# Application of the Boundary Element Method to Agricultural Engineering Problems : MICROCOMPUTER IMPLEMENTATION OF BEM


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# Application of the Boundary Element Method to Agricultural Engineering Problems

## MICROCOMPUTER IMPLEMENTATION OF BEM

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## Abstract

BEM were applied to solve interesting agricultural engineering problems. BEM computer program was run on a microcomputer (Fujitsu MICRO-8) to obtain specific results of the following problems. Tensile stresses developed in a structure of dam inspection gallery caused by a temperature drop were calculated. Pressure development of tomato fruit due to water potential change was also calculated.

## Introduction

Many differential equations describing physical phenomena which often relate to engineering problems can be solved numerically. The finite element method (FEM) and the finite difference method (FDM) are the leading techniques of numerical methods. Especially, the finite element method has reached such a stage of development that many workers would doubt that any equivalent technique might ever appear.

For a few decades integral equations have been used to model such mathematical problems as fluid mechanics, stress analysis, heat transfer, etc.[1] The boundary element method (BEM) is an updated method of solving integral equations. The weight residual approach can be used to formulate boundary element equations. BEM is well suited for the inexpensive microcomputers, which are nowadays being used in many engineering officies, since BEM solutions tend to requre less matrices and data storage than FEM.

There are a number of problems involving differential equations in the agricultural engineering. In this study, an attempt of solving a couple of example problems of agricultural engineering was made by using the boundary element method.

## Theory[1)2)]

The following is a brief description of boundary element equations for linear isotropic elasticity problems in an initial stress field. The thermoelasticity is a typical example of initial stress problems.

Thermal stress can be considered as a body force applied proportional to the temperature gradient. Fandamental boundary integral equations for a steady state thermal loading case is given by D. J. Danson[2] as

$$U_k(x) = \int_s U_{ki}(x,y)t_i(y)dS_y - \int_s T_{ki}(x,y)U_i(y)dS_y + \frac{\alpha E}{1-2\mu}\int_v U_{ki,i}(x,y)\theta(y)dV_y \quad \text{(1)}$$

where,
$U_k(x)$ : displacement at an initial point x in the k direction
$U_{ki}(x,y)$ : displacement in the i direction at y due to a unit point load in the k direction at x.
$T_{ki}(x,y)$ : traction in the i direction at y due to a unit point load in the k direction at x
$t_i(y)$ : traction at y
$U_i(y)$ : displacement at y
S : representation of the bonndary of the body
V : representation of the domain of the body
$\alpha$ : Expansion coefficient
E : Young's modulus
$\theta(y)$ : the temperature at y
,i : differentiation with respect to $y_i$.
$\mu$ : Poisson's ratio

The subscript y on $dS_y$ and $dV_y$ is to indicate that the coordinates $y_i$ and not the coordinates $x_j$ are the integration variables.

The volume integral which appears in the third term of the right hand side of Eq.(1) can be transformed into a surface integral by introducing the Galerkin tensor and some mathematical manipulations. The displacement kernel $U_{ki}(x,y)$ is given in the form of Galerkin tensor as

$$U_{ki}(x,y) = G_{ki,jj}(x,y) - \frac{G_{kj,ji}(x,y)}{2(1-\mu)} . \quad \text{(2)}$$

By substitution Eq.(2) into Eq.(1), we obtain

$$F_k(x) = \int_v [G_{ki,jj}(x,y) - \frac{G_{kj,ji}(x,y)}{2(1-\mu)}] b_i(y)dV_y . \quad \text{(3)}$$

Differentiating Eq.(2) with respect to $y_i$ and substituting into Eq.(3), we get

$$F_k(x) = \frac{\alpha E}{2(1-\mu)}\int_v \theta(y)G_{ki,ijj}(x,y)dV_y . \quad \text{(4)}$$

