DYNAMICAL PROPERTIES OF HICKSIAN STABILITY CONDITIONS

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I. Introduction, II. Market of Single Commodity,
III. Markets of Several Commodities, IV. Grapfical Representation.

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I

In his Value and Capital, Professor Hicks descibed the displacements of a price in a static system very clearly, but his method remained incomplete. Because, in the general case, it lacked of an explicit dynamic system. Treating with the case of a single commodity, he assumed that price tend to fall whenever supply exceeds demand, and to rise when demand exceeds supply. From this assumption, he derived a proposition that a market involving only one commodity is stable provided a rise in price above equilibrium creates an excess supply, while a fall in price below equilibrium creates an excess demand. For under such conditions a displacement of price from equiliburium tends to be self-corrective; it sets in motion forces which restore equilibrium. ([2] p. 278.)

After this he attempted to generalize this proposition. But, unfortunately, instead of deriving the stability condition, he simply extended this results, and said that an individual market within the system is stable, as in the single commodity case, provided a reduction in price below equilibrium creats an excess demand for that particular commodity, while an increase in price creates an excess supply.

But this stablity conditions cannot be accepted unless it is derived from some plausible hypothesis. The error of the Hicksian method were first demonstrated by Professor Samuelson in his pioneer article, on the significance of dynamics to static analysis. ([3] p. 279.) Professor Samuelson criticised as follows: The method of approach is postulational; stability conditions are not deduced from a dynamic model except implicity. ... It is true that on page 70 a hint of dynamical process creeps into the discussion. ([3] p. 270.)

Professor Hicks answered to this as follows: Professor Samuelson's

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theory is much too complex to be discussed at all adequately in the space now at my disposal; besides, I cannot pretend that I am yet sufficiently at home with it to have made my mind about it. ([2] p. 335.) I assumed the process of adjustment to temporary equilibrium to be completed within a short period (a week), while I neglected the movement of prices within the week, ...Professor Samuelson has ... undoubtedly made important progress ... He drops the assumption of a quick and easy passage to temporary equilibrium assuming instead that rates of price-change are functions of differences between demands and supplies. His whole theory thus becomes dynamic, ...

In terms of this new technique, my theory can be dynamized; it is possible to inquire into the stability of the ... system in the sense of investigating whether the movements set up when a system is initially out of equilibrium will converge upon an equilibrium position.

This paper is concerned with the explicit dynamization of the Hicksian System. In analyzing the market of single commodity, there arises no problem. These are explained in Section II. In the next section III, I will try to treat the general case involving several commodities. Here arised one problem, about the Hicksian special assumption concerning the behavior of the other market.

It is true that the stability condition of the markets comprising several commodities cannot be discussed without reference to the prices of the other commodities. To analize such market, he made following definition about the stability.

A market is defined as imperfectly stable if a fall in the price of a particular commodity creates an excess demand for that commodity, after all other prices have adjusted themselves so that supply is again equal to demand in all markets. And a market is defned as perfectly stable if a fall in price below equilibrium creates an excess demand after any given subset of prices in other markets is adjusted so that supply again equals demand. ([2] p. 279.)

Such treatment is criticised by Professor Metzler that it is inadmissible to assume ... a system of markets is ... stable if a fall in the price of a particular product leads to an increase in excess demand for that product after all other prices have been adjusted so that supply is again equal to demand for each of the other commodities. Or a market system is stable whenever a fall in the prices of a given commodity leads to excess demand for that product after any given subset of prices is adjusted so that supply again equals demand for these other commodities, with all remaining prices held constant. Thus the concept of stability developed by Professor Hicks, makes no allowance for the fact that all prices may be out of equilibrium at the same time. ([2] p. 291.) The one-thing-at-a-time method cannot always be applied to a multiple-market system in the manner proposed by Hicks.

Contrally to Metzler's criticism, I think I can find a way to avoid such a critisism. I will show this in Setion III. Moreover, another implicit assumption about the counting operational time will be made clear at the ends of this section. In section IV, using graphical representation, I will show that our method will corresponds very well to Professor Hicks' method.

