 - t_3}{2\,(Ft_2 - s_1 t_1)}$$

can be derived straightforwardly as part of the conditions which maximize the above generalized expected profit.

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