For the steady state case, $\theta,_{jj} = 0$, then Eq.(4) becomes

$$F_k(x) = \frac{\alpha E}{2(1-\mu)}\int_v \frac{\partial}{\partial y_j}[\theta(y)G_{ki,ij}(x,y) - \frac{\partial}{\partial y_j}[G_{ki,i}(x,y)\theta_{,j}(y)]\, dV_y . \quad \text{(5)}$$

Applying Gauss' theorem to Eq.(5), we can transform the volume integral of Eq.(5) into the surface integral, then we can write

$$F_k(x) = \frac{\alpha E}{2(1-\mu)} \int_s [\theta(y) G_{ki,ij}(x,y) - G_{ki,i}(x,y)\theta_{,j}(y)] n_j dS_y \quad \text{(6)}$$

For steady state thermal loads we write Eq.(6) as

$$F_k(x) = \int_s P_k(x,y)\theta(y) dS_y - \int_s Q_k(x,y)\theta_{,m}(y) n_m dS_y \quad \text{(7)}$$

where $P_k(x,y) = \frac{\alpha E}{2(1-\mu)} G_{ki,ij}(x,y) n_j$

and $Q_k(x,y) = \frac{\alpha E}{2(1-\mu)} G_{ki,i}(x,y)$ . (8)

The Galerkin tensor corresponding to the 2D fundamental solution is

$$G_{ki}(x,y) = \frac{1+\mu}{4\pi E} \delta_{ki} r^2 \ln\left(\frac{1}{r}\right) \quad \text{(9)}$$

where r is the distance between the force point x and the field point y and $\delta_{ki}$ is the Kronecker delta.
Differentiating Eq.(9) and substituting into Eq.(8), we obtain

$$P_k(x,y) = \frac{\alpha(1+\mu)}{4\pi(1-\mu)} [(\ln(\frac{1}{r}) - \frac{1}{2}) n_k - n_m r_m r_k] ,$$

$$Q_k(x,y) = \frac{\alpha(1+\mu)}{4\pi(1-\mu)} r_k r (\ln(\frac{1}{r}) - \frac{1}{2}) \quad \text{(10)}$$

Differentialting Eq.(1) and substitution in the stress-displacement relations give the stress at an initial point x.

Thus
$$\sigma_{ij}(x,y) = \int_s D_{kij}(x,y) t_k(y) dS_y - \int_s S_{kij}(x,y) u_k(y) dS_y + \int_s S^*_{ij}(x,y)\theta(y) dS_y - \int_s V^*_{ij}(x,y)\theta_{,m}(y) n_m(y) dS_y - \frac{\alpha E}{1-2\mu}\theta(x)\delta_{ij} \quad \text{(11)}$$

where $S^*_{ij}(x,y) = \frac{\alpha E}{4\pi(1-\mu) r} [n_m r_m \{\delta_{ij}/(1-2\mu) - 2 r_i r_j\} + n_i r_j + n_j r_i]$

and $V^*_{ij}(x,y) = \frac{\alpha E}{4\pi(1-\mu) r} [r_i r_j + \{\delta_{ij}/(1-2\mu)\}\{(1+2\mu)/2 - \ln(1/r)\}]$ .

The third order tensor $D_{kij}(x,y)$ and $S_{kij}(x,y)$ are given by Brebbia[1)] as

$$D_{kij} = (1/r^{\alpha}) [(1-2\mu)(\Delta_{ki} r_{,j} + \Delta_{kj} r_{,i} - \Delta_{ij} r_{,k}) + \beta r_{,i} r_{,j} r_{,k}] [1/\{4\alpha\pi(1-\mu)\}]$$

$$S_{kij} = (2\mu/r^{\beta}) [\beta(\partial r/\partial n)((1-2\mu)\Delta_{ij} r_{,k} + \mu(\Delta_{ik} r_{,j} + \Delta_{jk} r_{,i}) - \gamma r_{,i} r_{,j} r_{,k}) + \beta\mu(n_i r_{,j} r_{,k} + n_j r_{,i} r_{,k}) + (1-2\mu)(\beta n_k r_{,i} r_{,j} + n_j \Delta_{ik} + n_i \Delta_{jk}) - (1-4\mu) n_k \Delta_{ij}] [1/\{4\alpha\pi(1-\mu)\}]$$

where $r_{,i} = \dfrac{\partial r}{\partial x_i}$ on the boundary and $r_j = (y_j - x_j)/r$ (unit vector from the force point x to the field point y). For the two dimentional case the coefficients $\alpha$, $\beta$, and $\gamma$ are equal to 1, 2, and 4, respectively.