II

Let us now consider an individual in the market of X. He knows the shape of his own demand curve for X. But he does not know the shape of the market demand curve, which can be obtained by adding each individual's demand curve. He does not know how many quantities are to be supplied by other individuals, when a price of X is qouted. Also, he does not know the shape of market supply curve, which can be obtained by adding individual's supply curve. When a price is quoted in the market, he determines his own demand or supply, which is derived from his indifference map or production function. He tries to maximise his own utility or profit, but he never tries to make market demand equals to market supply.

The same thing can be said about each individual demander or supplier. Thus, when a price is quoted, each one does not know how much quantities are to be supplies or demanded in the market. There is no guarantee that, at that price level, market demand will be equal to market supply. Usually, there will be a gap between market demand and market supply. At this point, some one will have unfilled demand, so all people cannot be at their most preferable positions. Therefore, there will be competition among them to reach their more preferable positions. Transactions will not be able to preform here, and another price will be quoted. Again each individual determines the demand or supply at this new price level. Usually this quantity is different from the former one. But, as before, market demand does not equal to market supply, so transactions will not be able to perform here, and another price will have to be quoted. Thus we quote prices successively untill market demand becomes equal to market supply. When the market demand equals to market supply, every individual is in his most preferable position, so the transaction is actually performed.

Suppose that demand curve of X is falling, and supply curve is rising to the right. (See Fig. 1.) When a price is quoted as $p(0)$, the

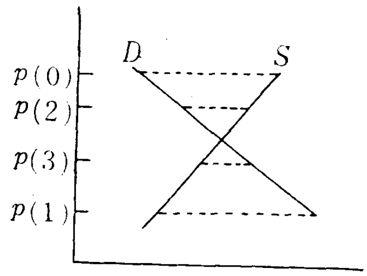

Fig. 1.

supply is greater than the demand. Some of the suppliers cannot find their demanders at this price level, so there will be competition among suppliers to reach their most preferable positions. Thus another price $p(1)$ will have to be quoted lower than before. $p(1) < p(0)$. At this new price, demand will be (say) greater than supply. The another price $p(2)$ will be quoted higher, $p(2) > p(1)$. Then supply will be greater than demand, therefore on the next time, price $p(3)$ will be quoted lower. At this price the demand will exceed supply, therefore another higher price will be quoted. The quoted price is changed by the demand and supply condition, i.e., it is an dependent variable. But we must be sure that, when we look at it from an individual view point, it is a constant (or independent variable) in the sense that any change in his demand or supply cannot produce any sensible effect on the quoted price. (Because there are so many individuals in the market.)

If a quoted price $p(t)$ on the t-th time is lower than that of the $t+1$-th, $p(t+1)$, this relation can be expressed as $p(t+1) > p(t)$. In the opposite case as $p(t+1) < p(t)$, and if they are equal, as $p(t+1) = p(t)$. At the price $p(t)$, the excess demand, i.e., demand minus supply will be expressed as $E[p(t)]$. Therefore the direction of the change of quoted price may be formulated as follows; the sign of $\{p(t+1) - p(t)\}$ is equal to the sign of $E[p(t)]$. When we determine a fraction F to connects quantitatively the variations of the quoted prices and the excess demand, we can express the above relation as

$$p(t+1) - p(t) = F \cdot E[p(t)].$$

To explain this relationship, let us consider a special case, where there is just one unit of positive excess demand $E[p(t)]=1$, at $p(t)$, then the increase in the quoted price will just equal to F. It is usually said that a price is flexible when it changes whenver there is a discrepancy between demand and supply. So the coefficient F may be called a price flexibility. In the above, we assumed that the quoted price will move in the same direction with the sign of the excess demand. This relation can be expressed as " F is positive ".

When market demand is unequal to market supply, a new price will be quoted following the above formula. When demand happens to become equal to supply, every individual is in his most preferable position, so competition will come to an end, and transactions will be performed.

