



Brand Selection and its Matrix Structure : Expansion to the Model Involving Customer Satisfaction

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Brand Selection and its Matrix Structure

—Expansion to the Model Involving Customer Satisfaction—

Kazuhiro Takeyasu · Yuki Higuchi

ABSTRACT: Focusing that consumers' are apt to buy superior brand when they are accustomed or bored to use current brand, new analysis method is introduced. Before buying data and after buying data is stated using liner model. When above stated events occur, transition matrix becomes upper triangular matrix. In this paper, equation using transition matrix is extended to the model involving customer satisfaction and the method is newly re-built. These are confirmed by numerical examples. *S*-step forecasting model is also introduced. This approach makes it possible to identify brand position in the market and it can be utilized for building useful and effective marketing plan.

Key Words: brand selection, matrix structure, brand position

1. INTRODUCTION

It is often observed that consumers select upper class brand when they buy next time after they are bored to use current brand. Suppose that former buying data and current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then transition matrix becomes upper triangular matrix under the supposition that former buying variables are set input and current buying variables are set output. If the top brand were selected from lower brand skipping intermediate brands, corresponding part in upper triangular matrix would be 0. These are verified in numerical examples with simple models.

If transition matrix is identified, *s*-step forecasting can be executed. Generalized forecasting matrix components' equations are introduced. Unless planners for products notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach makes it effective to execute marketing plan

and/or establish new brand.

Quantitative analysis concerning brand selection has been executed by Yamanaka^[5], Takahashi et al.^[4]. Yamanaka^[5] examined purchasing process by Markov Transition Probability with the input of advertising expense. Takahashi et al.^[4] made analysis by the Brand Selection Probability model using logistics distribution. In Takeyasu et al. (2007)^[6], matrix structure was analyzed for the case brand selection was executed for upper class.

In this paper, equation using transition matrix is extended to the model involving customer satisfaction and the method is newly re-built. These are confirmed by numerical examples. Such research is quite a new one.

Hereinafter, matrix structure is clarified for the selection of brand in section 2. Expansion to the model involving customer satisfaction is executed in section 3. S-step forecasting is formulated in section 4. Numerical calculation is executed in section 5. Section 6 is a summary.

2. BRAND SELECTION AND ITS MATRIX STRUCTURE

(1) Upper shift of Brand selection

Now, suppose that x is the most upper class brand, y is the second upper class brand, and z is the lowest class brand.

Consumer's behavior of selecting brand might be $z \rightarrow y, y \rightarrow x, z \rightarrow x$ etc. $x \rightarrow z$ might be few.

Suppose that x is current buying variable, and x_b is previous buying variable. Shift to x is executed from x_b, y_b , or z_b .

Therefore, x is stated in the following equation. a_{ij} represents transition probability from j -th to i -th brand.

$$x = a_{11}x_b + a_{12}y_b + a_{13}z_b$$

Similarly,

$$y = a_{22}y_b + a_{23}z_b$$

And

$$z = a_{33}z_b$$

These are re-written as follows.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \quad (1)$$

Set

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}, \quad \mathbf{X}_b = \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

then, \mathbf{X} is represented as follows.

$$\mathbf{X} = \mathbf{A}\mathbf{X}_b \quad (2)$$

Here,

$$\mathbf{X} \in \mathbf{R}^3, \mathbf{A} \in \mathbf{R}^{3 \times 3}, \mathbf{X}_b \in \mathbf{R}^3$$

\mathbf{A} is an upper triangular matrix.

To examine this, generating following data, which are all consisted by the data in which transition is made from lower brand to upper brand,

$$\mathbf{X}^i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \dots \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (3)$$

$$\mathbf{X}_b^i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \dots \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (4)$$

$$i = 1, \quad 2 \quad \dots \quad N$$

parameter can be estimated using least square method.

