Optimization of Soil Moisture Environment by Applying Simulation on Sub-irrigation Method (I) : Reappearance Trial in the Distribution of Soil Moisture Tension

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Optimization of Soil Moisture Environment by Applying Simulation on Sub-irrigation Method (I)
—Reappearance Trial in the Distribution of Soil Moisture Tension—

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Abstract

This study was carried out to clarify the reappearance method which shows the distribution of soil moisture in the sub-irrigation method by using negative pressure difference. On the other hand, there are many methods to reappear the distributions of soil moisture tension, but in this study, the conformal mapping method was applied to reappear them at the steady state. Consequently, it is found that the distributions of soil moisture tension could be reappeared to the different cases in the sub-irrigation method by using negative pressure difference. Also it is found that the different depth and different outer radius of porous pipe buried in the sub-irrigation method largely influence the distributions of soil moisture tension.

Introduction

Water which is necessary to grow the crops must be fully supplied by the sub-irrigation method by using negative pressure and the distribution of soil moisture tension in the root zone must be maintained at the optimum condition in accordance with the rate of evapotranspiration. On the other hand, if fixed negative pressure would be applied to the porous pipe, water could be automaticlly supplied into the soil in accordance with the rate of evapotranspiration. This phenomena are considered that soil moisture tension depends on the rate of evapotranspiration, and also that of water supply depends on the condition of soil moisture around the porous pipe buried. Therefore, it is a important theme to know the distributions of soil moisture tension. The prediction or the reappearance, however, on the distribution of soil moisture was not almost conducted to the sub-irrigation method by using the negative pressure difference util now. Then, this study was carried out to clarify the extent of influence factors by applying the reapperance ways which show the distribution of soil moisture tention.

I. Procedure and Method

(1) Fundamental development of conformal mapping

Consider the movement of soil water under the steady state and two dimensional flow, it can be represented by two dimensional coordinates wich are delt with Gaussian plane. Now, applying Gaussian plane z which x axis shows real number and axis imaginary number, the given point (x, y) on its plane can be written as follows.

$$z=x+iy \qquad (1)$$

where $(x+iy)$ shows the function of complex variables. Moreover, considering the different plane w wich is consisted of axis ϕ and axis ψ, the given point (ϕ, ψ) on its plane can be written as follows.

$$w=\phi+i\psi \tag{2}$$

where (ϕ+iψ) shows the function of complex variables. Then, considering the function w=f(z), the following relationships can be obtained.

$$\phi=\phi(x, y) \tag{3}$$

$$\psi=\psi(x, y) \tag{4}$$

Therefore, the given point (x, y)on plane z can be corresponded with any point (ϕ, ψ) on plane w. That is, the former function can be converted to the latter function.

Now, the given point (x, y) on plane z can be reflected to the point P′(ϕ, ψ) on plane w by applying $w=f(z)=f(x+iy)$, and then, differentiate the function w with x or y, its result can be written as of follows.

$$\frac{\partial w}{\partial x}=\frac{d\phi}{dx}+i\frac{\partial\psi}{\partial x}=\frac{dw}{dz}\ \frac{\partial z}{\partial x}=f'\ (z)\times 1 \tag{5}$$

or

$$\frac{\partial w}{\partial y}=\frac{\partial\phi}{\partial y}+i\frac{\partial\psi}{\partial y}=\frac{dw}{dz}\ \frac{\partial z}{\partial y}=f'\ (z)\times i \tag{6}$$

Therefore, the function f′ (z) can be rewritten as follows.

$$(\frac{\partial\phi}{\partial y}+i\frac{\partial\psi}{\partial y})\frac{1}{i}=\frac{\partial\psi}{\partial y}-i\frac{\partial\phi}{\partial y} \tag{7}$$

The part of real number and that of imaginary number in eq. (5) equal to the part of each number in eq. (7) respectively, so their relationship can be written as follows.

$$\frac{\partial\phi}{\partial x}=\frac{\partial\psi}{\partial y} \text{ and } \frac{\partial\phi}{\partial y}=\frac{\partial\psi}{\partial x} \tag{8}$$

Each differential equation is called "Cauchy-Rieman differential equation." So differentiate eq. (8) each other, eq. (8) can be written as follows.