It will be already clear that transactions will be performed at the quoted price which satisfies the relationship;

$$p(t+1) = p(t).$$

When a level of quoted price satisfies this relationship, we call it equilibrium price, and is denoted as p^0. And a market satisfies the above relationship, we call it is in equilibrium. The necessary and sufficient condition of the above relationship is $E=0$.

Let us start from a quoted price which is near to the equiliblium price. In this case, we can get

$$p(t+1) - p(t) = F \cdot E' \cdot [p(t) - p^0],$$

where $E' = dE(p^0)/dp$.

This means that, when a quoted price is different from the equilibrium price by one unit, there will be excess demand by E'.

Let us solve the above relationship. Putting $Am^t = p(t) - p^0$, we get

$$m = 1 + F \cdot E', \quad A = p(0) - p^0.$$

Therefore we get

$$p(t) = p^0 + [p(0) - p^0](1 + F \cdot E')^t.$$

Substituting 0, 1, 2,...... for t, we can get the price level quoted at the 0th, 1st, 2nd,...... time. We call this as the fundamental equation of the quoted price.

Following explanation about the time t will be helpful for readers. Suppose a market of X is opened from 9:00 to 11:60 noon. Any time of this period can be expressed as a point on the clock. This method of expression of time is called as " time by clock ".

But there are another method of expressing the same time. Now draw a horizontal line through the center of a roun clock to the left, mark dots at the same distance, for instance, at the length of radius, between themselves. Call these points as 0, 1, 2, ..., from the clock side. (See figure 2.). From these points draw tangent lines to the circle of the clock, and name the tangencies as C_0, C_1, C_2, ..., corresponding to 0, 1, 2, etc., on the line. In this way, every time

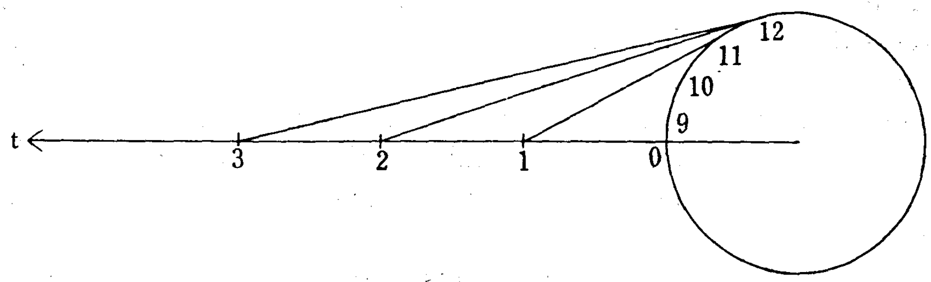

Fig. 2.

between 9 : 00 and 11 : 60 can be expressed by the points on the line. For instance, C_0 expresses 9 : 00 and C_1, 11 : 00. And C_t approaches to 11 : 60, as t increases infinitely. We quote prices after the following rule: At the begining of the market, i.e., at 9 : 00 by clock, or at the 0th point on the line, we quote $p(0)$, and at C_t time by by clock, i.e., at the t-th point on the line, we quote $p(t)$. By increasing t, the point C_t can be made to approache to 11 : 60 as near as we want. We call the time expressed by the point on the line as "operational time."

When the clock time approaches 11 : 60, i.e., when the operational time becomes indefinitely large, $p(t)$ approaches p^0, $p(t) \rightarrow p^0$, if and only if the following relation holds:

$$|1+F \cdot E'|<1$$

We recall that F is an increassng function of clock-time, hence of operational time, so if we take the interval of operational time short enough, we can make price flexibility F as small as we want. ($F>0$.) We take our operational time interval short enough. In this case, the necessary and sufficient condition for $p(t) \rightarrow p^0$ is $E'<0$.

Explanation of this relation in words is as follows. When an excess demand curve is falling to the right, market is stable. In other words, when the demand curve crosses the supply curve from the above, the market is stable, in the sense that starting from an arbitrary price (But it is implicitly assumed that it is not far from equilibrium price.), when we quoted prices successivly following above mentioned rule, we can approach the equilibrium price as close as we want. We call this condition as the stability condition.

