On the Curved Weir : On the shape of weit which is formed from two curves and whose discharge is a linear function of its head

On the Curved Weir

(On the shape of weir which is formed from two curves and whose discharge is a linear function of its head)

Minoru SHIBAHAYASHI* and Eitaro SUGINO*

(Received December 15, 1955)

When the shape of weir for measurement of discharge and its head are given, the discharge can be calculated readily. On the other hand, when discharge is given as an equation of head in particular form, it is generally difficult to give the shape of weir which fills this condition. However, if the shape of a weir, whose discharge is given as a linear function of its head, is presented, it may be not only convenient for the calculation, but very suitable for automatic control of discharge or for self-recording of measured discharge.

In this paper the shape of weir, which can be applied for such purposes, is presented by the combination of two curves. These curves are

$$x_1 = A y_1^{\frac{2m-3}{2}},$$

$$x_2 = A \sum_{n=2}^{m} \left[\frac{1\cdot3\cdot5\cdots(2m-3)}{1\cdot3\cdot5\cdots(2n-3)} \cdot \frac{a^{m-n}}{2^{m-n}(m-n)!} \cdot y_2^{\frac{2n-3}{2}} \right]$$

and the breadth of weir is $x_1 - x_2$. Thus, the discharge results a linear function of head.

$$Q = \pi A \sqrt{2g}\, \frac{1\cdot3\cdot5\cdots(2m-3)}{2^m (m-1)!} \cdot a^{m-1} \left(H - \frac{m-1}{m} a \right).$$

1. Introduction

The shape of weir, whose discharge is proportional to head, can be obtained by means of a simple curve. If the height from crest is y and the breadth of weir x, the cischarge must be presented in the form

$$Q = \int_0^H \sqrt{2g(H-y)}\, x\,dy = KH, \tag{1}$$

where H is head. Differentiate the both sides by H, then

$$\int_0^H \frac{x}{\sqrt{H-y}}\, dy = \frac{2K}{\sqrt{2g}}.$$

This is an integral equation of Abel type, when x is a function of y. Soluting this equation,

$$x = \frac{2K}{\pi\sqrt{2g}} \cdot \frac{1}{\sqrt{y}}. \tag{2}$$

Since $y \to 0$ results in $x \to \infty$ in the function (2), it is impossible to shape the weir with a finite breadth (Fig. 1). Namely, such weir as shaped with a simple curve, is not useful.

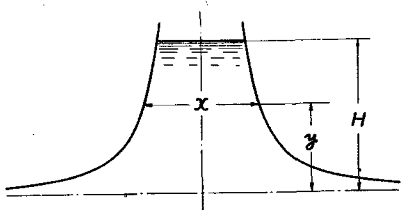

Fig. 1

* Department of Mechanical Engineering, Junior College of Engineering.

Consequently, the shape of weir which is formed from two curves and whose discharge is a linear function of its head must be considered as the possible form. For instance, Muto[1] had studied the weir whose lower part forms triangle and trapezoid, and Sugihara[2] had studied the one whose lower part is parabolic.

In this paper a shape of weir was studied, whose theoretical discharge is a linear function of its head by putting the weir plate, whose shape is $x_2=F(y_2)$, into the notch, $x_1=f(y_1)$.

2. Calculation

In the Fig. 2, (I) indicates the notch whose breadth is x_1 and whose height from crest is y_1, (II) indicates the weir plate whose breadth is x_2 and whose height from lower end is y_2. The discharge Q_1 from the notch (I) is obstructed partialy by the weir plate (II).

Considering x_1 and x_2 respectively as the functions of y_1 and y_2,

$$x_1=f(y_1)\,, \tag{3}$$

$$x_2=F(y_2)\,. \tag{4}$$

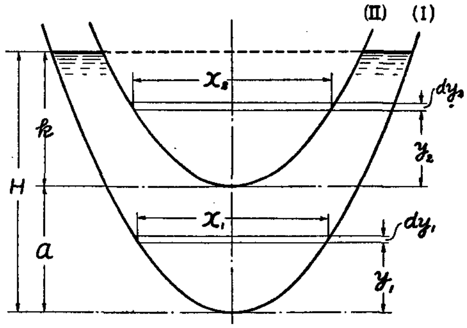

Fig. 2

Discharge Q_1 from the notch (I) only is

$$Q_1=\sqrt{2g}\int_0^H (H-y_1)^{\frac{1}{2}}x_1\,dy_1\,, \tag{5}$$

where H is a head on crest. Discharge Q_2 obstructed by the weir plate (II) is

$$Q_2=\sqrt{2g}\int_0^k (k-y_2)^{\frac{1}{2}}x_2\,dy_2\,, \tag{6}$$

where k is a head on the lower end of the plate (II). Therefore, discharge Q, which flows out between notch (I) and weir plate (II), is

$$Q=Q_1-Q_2\,. \tag{7}$$

Q must be a linear function of H, so

$$Q=MH+N=M(k+a)+N\,, \tag{8}$$

where a is a height of the lower end of weir plate (II) from the crest of notch (I), M and N are constants.