Suppose

$$\mathbf{X}^i = \mathbf{A}\mathbf{X}_b^i + \boldsymbol{\varepsilon}^i \quad (5)$$

Where

$$\boldsymbol{\varepsilon}^i = \begin{pmatrix} \varepsilon_1^i \\ \varepsilon_2^i \\ \varepsilon_3^i \end{pmatrix} \quad i = 1, 2, \dots, N$$

and minimize following J

$$J = \sum_{i=1}^N \boldsymbol{\varepsilon}^{iT} \boldsymbol{\varepsilon}^i \rightarrow \text{Min} \tag{6}$$

$\hat{\mathbf{A}}$ which is an estimated value of \mathbf{A} is obtained as follows.

$$\hat{\mathbf{A}} = \left(\sum_{i=1}^N \mathbf{X}^i \mathbf{X}_b^{iT} \right) \left(\sum_{i=1}^N \mathbf{X}_b^i \mathbf{X}_b^{iT} \right)^{-1} \tag{7}$$

In the data group which are all consisted by the data in which transition is made from lower brand to upper brand, estimated value $\hat{\mathbf{A}}$ should be upper triangular matrix.

If following data which shift to lower brand are added only a few in equation (3) and (4),

$$\mathbf{X}^i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_b^i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\hat{\mathbf{A}}$ would contain minute items in the lower part triangle.

(2) Sorting brand ranking by re-arranging row

In a general data, variables may not be in order as x, y, z . In that case, large and small value lie scattered in $\hat{\mathbf{A}}$. But re-arranging this, we can set in order by shifting row. The large value parts are gathered in upper triangular matrix, and the small value parts are gathered in lower triangular matrix.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} \circ & \circ & \circ \\ \varepsilon & \circ & \circ \\ \varepsilon & \varepsilon & \circ \end{pmatrix} \leftarrow \begin{pmatrix} z \\ x \\ y \end{pmatrix} \begin{pmatrix} \varepsilon & \varepsilon & \circ \\ \circ & \circ & \circ \\ \varepsilon & \circ & \circ \end{pmatrix} \tag{8}$$

(3) Matrix structure under the case skipping intermediate class brand is skipped

It is often observed that some consumers select the most upper class brand from the most lower class brand and skip selecting the intermediate class brand.

We suppose v, w, x, y, z brands (suppose they are laid from upper position to lower position as $v > w > x > y > z$).

In the above case, selection shifts would be

$$v \leftarrow z$$

$$v \leftarrow y$$

Suppose they do not shift to y, x, w from z , to x, w from y , and to w from x , then Matrix structure would be as follows.

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{pmatrix} \begin{pmatrix} v_b \\ w_b \\ x_b \\ y_b \\ z_b \end{pmatrix} \quad (9)$$

3. EXPANSION OF THE MODEL INVOLVING CUSTOMER SATISFACTION

We extend Eq.(2) to the model involving customer satisfaction item in this section. It is considered that consumers buy higher ranked brand as well as keeping the same ranked brand when they are satisfied with the former buying goods. Then, the following expansion of the model may be introduced.

$$y_t^j = a_{j1}y_{t-1}^1 + a_{j2}y_{t-1}^2 + \cdots + a_{jp}y_{t-1}^p + b_{j1}x_{t-1}^1 + b_{j2}x_{t-1}^2 + \cdots + b_{jp}x_{t-1}^p \quad (10)$$

where

$$y_t^j : j\text{-th brand selection at time } t \quad (j=1,2,\cdots,p)$$

$$x_{t-1}^i : i\text{-th customer satisfaction item at time } t-1 \quad (i=1,2,\cdots,p)$$

And the brand position is supposed to be :

$$y_t^1 > y_t^2 > \cdots > y_t^p \quad (11)$$

Then it can be written in the following matrix form.

$$\begin{pmatrix} y_t^1 \\ y_t^2 \\ \vdots \\ y_t^p \end{pmatrix} = \begin{pmatrix} a_{11}, & a_{12}, & \cdots, & a_{1p} \\ a_{21}, & a_{22}, & \cdots, & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{p1}, & a_{p2}, & \cdots, & a_{pp} \end{pmatrix} \begin{pmatrix} y_{t-1}^1 \\ y_{t-1}^2 \\ \vdots \\ y_{t-1}^p \end{pmatrix} + \begin{pmatrix} b_{11}, & b_{12}, & \cdots, & b_{1p} \\ b_{21}, & b_{22}, & \cdots, & b_{2p} \\ \vdots & \vdots & & \vdots \\ b_{p1}, & b_{p2}, & \cdots, & b_{pp} \end{pmatrix} \begin{pmatrix} x_{t-1}^1 \\ x_{t-1}^2 \\ \vdots \\ x_{t-1}^p \end{pmatrix} \quad (12)$$