III

Now we consider a market where three commodities are exchanged. We arbitrary quote the price of X as $p_x(0)$. Keeping it constant, quote a price of Y, then we can find the quantities of Y demanded and supplied. Changing the quoted price of Y (but keeping the price of X constant, $p_x(0)=$constant) we can draw the demand and supply curves of Y. Therefore, we can draw a excess demand curve of Y at $p_x(0)$. Temporarily, we assume that the excess demand curve of Y is downward sloping. In this case, when we quote the price of Y successively, we can reach the price of Y which will equilibrate the demand and supply of Y at $p_x(0)$. We denote this price as $p_y(0)$. When we quote this set of prices $p(0)=[p_x(0), p_y(0)]$ in the market of Y, the demand for Y is equal to the supply of Y, i.e., the excess demond for Y is zero. But the excess demand for X is usually not zero.

$$E_x[p(0)]\gtrless 0, \quad E_y[p(0)]=0.$$

Here, keeping the price of Y constant, find the price of X which will make the excess demand for X zero. When we quote the price of X successivly, we can find such price if the excess demand function for X is downward sloping at $p_y(0)$. We denote this price as $p_x(1)$. When we quote the set of prices $p(1)=[p_x(1), p_y(1)]$, where $p_y(1)=p_y(0)$, the market of X will be in equilibrium, but the market of Y will usually out of equilibrium;

$$E_x[p(1)]=0, \quad E_y[p(1)]\gtrless 0.$$

When we quote the set of prices successibly by this procedure, we can get the following series of excess demand:

$$E_x[p(0)]\gtrless 0, \qquad E_x[p(1)]=0, \qquad E_x[p(2)]\gtrless 0, \ldots\ldots$$
$$E_y[p(0)]=0, \qquad E_y[p(1)]\gtrless 0, \qquad E_y[p(2)]=0, \ldots\ldots$$

I.e., first, keeping arbitrarily the quoted price of X constant, $p_x(0)=$ constant, find a price of Y which will equilibrate the market of Y. We can find it, when the excess demand curve of Y is downward sloping at $p_x(0)$, i.e., when the stability condition of the first order is satisfied in the market of Y at $p_x(0)$. We quote the set of prices thus determined in the both market of X and Y. In this case, excess demand for Y is zero, $E_y[p(0)]=0$, but the excess demand for X is usually not zero. There we find a price of X, which will make the excess demand for X equal to zero, keeping the price of Y constant. We quote new set of prices in the both markets X and Y. Thus we find a series of prices which will equilibrate the markets X and Y alternatively.

But you must be carefull about the following assumption that, the excess demand for X affects only the price of X which will be quoted in the next time, but it does not change the price of Y. It changes only the price of X and does not change the price of Y. Of course, at the new price system, in which the price of Y is the same as before, the excess demand for Y may become non-zero, therofore on the next time, the price of Y will be changed. The price of X will be quoted differently only when the excess demand for X is not zero. The excess demand of X changes only the price of X on the next time, it prodnce no direct effect on the price of Y. A higher or lower price will be quoted according as there is prositive or negative excess demand, i.e., the sign of $p(t+1)-p(t)$ is determined by the sign of $E[p(t)]$,

$$sign\ [p_i(t+1)-p_i(t)]=sign\ E_i[p(t)] \qquad (i=x,\ y)$$

When a set of price $p(t)$ is quoted on the t-th time, each individual determines the quantities to demand and supplies following respective most preferable position, which are expressed by demand and supply functions. And usually, new set of price $p(t+1)$ will be quoted following the excess demand $E[p(t)]$.

Now we define price flexibilities H, K. These coefficients satisfy the following relations.

$$p_x(t+1)-p_x(t)=H\cdot E_x[p(t)],$$
$$p_y(t+1)-p_y(t)=K\cdot E_y[p(t)].$$

This shows the relation of prices which will be quoted on the t-th and t+1-th time. We are assuming that, when demand is larger than supply, a new price will be quoted higher than before, and lower price will be quoted in the opposite case. This assumption will be expressed as $H>0$, and $K>0$.