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \text{ and } \frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0 \tag{9}$$

Each equation shows the Laplace equation, so each part of real numder and imaginary number can be statisfy the Laplace equation. On the other hand, the following equation is obtained from eq. (8)

$$(\frac{\partial\phi}{\partial x})(\frac{\partial\psi}{\partial x})+(\frac{\partial\phi}{\partial y})(\frac{\partial\psi}{\partial x})=0 \tag{10}$$

Eq. (10) means the orthogonal condition of curves ϕ and curves ψ.

Now, considering curves ϕ as the group of isopotential, curves ψ represent the group of flow lines. So, considering the function w(=ln z) as a simple example pf conformal mapping as follows.

$$w=\phi+i\psi \tag{11}$$

$$z=x+iy \tag{12}$$

Therefore, eq. (11) and eq. (12) can be converted as follows.

$$\phi+i\psi=\ln(x+iy) \tag{13}$$

$$e^{\phi+i\psi}\psi=x+iy \tag{14}$$

Then, applying the Euler's equation to eq. (13) and eq. (14), the following equation is obtained.

$$e^{\phi} \cdot e^{i\psi} = e^{\phi}(\cos\psi + i\ \sin\psi) = x + iy \tag{15}$$

On the other hand, each part of real number and imaginary number in the left side equal to that in the right side respectively, the following relationships can be obtained.

$$e^{\phi}\ \cos\psi = x \text{ and } e^{\phi} i\ \sin\psi = y \tag{16}$$

$$e^{2\phi} = x^2 + y^2 \text{ and } \tan\psi = \frac{y}{x} \tag{17}$$

Consequently, ϕ and ψ can be represented as follows.

$$\phi = \frac{1}{2}\ \{\ln(x^2 + y^2)\} \text{ and } \psi = \tan^{-1}(\frac{y}{x}) \tag{18}$$

(2) Constitution of model and analysis

Components in model are consisted of water supply, evaporation, transpiration, percolation, and others. Water supply in the model means source and other factors mean sink. So applying the image method by inverse by using their factors, the model which is consisted of their factoes is shown in Fig. 1.

Where the point $P_1(-a, 0)$ in Fig. 1 shows the source place with the intensity $m_1(Q/2\pi$, Q: water supply, π: circular constant) of water supply, the point $P_2(a, 0)$ the sink place with the intensity m_2 of evaporation, the point $P_3(21-a, 0)$ the sink place with the intensity m_3 of percolation, and the point $P_4(-\alpha, -\beta)$ and $P_5(-\alpha, \beta)$ the sink place with the intensity m_4 of transpiration respectively. Therefore, the function w can be represented as follows.

$$w_1 = f_1(z) = -m_1 \ln(z+a) \tag{19}$$
$$w_2 = f_2(z) = m_2 \ln(z-a) \tag{20}$$
$$w_3 = f_3(z) = m_3 \ln(z+21-a) \tag{21}$$
$$w_4 = f_4(z) = m_4 \ln(z+\alpha-\beta)(z+\alpha+\beta)\} \tag{22}$$

These functions are the linear conformal mapping, so their equations can be put upon another. It's result can be expressed follows.