Re-writing this, we can get the following equation.

$$\mathbf{Y}_t = (\mathbf{A}, \mathbf{B}) \begin{pmatrix} \mathbf{Y}_{t-1} \\ \mathbf{X}_{t-1} \end{pmatrix} \quad (13)$$

where

$$\mathbf{Y}_t = \begin{pmatrix} y_t^1 \\ y_t^2 \\ \vdots \\ y_t^p \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} x_{t-1}^1 \\ x_{t-1}^2 \\ \vdots \\ x_{t-1}^p \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} a_{11}, & a_{12}, & \cdots, & a_{1p} \\ a_{21}, & a_{22}, & \cdots, & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{p1}, & a_{p2}, & \cdots, & a_{pp} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_{11}, & b_{12}, & \cdots, & b_{1p} \\ b_{21}, & b_{22}, & \cdots, & b_{2p} \\ \vdots & \vdots & & \vdots \\ b_{p1}, & b_{p2}, & \cdots, & b_{pp} \end{pmatrix}$$

$$\mathbf{Y}_t \in \mathbf{R}^p \quad (t=1,2,\dots), \quad \mathbf{X}_{t-1} \in \mathbf{R}^p \quad (t=1,2,\dots), \quad \mathbf{A} \in \mathbf{R}^{p \times p}, \quad \mathbf{B} \in \mathbf{R}^{p \times p}$$

In order to estimate \mathbf{A}, \mathbf{B} , we set the following equation in the same way as before.

$$\mathbf{Y}_t^i = \mathbf{A} \mathbf{Y}_{t-1}^i + \mathbf{B} \mathbf{X}_{t-1}^i + \boldsymbol{\varepsilon}_t^i \quad (i=1,2,\dots,N) \quad (14)$$

$$J = \sum_{i=1}^N \boldsymbol{\varepsilon}_t^{iT} \boldsymbol{\varepsilon}_t^i \rightarrow \text{Min} \quad (15)$$

Eq.(14) is expressed as follows.

$$\mathbf{Y}_t^i = (\mathbf{A}, \mathbf{B}) \begin{pmatrix} \mathbf{Y}_{t-1}^i \\ \mathbf{X}_{t-1}^i \end{pmatrix} + \boldsymbol{\varepsilon}_t^i \quad (i=1,2,\dots,N) \quad (16)$$

$(\hat{\mathbf{A}}, \hat{\mathbf{B}})$ which is an estimated value of (\mathbf{A}, \mathbf{B}) is obtained as follows in the same way as Eq.(7).

$$(\hat{\mathbf{A}}, \hat{\mathbf{B}}) = \left(\sum_{i=1}^N \mathbf{Y}_t^i (\mathbf{Y}_{t-1}^{iT}, \mathbf{X}_{t-1}^{iT}) \right) \left(\sum_{i=1}^N \begin{pmatrix} \mathbf{Y}_{t-1}^i \\ \mathbf{X}_{t-1}^i \end{pmatrix} (\mathbf{Y}_{t-1}^{iT}, \mathbf{X}_{t-1}^{iT}) \right)^{-1} \quad (17)$$

