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NET- *versus* GROSS-APPROACH IN MACRO-DYNAMIC ECONOMIC ANALYSIS

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It is a convention which is followed in most of contemporary economic analysis to treat macroscopic economic relations in *net terms*. This mode of investigation, which we shall call the *net*-approach, seems to be inherited from Keynes' technique in his *General Theory*. Keynes, however, applied it to static analysis, while proponents of *Keynesian Economics* have extended it to dynamic theories. Though with a few exceptions, this extension of application has been thought as self-evidently valid and seldom questioned. This paper is concerned with this problem.

I

We, at first, define some economic relations, using the following notations

D .....total demand
Y^* .....gross output (gross income)
Y .....net output (net income)
C .....consumption
S .....net saving
G .....total investment demand
I .....new investment outlay
R .....reinvestment
K .....gross stock of capital (at the beginning of the period)
W .....depreciation of capital [1]
H .....part of K which can operate for one more period
$Q_{(\tau)}$ .....part of new capital equipment which can operate for τ periods

These items are reckoned in terms of constant price. And subscript t is used to mean the t-th period.

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[1] Definitionally, it contains the wear and tear of materials.

Total demand is composed of consumption and investment demand, while gross income is divided into consumption and gross saving, which in turn consists of net saving and depreciation of capital. So

$$(1) \qquad D_t \equiv C_t + G_t,$$

$$(2) \qquad Y_t^* \equiv C_t + (S_t + W_t).$$

Herin, depreciation is not exactly synonymous with decrease in value of capital, but it means also reserved value for the purpose of outlay in future. Next, since gross income substracted by depresiation is net income,

$$(3) \qquad Y_t \equiv Y_t^* - W_t.$$

We, also, define new investment as total investment minus reinvestment which is financed by amortization funds. Therefore

$$(4) \qquad G_t \equiv I_t + R_t.$$

According to the net-approach, the temporary equilibrium of market is defined as the equality of net output and demand. Generally, however, equilibrium in aggregated terms, if it is to be meaningful, must mean balance between total demand and total output, for it is considered to be the situation where both intended demand and planned supply are realized. Supplyers of commodities expect to make proceeds reckoned in grand total, and are not concerned with the usage of commodities, that is, whether they are used for consumption, net addition to capital or replenishment of equipment. Also, buyers of commodities decide volume of their purchase for their own calculation regardless of depreciation caused by production of goods which they would buy. Therefore, at any rate, natural is to define the temporary equilibrium as

$$(5) \qquad D_t = Y_t^*.$$

And, in so far as equilibrium defined in net terms is consistent with this difinition, it must mean

$$(6) \qquad D_t - W_t = Y_t.$$

Identities (1) and (4) show that demand consists of consumption, new investment and reinvestment, and that it does not contain depreciation itself. Depreciation is related to the side of producers, but not of buyers. So, though it may be possible to be calculated arithmetically, the left side of (6) can not be endowed with any economic meaning, without additional assumption.

Regarding (5), which means the temporary equilibrium, and identities (1), (2), (4), we obtain

$$(7) \qquad I_t - S_t = W_t - R_t.$$

Also, we are able to get (5) taking account of (7) and these identities. So (5) and (7) are equivalent. (7) can be taken as definition of the temporary equilibrium. Therefore the notion in the net-approach that when there is the temporary equilibrium new investment and net saving are equal is valid only if $W_t = R_t$. And, in so far as this equation holds good, the left side of (6) is no more meaningless.[2] However, if depreciation is not equal to reinvestment, new investment must deviate appropriately from net saving for equilibrium. Thus, we can suppose that the net-approach does assume the equality of depreciation and reinvestment, at least concerning equilibrium. This presupposition may not be so unreal for non-durable capital goods, in so far as entrepreneurs replenish wear and tear of them. But it is not applicable to durable capital goods, for generally they do not loose their capacity proportionally to their age. So, even if productive capacity of equipment is maintained intact, depreciation and outlay for replenishment do not be equalized. This fact makes it necessary to examine the supposition, with results of it, in the net-approach that they are always identical.

II

As explained in the preceding section, when there can be discrepancy between depreciation and reinvestment, the notion of the temporary equilibrium defined in net terms is somewhat vague and must be amended. We, now, examine how this amendment affects the analysis of income-effect investment.

For expediency, we assume as follows, though these postulates are not so rigid that some of them could be altered in appropriate mode.

1 Capital goods continue to have constant productivity during whole periods of their life time, the length of which is not affected by the amount of their products.
2 Depreciation is calculated by means of the straight line method.
3 Brand-new capital goods begin operating in the following period of their installation.
4 Orders for reinvestment are made in the last period of life of

[2] Some writers, preferring the *gross*-approach to the *net*-approach, consider the temporal equilibrium as the equality between gross output and the sum of consumption, new investment and depreciation. If, however, depreciation is not equal to reinvestment, this notion is not clear. And if they are equal, there is no reason for rejecting the net-approach.

the original capital goods.