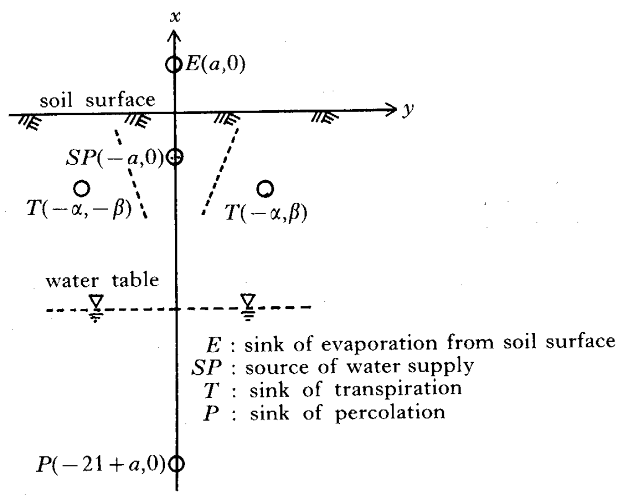

Fig. 1 A model diagram of source and sinks

$$w = -m_1 \ln(z+a) + m_2 \ln(z-a) + m_3 \ln(z+21-a) + m_4 \ln \{(z+\alpha-i\beta)(z+\alpha+i\beta)\} \tag{23}$$

Consequently, the function ϕ (x, y) of velocity potential and the function ψ(x, y) of flow lines can be estimated as follows.

$$\phi(x, y) = (-m_1/2) ln \{(x+a)^2+y^2\} + (m_2/2) \ln \{(x-a)^2+y^2\} + (m_3/2) \ln \{(x+21-a)^2+y^2\} + (m_4/2) \ln [\{(x+\alpha)^2+(y-\beta)^2\} \{(x+\alpha)^2+(y+\beta)^2\}] + \phi_0 \tag{24}$$

$$\psi(x, y) = -m_1 \tan^{-1}(\frac{y}{x+a}) + m_2 \tan^{-1}(\frac{y}{x-a}) + m_3 \tan^{-1}(\frac{y}{x+21-a}) + m_4 \{\tan^{-1}(\frac{y-\beta}{x+a}) + \tan^{-1}(\frac{y+\beta}{x+a})\} + \psi_0 \tag{25}$$

Where ϕ_0 and ψ_0 are constant. On the other hand, setting total potential H in soil, gravity potential x, and moisture potential h, the following equations can be obtained.

$$H = h - x \tag{26}$$

$$\phi(x, y) = -K_s \cdot H \tag{27}$$

Where K_s is the permeabillity of soil. Therefore, ϕ_0 in eq. (24) can be estimated by applying the boundary condition ($\phi(-l, 0) = K_s(-l)$) as follows.

$$\phi_0 = m_1 \ln(-l+a)^2 - m_2 \ln(-l+a)^2 - m_3 \ln(l-a)^2 - m_4 \ln(-l+\alpha)^2 + \beta^2\} - K_s \cdot l \tag{28}$$

ψ_0 in eq.(25) can be estimated by applying the flow between the point $(-a, 0)$ of water supply and the original point (0, 0) as follows.

$$\psi_0 = 0 \tag{29}$$

Consequently, soil moisture tension h(x, y) in the given place (x, y) can be estimated by applying eq.(26) and eq.(27) as follows.

$$h(x, y) = - \{\phi(x, y)/K_s - x\} \tag{30}$$

And also, the permeability of soil can be estimated by applying the fixed perssure H_p and the outer radius r of porous pipe as follows.

$$K_s(-H_p - a) = \phi(-a, r) \tag{31}$$

(3) Decision of parameters in model and calculation conditions

Various parameters in sumalation model are shown in Table 1.

Were Q in Table 1 is the amount of water, E evaporation, T transpiration, amd ET evapotranspiration. The relationships of each parameters are written as follows.