We must determine the forms of x_1 and x_2, so that the two formulas (7) and (8) may be satisfied, but it is difficult to determine simultaneously x_1 and x_2. Thus, at first x_1 must be determined suitably. We gave the shape of notch (I) as follows:

$$x_1=f(y_1)=Ay_1^{\frac{2m-3}{2}}\,, \tag{9}$$

where A is constant which concerns the size of notch, m is a positive integer except 1. Substituting this x_1 into (5), we have

$$Q_1=\pi A\sqrt{2g}\frac{1\cdot3\cdot5\cdots(2m-3)}{2^m m!}\cdot H^m. \tag{10}$$

Namely, when the notch given by the function (9) is used, discharge is expressed with an exponential function of head. If $m=1$ is substituted into the function (9) and (10), the previous expressions (1) and (2) are obtained and Q becomes proportional to H. However, as already described, since the breadth of notch becomes infinitive, we now except $m=1$.

Setting

$$K=\pi A\sqrt{2g}\frac{1\cdot3\cdot5\cdots(2m-3)}{2^m m!}$$

and since $H=k+a$, we get from (10)

$$Q_1=K(k+a)^m. \tag{11}$$

From (4), (6) and (7)

$$\sqrt{2g}\int_0^k (k-y_2)^{\frac{1}{2}}F(y_2)dy_2=Q_1-Q.$$

Substituting (8) and (11) into this,

$$\sqrt{2g}\int_0^k (k-y_2)^{\frac{1}{2}}F(y_2)dy_2=K(k+a)^m-M(k+a)-N. \tag{12}$$

This integral equation can be solved by Abel's method,

$$F(y_2)=A\sum_{n=2}^{m}\left[\frac{1\cdot3\cdot5\cdots(2m-3)}{1\cdot3\cdot5\cdots(2n-3)}\cdot\frac{a^{m-n}}{2^{m-n}(m-n)!}\cdot y_2^{\frac{2n-3}{2}}\right]+\frac{2}{\pi\sqrt{2gy_2}}\cdot(mKa^{m-1}-M). \tag{13}$$

This result gives the shape of weir plate (**II**). In order to simplify the calculation, we moreover set the following formula into the function (13).

$$M=mKa^{m-1}. \tag{14}$$

Thus, the second term of the right side of the function (13) disappears.

$$x_2=A\sum_{n=2}^{m}\left[\frac{1\cdot3\cdot5\cdots(2m-3)}{1\cdot3\cdot5\cdots(2n-3)}\cdot\frac{a^{m-n}}{2^{m-n}(m-n)!}\cdot y_2^{\frac{2n-3}{2}}\right]. \tag{15}$$

When the weir plate (**II**), whose shape is determined with (15), is used, Q_2 is obtained by (6).

$$Q_2=\pi A\sqrt{2g}\sum_{n=2}^{m}\left[\frac{1\cdot3\cdot5\cdots(2m-3)}{2^m m!}\cdot a^{m-n}\cdot k^n\right]. \tag{16}$$

By the two substitutions, $k=H-a$ into (16) and the two formulas (10) and (16) into (7), discharge Q, which flows out between the notch (**I**) given by the function (9) and the weir plate (**II**) by the function (15), is obtained.

$$Q=\pi A\sqrt{2g}\frac{1\cdot3\cdot5\cdots(2m-3)}{2^m(m-1)!}\cdot a^{m-1}\left(H-\frac{m-1}{m}a\right). \tag{17}$$

Thus, discharge Q is given as a linear function of its head H.

However, since the setting of the weir plate (**II**) is actually inconvenient, $z=x_1-x_2$

may be selected as shown in Fig. 3 for the breadth of upper part, where the weir plate must be set. Thus, we ought to find the shape of weir from the two functions (9) and (15), and calculate the discharge by (17). In this case the function (17) can be applied only for $H>a$. Therefore, when this weir is used for the automatic control of discharge or for the self-recording of measured discharge, the head must be kept in range $H>a$. However, in order to enlarge the applied range of the function (17), we ought to select small value of a. When $H<a$, discharge Q can be calculated only by the formula (10). Consequently, if this weir is used merely for measurement of discharge, we can use it in any range of H.