This is re-written as :

$$\left(\hat{\mathbf{A}}, \hat{\mathbf{B}} \right) = \left(\sum_{i=1}^N \mathbf{Y}_t^i \mathbf{Y}_{t-1}^{iT}, \sum_{i=1}^N \mathbf{Y}_t^i \mathbf{X}_{t-1}^{iT} \right) \left(\begin{array}{cc} \sum_{i=1}^N \mathbf{Y}_{t-1}^i \mathbf{Y}_{t-1}^{iT}, & \sum_{i=1}^N \mathbf{Y}_{t-1}^i \mathbf{X}_{t-1}^{iT} \\ \sum_{i=1}^N \mathbf{X}_{t-1}^i \mathbf{Y}_{t-1}^{iT}, & \sum_{i=1}^N \mathbf{X}_{t-1}^i \mathbf{X}_{t-1}^{iT} \end{array} \right)^{-1} \quad (18)$$

We set this as :

$$\left(\hat{\mathbf{A}}, \hat{\mathbf{B}} \right) = \left(\hat{\mathbf{C}}, \hat{\mathbf{D}} \right) \left(\begin{array}{cc} \hat{\mathbf{E}} & \hat{\mathbf{F}} \\ \hat{\mathbf{F}}^T & \hat{\mathbf{G}} \end{array} \right)^{-1} \quad (19)$$

In the data group of upper shift brand, $\hat{\mathbf{C}}, \hat{\mathbf{D}}$ become upper triangular matrices (or diagonal matrices). While $\hat{\mathbf{E}}$ and $\hat{\mathbf{G}}$ are diagonal matrices in any case. If the shift of purchasing goods corresponds to the shifting result of customer satisfaction item, $\hat{\mathbf{F}}$ becomes an upper triangular matrix (or diagonal matrix). This will be made clear in the numerical calculation later.

4. S-STEP FORECASTING

After transition matrix is estimated, we can make forecasting. S -step forecasting equation as follows.

$$\mathbf{Y}_t = \hat{\mathbf{A}}_1 \mathbf{Y}_{t-1} + \mathbf{B} \mathbf{X}_{t-1} \quad (20)$$

$$\mathbf{Y}_{t+1} = \mathbf{A}^2 \mathbf{Y}_{t-1} + \mathbf{A} \mathbf{B} \mathbf{X}_{t-1} + \mathbf{B} \mathbf{X}_t \quad (21)$$

$$\mathbf{Y}_{t+2} = \mathbf{A}^3 \mathbf{Y}_{t-1} + \mathbf{A}^2 \mathbf{B} \mathbf{X}_{t-1} + \mathbf{A} \mathbf{B} \mathbf{X}_t + \mathbf{B} \mathbf{X}_{t+1} \quad (22)$$

$$\mathbf{Y}_{t+3} = \mathbf{A}^4 \mathbf{Y}_{t-1} + \mathbf{A}^3 \mathbf{B} \mathbf{X}_{t-1} + \mathbf{A}^2 \mathbf{B} \mathbf{X}_t + \mathbf{A} \mathbf{B} \mathbf{X}_{t+1} + \mathbf{B} \mathbf{X}_{t+2} \quad (23)$$

$$\mathbf{Y}_{t+4} = \mathbf{A}^5 \mathbf{Y}_{t-1} + \mathbf{A}^4 \mathbf{B} \mathbf{X}_{t-1} + \mathbf{A}^3 \mathbf{B} \mathbf{X}_t + \mathbf{A}^2 \mathbf{B} \mathbf{X}_{t+1} + \mathbf{A} \mathbf{B} \mathbf{X}_{t+2} + \mathbf{B} \mathbf{X}_{t+3} \quad (24)$$

Generalizing this, s -step forecasting equation is derived as follows.

$$\mathbf{Y}_{t+s} = \mathbf{A}^{s+1} \mathbf{Y}_{t+s-1} + \sum_{i=1}^{s+1} \mathbf{A}^{s-i+1} \mathbf{B} \mathbf{X}_{t+i-2} \quad (25)$$

5. NUMERICAL EXAMPLE

In this section, we consider the case there is no shift to lower brand. We consider the case that brand selection shifts to the same class or upper classes. As above referenced, corresponding part of transition matrix must be an upper triangular matrix. Suppose following events occur. Here we set $p = 5$ in Eq.(12).