Table 1. Values of different calculation conditions.

name of condition	value of calcuration condition		
amount of water (ml/s/cm)	0.0003	0.0005	0.0007
negative pressure setted (cm)	10	20	30
depth of porous pipe buried (cm)	5	10	15
outer radius of porous pipe (cm)	2.5	5.0	7.5
ratio of evapotranspiration to water supply	0.9		
ratio of transpiration to evapotranspiration	0.7		
level of water table (cm)	500		
point of transpiration sink $(-\alpha, \pm\beta)$	$(-5, \pm 30)$		

$$m_1=Q/2\pi,\ m_2=(ET-T)/2\pi,\ m_3=(Q-ET)/2\pi,\ m_4=T/4\pi \tag{32}$$
$$m_1=m_2+m_3+2m_4 \tag{33}$$

And also K_p is the coefficient of permeability with porous pipe, and it's valus is obtained from the actual measurement as follows.

$$K_p=7\times10^{-5}\ cm/s \tag{34}$$

On the hand, total parameters in this model are amounted 9 kinds. That is, their parameters are divided to the amount of water supply, given negative pressure in porous pipe, the buried depth of porous pipe, the inner or outer radius of porous pipe, the ratio of water supply and evapotranspiration, ground water level, the souce place of water supply, and the sink place of evapotranspiration. Each value decided is shown in Table 2. But the sink place (a, 0) of evaporation can be decided from the principle of the image by inversion by applying the source place (−a, 0) of water supply.

II. Distributions of Soil Moisture Tension Influenced by Different Conditions

(1) Intensity of water supply

The intensities of water supply were selected three kinds, that is, Q=0.0259, 0.0432, and 0.060 (l/d/cm). The results of the distributions obtained are shown in Fig. 2.

Judging from the results in Fig. 2, each distribution of soil moisture tension is found not to be almost different under the various intensities of water supply because they are analyzed at the steady state. Therefore, it is found that the different intensity of water

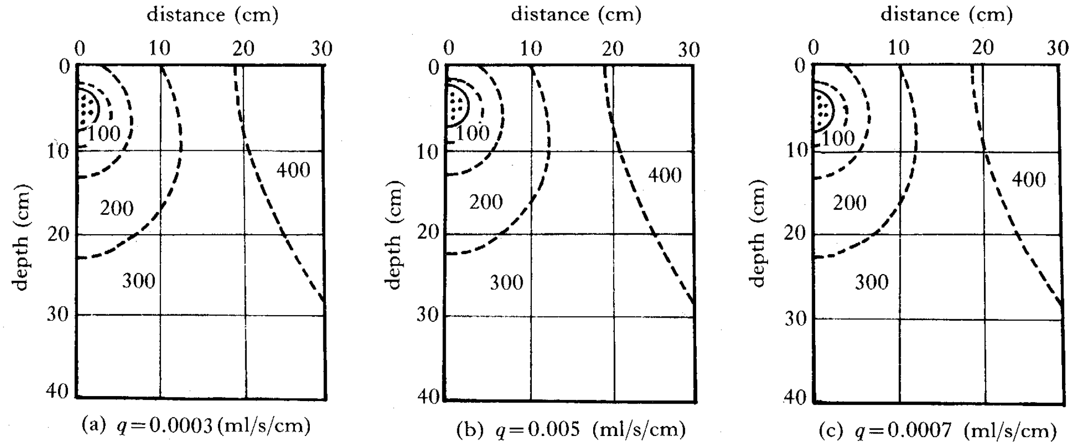

Fig. 2 Distributions of soil moisture tension under different water supply. (unit: cm)