When we look at the above series of excess demand, we can easily find that, when t is even the price of Y is quoted at the level which make the market of Y equilibrium. Of course we are assuming that when we denote the set of prices to be quoted on the t-th time as $p(t)=[p_x(t),\ p_y(t)]$, this of prices is quoted on the both markets. And when it is quoted on the even time, the market of Y will be in equilibrium, but usully, the market of X is out of equilibrium.

$$E_x[p(t)] \gtrless 0, \quad E_y[p(t)] = 0.$$

Therfore, when t is even, the set of prices to be quoted on the next time, are determined by

$$\begin{aligned} p_x(t+1) - p_x(t) &= H \cdot E_x[p_x(t),\ p_y(t)] \\ 0 &= K \cdot E_y[p_x(t),\ p_y(t)], \qquad (t: \text{even}) \end{aligned}$$

and t is odd, the set of prices are determined by

$$\begin{aligned} 0 &= H \cdot E_x[p_x(t),\ p_y(t)] \\ p_y(t+1) - p_y(t) &= K \cdot E_y[p_x(t),\ p_y(t)]. \qquad (t: \text{odd}) \end{aligned}$$

Thus we quote the set of prices successibly as long as the either of the market is out of equiliblium. When both markets are in equilibrim, by the Walras' Law, (which will be explained in the following) the market of the last commodity Z will be in equilibrium. In this case all individuals are at their most preferable positions, so exchanges are actually performed at this prices. I.e., the set of the prices which establish the relation $p_i(t) = p_i(t+1)$ is the one by which all the contracts are actually performed. The necessary and sufficient conditions for this relation is $E_i = 0$. $(i = x, y.)$ When these conditions are satisfied, the makets are called to be in temporary equilibrium, and the prices which establishes these relations are called as the temporary equilibrium prices. We denote these prices as $p^0 = [p_x^0,\ p_y^0]$.

As three kinds of commodities X, Y, Z are exchanged, this gives us two prices to be determined, i.e., for instance the price of X and Y in terms of Z. $(p_z \equiv 1)$ For the exchange of two commodities we have one price to determine; similarly for the exchange of three commodities we have two prices. This can be seen at once if we select Z as a standard of value, then the two prices are the price of X and Y in terms of Z. Of course, X may be exchanged with Y by direct exchange without recourse to Z: but in equilibrium the rate of exchange between X and Y must always be equal to the relative prices of Z. For, if not, one would always be able to benefit himself by abondoning direct exchange, and spliting the transaction into two parts; first an exchange of X for the Z, and then an exchange of Z for Y.

It might appear at first sight that there are three equations to determine them, demand-and-supply equations on the markets for the three commodities. But this is not the case. In bilateral economy the equation of demand and supply for Z follows from the X and Y. Once an individual has decided how much of X, Y he will sell or he will by, he will automatically have decided how much of Z he will buy or sell. Thus

Demand for Z= Receipts from sale of X and Y − Expenditures on purchase of X and Y.

or Supply of Z= Expenditures on purchase of X and Y − Receipts from of sale of X and Y.

Therefore, for the whole community,

Demand for − supply of Z= Total Receipts from sales of X, Y − Total Expenditure on purchase of X, Y.

Once the demand for X, Y equals the supply, the excess demand for Z must be zero. There are two independent equations to determine the two independent prices. We call this relation as Walras' Law.

For the simplicity of treatment, we assume that the set of the prices which equilibrate demands and supplies on the markets for X and Y is one and only one. Now we assume that the quoted prices are sufficiently near to the equilibrium prices, so we can neglect the parts of more than second power of $p(t)-p^0$. In this case, we get

$$p_x(t+2)-p_x(t)=H\cdot E_{xx}\cdot[p_x(t)-p_x^0]+H\cdot E_{xy}\cdot[p_y(t)-p_y^0]$$
$$0=K\cdot E_{yx}\cdot[p_x(t)-p_x^0]+K\cdot E_{yy}\cdot[p_y(t)-p_y^0].$$