$\langle \mathbf{Y}_{t-1}$ to $\mathbf{Y}_t \rangle$	$\langle \mathbf{X}_{t-1}$ to $\mathbf{Y}_t \rangle$	average	total
① Shift from 5th position to 1st position : 5 events	4/5, 4/5, 1, 1, 1	0.92	4.6
② Shift from 4th position to 1st position : 2 event	4/5, 4/5	0.8	1.6
③ Shift from 3rd position to 1st position : 1 event	4/5	0.8	0.8
④ Shift from 1st position to 1st position : 1 event	4/5	0.8	0.8
⑤ Shift from 5th position to 2nd position : 5 events	4/5, 3/5, 7/10, 7/10, 7/10	0.7	3.5
⑥ Shift from 4th position to 2nd position : 1 event	4/5	0.8	0.8
⑦ Shift from 2nd position to 2nd position : 1 event	3/5	0.6	0.6
⑧ Shift from 5th position to 3rd position : 2 events	4/5, 4/5	0.8	1.6
⑨ Shift from 3rd position to 3rd position : 1 event	3/5	0.6	0.6
⑩ Shift from 4th position to 4th position : 1 event	3/5	0.6	0.6
⑪ Shift from 5th position to 5th position : 1 event	3/5	0.6	0.6
⑫ Shift from 5th position to 4th position : 1 event	4/5	0.8	0.8
⑬ Shift from 4th position to 3th position : 1 event	3/5	0.6	0.6
⑭ Shift from 3rd position to 2nd position : 2 events	4/5, 4/5	0.8	1.6
⑮ Shift from 2nd position to 1st position : 2 events	1, 1	1	2

Vector $\mathbf{Y}_t, \mathbf{Y}_{t-1}, \mathbf{X}_{t-1}$ in these cases are expressed as follows.

$$\begin{aligned}
 \text{① } \mathbf{Y}_t &= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, & \mathbf{Y}_{t-1} &= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, & \mathbf{X}_{t-1} &= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.92 \end{pmatrix} \\
 \text{② } \mathbf{Y}_t &= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, & \mathbf{Y}_{t-1} &= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, & \mathbf{X}_{t-1} &= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.8 \\ 0 \end{pmatrix}
 \end{aligned}$$

$$\textcircled{3} \quad \mathbf{Y}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0.8 \\ 0 \end{pmatrix}$$

$$\textcircled{4} \quad \mathbf{Y}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0.8 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\textcircled{5} \quad \mathbf{Y}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.7 \end{pmatrix}$$

$$\textcircled{6} \quad \mathbf{Y}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0.8 \\ 0 \end{pmatrix}$$

$$\textcircled{7} \quad \mathbf{Y}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0.6 \\ 0 \\ 0 \end{pmatrix}$$

$$\textcircled{8} \quad \mathbf{Y}_t = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.8 \end{pmatrix}$$

$$\textcircled{9} \quad \mathbf{Y}_t = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0.6 \\ 0 \\ 0 \end{pmatrix}$$

$$\textcircled{10} \quad \mathbf{Y}_t = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.6 \\ 0 \end{pmatrix}$$

$$\textcircled{11} \quad \mathbf{Y}_t = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.6 \end{pmatrix}$$

$$\textcircled{12} \quad \mathbf{Y}_t = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.8 \end{pmatrix}$$

$$\textcircled{13} \quad \mathbf{Y}_t = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.6 \\ 0 \end{pmatrix}$$

$$\textcircled{14} \quad \mathbf{Y}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0.8 \\ 0 \\ 0 \end{pmatrix}$$

$$\textcircled{15} \quad \mathbf{Y}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{Y}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Substituting these to Eq.(18), we obtain the following equation.