Brebbia[1) ] utilized triangular internal celles to discretize the domain in order to calculate the volume integral in Eq.(1). In this study, however, the volume integral was transformed into the surface integral which can readily become boundary element equations as described by D. J. Danson[2)].

## Computer implementation

The computer flow diagram presented in Fig. 1 is a outline of the computer implementation of BEM. The subroutine EXPAN provides calculations of vecter entities due to thermal loads.

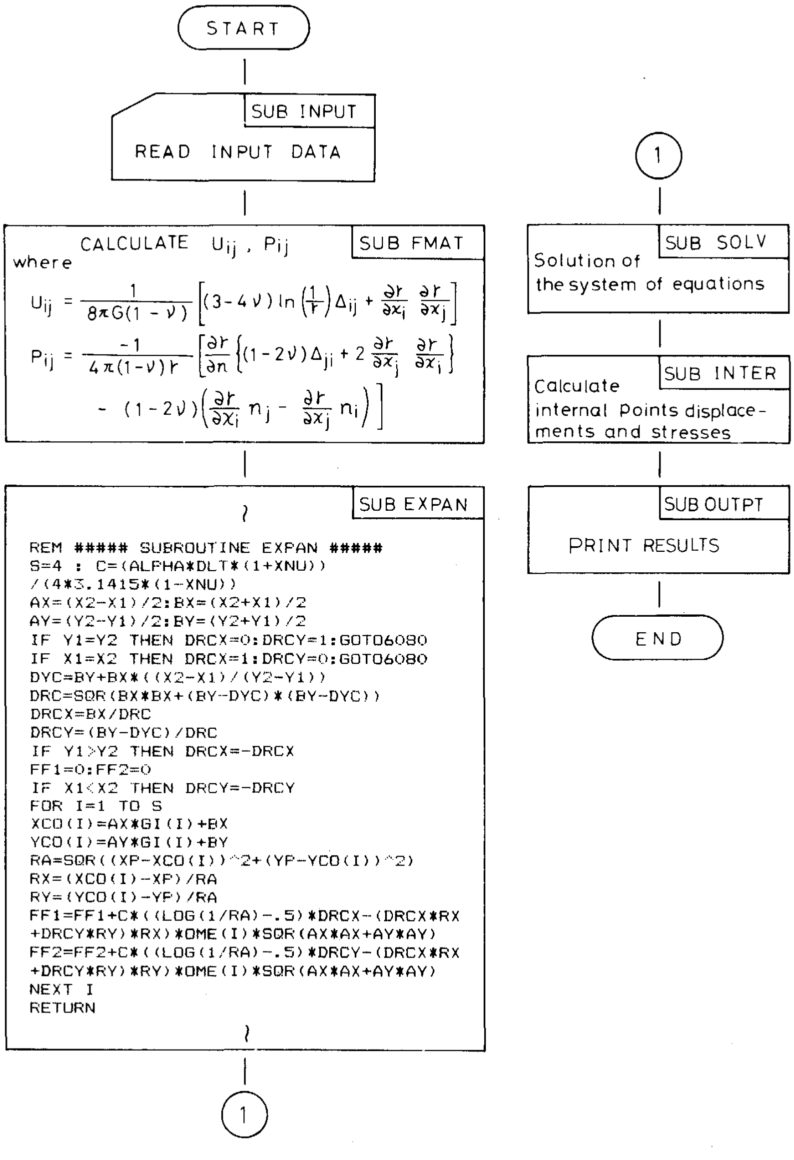


Fig. 1. Flow diagram of BEM program

Example 1　Stress development due to tomato fruit water potential change.