These relations express the fact that, the difference between the t-th quoted price and the temporary equilibrium prices produce repercussions to the $t+1$-th quoted prices through both substitutions between commodities and price flexibilities. In this case, when t is even, the price which will be quoted on the $t+1$-th (therefor $t+2$-th) time is determined as

$$p_x(t+2)=p_x^0+[p_x(0)-p_x^0]\lambda_x^t,$$

where $$\lambda_x=1+\frac{H}{E_{yy}}\begin{vmatrix} E_{xx} & E_{xy} \\ E_{yx} & E_{yy}\end{vmatrix}.$$

Similarly, when t is odd, we get

$$p_y(t+2)=p_y^0+[p_y(0)-p_y^0]\lambda_y^t,$$

where $$\lambda_y=1+\frac{K}{E_{xx}}\begin{vmatrix} E_{xx} & E_{xy} \\ E_{yx} & E_{yy}\end{vmatrix}.$$

Starting from an arbitrary quoted price, in order to approach the temporary equilibrium price, when we quote prices successibly i.e., in order to satisfy the relations

$$p_x(\infty)=p_x^0, \quad p_y(\infty)=p_y^0$$

the inequalities $|\lambda_x|<1$, $|\lambda_y|<1$

must hold. Taking the intervals of operational time short enough, we can make the price flexibilities H, K as small as we want. $(H, K>0.)$ Therefore, in this case the above inequalities become

$$\frac{\begin{vmatrix} E_{xx} & E_{xy} \\ E_{yx} & E_{yy} \end{vmatrix}}{E_{xx}}<0, \qquad \frac{\begin{vmatrix} E_{xx} & E_{xy} \\ E_{yx} & E_{yy} \end{vmatrix}}{E_{yy}}<0.$$

In the above section, taking all other things as given, and changing only the quoted price of X, we got the stability condition of first order, $E_{xx}<0$ i.e., the condition that the quoted price approaches the temporary equilibrium price. Here I got the condition that the set of quoted prices approaches to the temporary equilibrium prices. We call this condition as the stability condition of second order. We must note about the assumption that, when we derive the stability condition of the second order for the market of X, we assumed that the stability conditions of the first order for the market of X and Y are satisfied at arbitrarily quoted prices of X and Y. I.e., in order to satisfy the stability condition of the second order for the market of X, the stability conditions of the first order for the market for X and Y must be satisfied at arbitrarily quoted prices. In this case the stability conditions of the second order are expressed as

$$E_{ii}<0, \quad \begin{vmatrix} E_{ii} & E_{ij} \\ E_{ji} & E_{jj} \end{vmatrix}>0. \qquad (i, j=x, y)$$

Above we quoted the prices following the two rules:

(1) When t is even, keeping the price of X constant, we find a price of Y which equilibrates the market of Y.

(2) When t is odd, keeping the price of Y constant, we find a price of X, which equilibrates the market of X.

We quote the set of prices thus determined, in the both markets successively. Therefore, when we look at the markets only when t is even, market of Y is always in equilibrium. In this sense, we can say that price of Y is quoted so as to equilibrate the market of Y. The same thing can be said about the market of X, if we look at the markets only when t is odd.

IV

Let us try to represent the above discussion graphically. Measure the price of X on the horizontal axis and the price of Y on the vertical axis. Any point on this plane shows a set of (quoted) prices. Now quote the price of X arbitrarily. Keeping it constant, we can find a price of Y, which will equilibrate the market of Y. Represent this set of prices by a point in our price plane. Similary, when we keep another (arbitrary) price of X constant, we can find a price of Y, which will equilibrate the market of Y. And again we represent this set of price by a point in our price plane. These sets of prices form a curve YY' in the plane. All of these sets of prices equilibrate the market of Y, therefore they satisfies the relation $E_y[p_x, p_y]=0$. We are thinking about the prices which lie very close to the equilibrium price, so we can regard the curve YY' as a straight line. From the second equation of (4), we get the slope of this line. (messured from the p_y axis).

$$\frac{p_x(t)-p_x^0}{p_y(t)-p_y^0}=-\frac{E_{yy}}{E_{yx}}. \qquad (t: \text{even})$$

Here we assume, as usual, the market of Y is stable of the first order, i.e., when we raise the quoted price of Y, keeping the price of X constant, the excess demand for Y decrease, $E_{yy}<0$. And then, if Y is substitute for X (including income effect) i.e., $E_{yx}>0$, then the slope of YY' is positive, and when it is complimentary with X (including income effect) i.e., $E_{yx}<0$, the slope of it is negative.