$$(\hat{\mathbf{A}}, \hat{\mathbf{B}}) = \begin{pmatrix} 1 & 2 & 1 & 2 & 5 & | & 0.8 & 2 & 0.8 & 1.6 & 4.6 \\ 0 & 1 & 2 & 1 & 5 & | & 0 & 0.6 & 1.6 & 0.8 & 3.5 \\ 0 & 0 & 1 & 1 & 2 & | & 0 & 0 & 0.6 & 0.6 & 1.6 \\ 0 & 0 & 0 & 1 & 1 & | & 0 & 0 & 0 & 0.6 & 0.8 \\ 0 & 0 & 0 & 0 & 1 & | & 0 & 0 & 0 & 0 & 0.6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & | & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & | & 0 & 2.6 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & | & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & | & 0 & 0 & 0 & 3.6 & 0 \\ 0 & 0 & 0 & 0 & 14 & | & 0 & 0 & 0 & 0 & 11.1 \\ \hline 0.8 & 0 & 0 & 0 & 0 & | & 0.64 & 0 & 0 & 0 & 0 \\ 0 & 2.6 & 0 & 0 & 0 & | & 0 & 2.36 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & | & 0 & 0 & 2.28 & 0 & 0 \\ 0 & 0 & 0 & 3.6 & 0 & | & 0 & 0 & 0 & 2.64 & 0 \\ 0 & 0 & 0 & 0 & 11.1 & | & 0 & 0 & 0 & 0 & 9.03 \end{pmatrix}^{-1} \quad (26)$$

As we have seen before, we can confirm that

$\hat{\mathbf{C}}, \hat{\mathbf{D}}, \hat{\mathbf{F}}$ part in Eq.(19) is an upper triangular matrix (or diagonal matrix) and $\hat{\mathbf{E}}, \hat{\mathbf{G}}$ part in Eq.(19) are diagonal matrices.

We can find that if $\hat{\mathbf{C}}$ part becomes an upper triangular matrix (or diagonal matrix), then the items compose upper shift or the same level shift. Calculation results of $(\hat{\mathbf{A}}, \hat{\mathbf{B}})$ become as follows.

$$(\hat{\mathbf{A}}, \hat{\mathbf{B}}) = \left(\begin{array}{ccccc|ccccc} 1 & -1.5 & -1 & -2 & -1.841 & 0 & 2.5 & 1.667 & 3.333 & 2.773 \\ 0 & 2.5 & -2 & -1 & 1.963 & 0 & -2.5 & 3.333 & 1.667 & -2.025 \\ 0 & 0 & 4 & 2 & 0.093 & 0 & 0 & -5.0 & -2.5 & 0.062 \\ 0 & 0 & 0 & 2 & 0.047 & 0 & 0 & 0 & -2.5 & 0.031 \\ 0 & 0 & 0 & 0 & 0.738 & 0 & 0 & 0 & 0 & -0.841 \end{array} \right) \quad (27)$$

6. CONCLUSION

Consumers often buy higher grade brand products as they are accustomed or bored to use current brand products they have.

Formerly we have presented the paper and matrix structure was clarified when brand selection was executed toward higher grade brand. In Takeyasu et al. (2007)^[6], matrix structure was analyzed for the case brand selection was executed for upper class. In this paper, equation using transition matrix was extended to the model involving customer satisfaction and the method was newly re-built. These were confirmed by numerical examples. In the numerical example, matrix structure's hypothesis was verified. We can utilize the customer satisfaction data by this new model and estimation accuracy of parameter becomes more accurate and forecasting becomes more precise. Such research as questionnaire investigation of consumers' activity in automobile purchasing should be executed in the near future to verify obtained results.

REFERENCES

- [1] Aker, D.A, Management Brands Equity, Simon & Schuster, USA, 1991.
- [2] Katahira, H., Marketing Science (In Japanese), Tokyo University Press, 1987.
- [3] Katahira, H., Y. Sugita., "Current movement of Marketing Science" (In Japanese), Operations Research, 1994; 14: 178-188
- [4] Takahashi, Y., T. Takahashi, "Building Brand Selection Model Considering Consumers Royalty to Brand" (In Japanese), Japan Industrial Management Association 2002; 53(5): 342-347
- [5] Yamanaka, H., "Quantitative Research Concerning Advertising and Brand Shift" (In Japanese), Marketing Science, Chikra-Shobo Publishing, 1982.
- [6] Takeyasu, K., Y. Higuchi, "Analysis of the Preference Shift of Customer Brand Selection", International Journal of Computational Science 2007; 1(4): 371-394