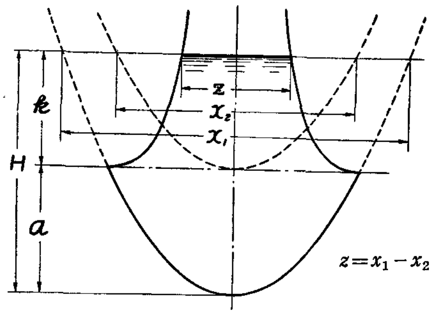

Fig. 3

As shown in the two functions (9) and (15), large value of A results in wide breadth of the weir and the discharge increases in proportion to the value of A. Therefore, the value of A must be determined pertinently, according to discharge in channel.

3. Examples

(1) $m=2$

$$x_1=Ay_1^{\frac{1}{2}}, \qquad x_2=Ay_2^{\frac{1}{2}},$$

$$Q=\frac{1}{4}\pi A\sqrt{2g}\,a\left(H-\frac{1}{2}a\right).$$

(2) $m=3$

$$x_1=Ay_1^{\frac{3}{2}}, \qquad x_2=A\left(\frac{3}{2}ay_2^{\frac{1}{2}}+y_2^{\frac{3}{2}}\right),$$

$$Q=\frac{3}{16}\pi A\sqrt{2g}\,a^2\left(H-\frac{2}{3}a\right).$$

(3) $m=4$

$$x_1=Ay_1^{\frac{5}{2}}, \qquad x_2=A\left(\frac{15}{8}a^2y_2^{\frac{1}{2}}+\frac{5}{2}ay_2^{\frac{3}{2}}+y_2^{\frac{5}{2}}\right),$$

$$Q=\frac{5}{32}\pi A\sqrt{2g}\,a^3\left(H-\frac{3}{4}a\right).$$

These results are shown in Fig. 4, where the maximum breadth and a are constant. The breadth of weir decreases and the discharge in the same head becomes also smaller for large value of m. In other words, when the discharge is small, the shape of weir with large m is effective. The chain line in this figure indicates the shape with triangular lower part[1] for comparison. Sugihara[2] gave the one with parabolic lower part, but this result can be induced from $m=2$ in our calculation.

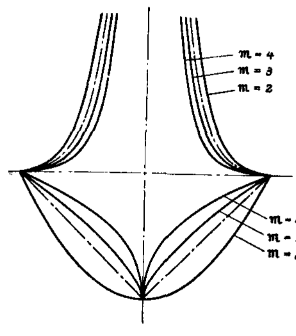

Fig. 4

4. Coefficient of Discharge

The discharge Q treated in the calculation above is the theoretical one. We may readily consider that the actual discharge Q_a for such weir is smaller than Q, causing by the contraction of flow and so on. Here consider what influence of head H exists on coefficient of discharge Q_a/Q.

(1) When the coefficient of discharge is constant

According to the result of experiment by Muto[1], which concerned with several kinds of weir made from the two curves, the coefficient of discharge is almost constant except very small value of H. Therefore, changing the form (8) by using the coefficient of discharge C_d,

$$Q_a = C_d(MH+N) = M'H+N'. \tag{18}$$

This form is always a linear function of the head and there is no objection in actual case.

(2) When the coefficient of discharge is variable

Generally it is a matter of course that C_d varies a little with the variation of H. This variance differs according to the size and shape of weir, but it is common that C_d is large in very small value of H, that C_d decreases in conformity with increasing of H and that decreasing rate of C_d is small for large H. The study by Sugihara[2], who did the same kind of experiment as Muto did, presents the following relation between H and C_d.

$$C_d = \lambda + \mu/H, \tag{19}$$

where λ and μ are constant values given by the weir. Substituting this C_d into the formula (8),

$$Q_a = M\lambda H + (M\mu + N\lambda) + N\mu/H.$$

If we select small value of a, as mentioned above, N becomes smaller as compared with M. From the results of experiment, the value of μ is smaller as compared with the value of λ. Consequently, the final term of the right side of the upper formula can be neglected. Setting $M\lambda = M''$, $M\mu + N\lambda = N''$, so

$$Q_a = M''H + N''. \tag{20}$$

Namely, Q_a is practically a linear function of H except when H is very small.

5. Conclusions

(1) Theoretical discharge Q is given as a linear function of H in the range $H > a$, as shown in the expression (17), when the breadth of weir is determined by means of the two functions (9) and (15).

(2) In order to enlarge the range for application of the formula (17), it is effective to make the value of a smaller in the function (15).

(3) When coefficient of discharge C_d can be expressed as the form (19), actual discharge Q_a also can be found as a linear function of H, as shown in the form (20).

References

1) F. Muto, Jour. Japan Soc. Mech. Engr., **39**, 228, p. 205 (1936).
2) T. Sugihara, Jour. Japan Soc. Mech. Engr., **39**, 234, p. 568 (1936).