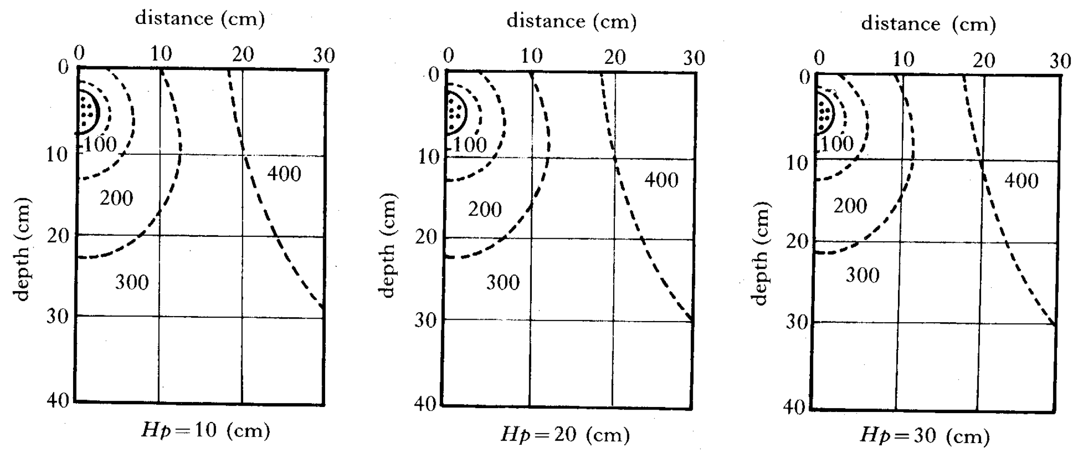

Fig. 3 Distributions of soil moisture tension under different negative pressure (unit: cm)

supply does not influence the distribution of soil moisture tension.

(2) Negative pressure given in porous pipe buried

The extents of negative pressure(H_p) given in porous pipe buried were seleced three kinds, that is, $H_p = 10$, 20, and 30 (cmH_2O). The results of the distributions obtained are shown in Fig. 3.

Judging from the results shown in Fig. 3, each distribution of soil moisture tension is considered not to be different under the various negative pressure. That is, the isopotential line of 300 (cmH_2O) under negative presure of 10 (cmH_2O) given shows a tendency to spread downward more than that under other negative presure, and also the isopotential line of 400 (cmH_2O) under negative presure of 10 (cmH_2O) given shows a tendency to spread upward more than that under other negative presure, However, the difference of negative presure given in porous pipe does not so much influence the distributions of soil moisture tension.

(3) Depth of porous pipe buried

The depth of porous pipe buried was measured from the soil surface to the centre of porous pipe. It's depth d was celected three kinds, that is, d=5, 10, and 15 cm. The results of the distributions are shown Fig. 4.

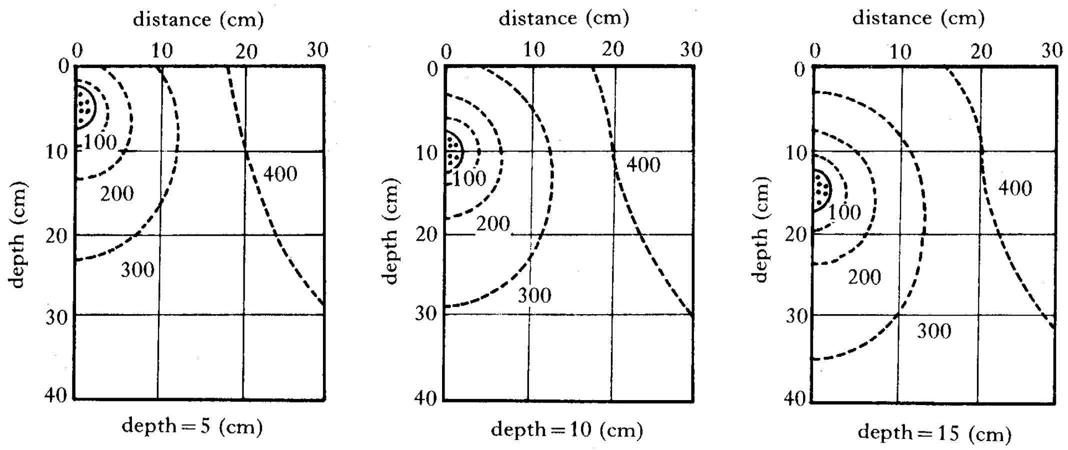

Fig. 4 Distributions of soil moisture tension under different depth of porous pipe buried. (unit: cm)