Y is substitute for *X*. *Y* is complimentary with *X*.

Fig. 3.

Exact similarly, the set of prices which equilibrates the market of X will satisfy the relation $E_x=0$. When we express this relation approximately by a line XX', then the slope of this line is obtained by our first equation I.e.,

$$\frac{p_x(t)-p_x^0}{p_y(t)-p_y^0}=-\frac{E_{xy}}{E_{xx}}, \qquad (t: \text{odd})$$

is the slope of XX'. When the market of Y is in stable of the first order $E_{xx}<0$, the slope of XX' is positive when X is substitute (including income effect) for Y, and negative when X is complimentary with Y. (including income effect).

X is substitute for *Y*. *X* is complimentary with *Y*.

Fig. 4.

Consider the case where XX' is rising. i.e., $E_{yx}>0$. Now at the T-th time (assume T is even) we quote price of X as $p_x(T)$, and here we assume that $p_x(T)>p_x^0$ (We can treat exactly similarily, the case where $p_x(T)<p_x^0$). As before, keeping this price constant, find a price of Y which will eqilibrate the market of Y, and call this as $p_y(T)$. The set of prices, $p(T)=[p_x(T),\ p_y(T)]$ is on YY'. But when we quote this set of prices, on the market of X, there will usually be negative excess demand. Therefore, in order to equilibrate market of X, a lower price of X must be quoted. $p_x(T+1)<p_x(T)$. This set of prices $p(T+1)=[p_x(T+1),\ p_y(T+1)]$, where $p_y(T+1)=p_y(T)$, lies on the line XX'. On our price plane, draw a horizontal line through $p(T)$, then $p(T+1)$ is the intersection of this line and XX'. We can calculate the difference of the slope of XX' and YY' from the two equations shown in the preceeding page.

$$\frac{p_x(T+1)-p_x^0}{p_y(T+1)-p_x^0}-\frac{p_x(T)-p_x^0}{p_y(T)-p_y^0}=-\frac{\begin{vmatrix} E_{xx} & E_{xy} \\ E_{yx} & E_{yy} \end{vmatrix}}{E_{xx}E_{yx}}. \quad (T\text{: even})$$

By assumption, $E_{yx}>0$. Therefore, the stability condition exactly corresponds to the difference of the slopes of XX'. In other words, when we quote the sets of prices following our rule, the stability condition is expressed by the difference of the slope of XX' and YY'.

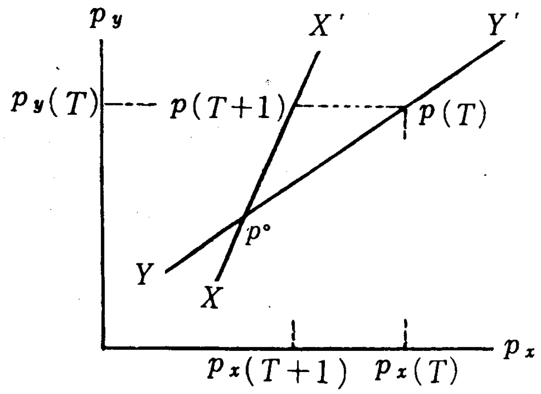

Fig 5.

Now let us consider the above case with the help of Figure 5. At T-th time (assume T is even) we quoted the price of X as $p_x(T)$, and here we assume that $p_x(T) > p^0$. The set of prices $p(T) = [p_x(T), p_y(T)]$ is on YY'. And new set of prices $p(T+1) = [p_x(T+1), p_y(T+1)]$, where $p_y(T+2) = p_y(T)$, lies on the XX'. When we draw a horizontal line through $p(T_y)$, then $p(T+1)$ is on the intersection of this line and XX'.

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