Murase[3] reported that stress-strain relations of plant tissue in a water potential field can be expressed by the Duhmel-Neumann relation. As an illustrative example of BEM, stress developments of solid tomato fruit confined by rigid skin due to water potential elevation ($\Delta\Psi$) by 2 bars were calculated. Fifteen boundary elements were used to simulate the given structure. The following constants were used for this calculation.

$$\Delta\Psi = 2 \text{ bars}$$

$$\alpha = 0.069 \text{ bars}^{-1}$$

$$E = 1005 \text{ KPa}$$

$$\mu = 0.131$$

Calculated compressive stresses at five internal points shown in Fig. 2 are as large as 2 MPa.

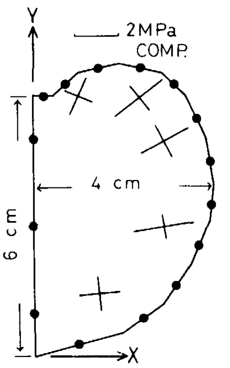


Fig. 2. Compressive stress development due to water potential change

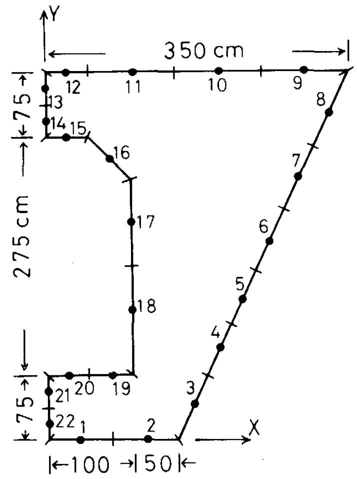


Fig. 3. Boundary element discritization of dam inspection gallery

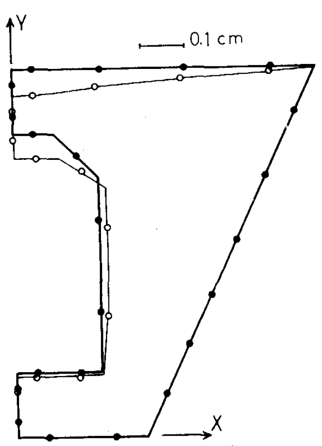


Fig. 4 (a). Displacement of the boundary due to the temperature drop

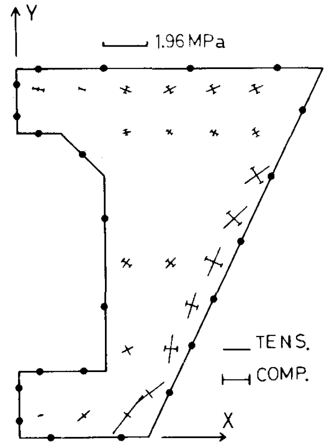


Fig. 4 (b). Pricipal stress at internal points

Example 2 Analysis of confined stresses due to temperature drops.

Tensile stresses may develop in a structure with a confined external boundary of dam inspection gallery when the structure experiences temperature drops. As another illustrative example of BEM application, the confined stress analysis was chosen. The BEM model of the gallery using 22 boundary elements is shown in Fig. 3. An uniform temperature drop by 10°C is assumed for this analysis. Node 1 to 8 are fixed boundary nodes. Using the following material constants, the displacements and the stresses were calculated. Fig. 4 (a) and (b) show the results of the BEM computer simulation.

$$\alpha = 0.00001\ (^{\circ}C^{-1})$$

$$E = 1.0 \times 10^{7}\ \text{KPa}$$

$$\mu = 0.2$$

## Concluding remarks

In this paper, we were able to present the BEM computer program for steady state thermal loading in the two-dimensional elasticity by a micro-computer, which is being used in many laboratory. However, the time of calculation by the microcomputer was pretty late. This matter is one of the present pending problems.

## Acknowledgement

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