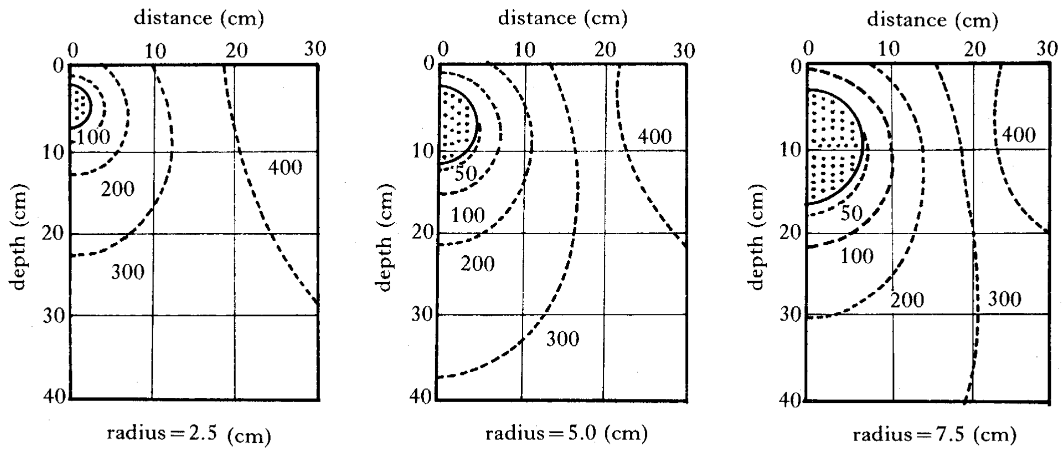

Fig. 5 Distributions of soil moisture tension under radius of porous pipe buried. (unit: cm)

Judging from the results shown in Fig. 4, each distribution of soil moisture tension is considered to be very different under the various depthes of porous pipe buried. That is, the isopotential line under the lower depth of porous pipe buried shows a tendency to spread out largely to the vertical direction and also to the horizontal more than that under other depthes. Therefore, it is found that the depth of porous pipe buried influences the distributions of soil moisture tension, so it is very important to decide the space of crops planted or the space of crops pipe buried.

(4) Outer radius of porous pipe buried

The radius of porous pipe buried is selected three kinds, that is, 2.5, 5.0, and 7.5 cm. The results of the distributions obtained are shown in Fig. 5.

Judging from the results shown in Fig. 5, each distribution of soil moisture tension is considered to be very different under the various radius of porous pipe buried. That is, the more the radius of porous pipe increases, the more the isopotential line shows a tendency to spread out to the vertical and also horizontal direction. Therefore, to obtain the optimum distributions of soil moisture tension to the group of crops, it is better to increase the radius of porous pipe buried. Futhermore, it is importan factor to decide the radius of porous pipe buried under the sub-irrigation method by using negative pressure difference.

Conclusion

This study was conducted to investigate the appearance method which shows the distribution of soil moisture tension in the sub-irrigation method by using negative pressure difference. There are many methods to rappear the distributions of soil moisture tension, but in this study, the conformal mapping method was tried to reappear. On the other hand, to investigate the distributions of soil moisture tension, many kinds of factors, that is, the intensity of water supply, negative pressure given in porous pipe buried, the depth of porous pipe buried, and the outer radius of porous pipe were selected. Their conditions investigated were three kinds. Consequently, it is found that the distributions of soil moisture tension can be reappeared by applying the different conditions. Futhermore, it is found that the different intensities of water supply and negative pressure given in porous pipe does not almost influence the distributions of soil moisture tension, but the different depthes and the outer radius of porous pipe buried influence very largely. Therefore, in the sub-irrigation method by using negative pressure difference, to decide the depth of porous pipe buried and the outer radius of porous pipe, they should be decided rightly in accordance with their objectives.

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