5 Consumption is a linear function of net income of the preceding period.

6 Gross output is equal to total demand.

Postulate 1 is so-called "assumption of one-horse-shay". Actually durable capital goods may become ineffecient as they be timeworn. Brand-new equipment may have more productivity than old one. Then renewal of equipment may increase capacity, even if there is no technological progress. We, however, negect such possibility for simplicity. Moreover, this postulate may not be applied to non-durable capital goods. However, as mentioned in the preceding section, there is no discrepancy between depreciation and reinvestment as far as we are concerned with non-durable goods if wear of them is replenished. So we can safely neglect this category of capital without affecting our argument.

Postulate 2, with 3, allows us to suppose that

$$(8) \qquad W_t=\sum_{\tau=1}^{m}\sum_{\rho=1}^{\tau}\frac{1}{\tau}Q_{(\tau)t-\rho},$$

where m is the longest life time of equipment.[3]

Postulate 5 means

$$(9) \qquad C_t=cY_{t-1}+B.$$

Herein c stands for marginal propensity to consume, and B is a constant.

Assuming Postate 6, we are able to restrict our analysis only to the temporary equilibrium, supposing (5) and (7) to hold always good. The reason for this restriction follows what we have mentioned in the preceding section.

By the way, we have following definite relations;

$$(10) \qquad K_t=\sum_{\tau=1}^{m}\sum_{\rho=1}^{\tau}Q_{(\tau)t-\rho},$$

$$(11) \qquad H_t=\sum_{\tau=1}^{m}Q_{(\tau)t-\tau}.$$

We, now, inquire how a given autonomous investment demand affects each item. We shall reckon all items as deviation from their stationary level per unit of autonomous investment.

(1), (2), (5), (6) and (9) give us

$$(12) \qquad Y_t=cY_{t-1}+G_t-W_t.$$

If depreciation allowance is never reserved in order to outlay in future,

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[3] Capital goods which can operate only for one period, also, may be classfied as durable if our postulates are valid to them.

W and R are zero, and (12) and (4) give a ordinary model of investment multiplier.[4,5] This is a special case in which depreciation and reinvestment are equal. We, here, assume all capital goods once installed are maintained for ever by replemishment. So we have, considering Postulate 4,

$$R_t=H_t. \tag{13}$$

Let us suppose now

7a Each part of capital equipment can operate for θ periods.

Then

$$Q_{(\tau)t}=\begin{cases}G_t & (\tau=\theta)\\ 0 & (\tau\neq\theta),\end{cases} \tag{14}$$

and, inserting it into (10) and (11) we obtain

$$K_t=\sum_{\rho=1}^{\theta}G_{t-\rho}, \tag{15}$$

$$H_t=G_{t-\theta}. \tag{16}$$

Also we get, considering (13) and (16),

$$R_t=G_{t-\theta}, \tag{17}$$

and, taking account of (8), (14) and (15),

$$W_t=\frac{1}{\theta}K_t. \tag{18}$$

If we assume that there appears autonomous investment only in period 0, that is,

$$I_t=\begin{cases}1 & (t=0)\\ 0 & (t\neq0),\end{cases} \tag{19}$$

then we get, regarding also (4), (17) and (18),

$$R_t=\begin{cases}1 & (t=n\theta)\\ 0 & (t\neq n\theta)\end{cases}\qquad (n=1,\ 2,\ 3.....), \tag{20}$$

$$W_t=\frac{1}{\theta}\qquad (t\geqq1). \tag{21}$$

Therefore, unless θ is unity, depreciation and reinvestment are not equal in every period. Comparing sum of depreciation and total of reinvestment every θ period, we should observe both to be equal. But usually

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[4] See, for example, J. R. Hicks, *A Contribution to the Theory of the Trade Cycle*, pp. 11-23, 170-171.

[5] Prof. Takata has shown, in his work in 1955, that the theory of investment multiplier is applicable only to government expenditures which are not reinvested in subsequent periods.

such lumping of periods may not be legitimate in sequential analysis which aims to explain causal relations chronologically.

The time path of net income is

$$(22a) \qquad Y_{n\theta}=Y_0+A\frac{1-C^{n\theta}}{1-C^{\theta}} \qquad (n=1,\ 2,\ 3.....),$$

$$(22b) \qquad Y_{n\theta+t_1}=c^{t_1}Y_{n\theta}-\alpha(1-c^{t_1}) \qquad (n=0,\ 1,\ 2\ .....,\ 0\leqq t_1\leqq\theta-1),$$

where

$$\alpha=\frac{1}{\theta(1-c)}$$

$$A=1-(1-c^{\theta})(1+\alpha)\ .^{6}$$

If a given amount of autonomous investment appears in every period, we have, instead of (19),

$$(23) \qquad I_t=1 \qquad (t\geqq 0).$$

Then follow

$$(24) \qquad R_{n\theta+t_1}=\begin{cases} 0 & (n=0,\ t_1=0) \\ n & (n=0,\ 0<t_1\leqq\theta-1 \text{ or } \\ & \quad n=1,\ 2,\ 3\ .....,\ 0\leqq t_1\leqq\theta-1), \end{cases}$$

$$(25) \qquad W_t=\frac{t}{\theta} \qquad (t\geqq 0).$$

So reinvestment does not exceed depreciation in each period and becomes equal to the latter every θ period.

The path of net income in this case is

$$(26a) \qquad Y_{n\theta}=Y_0+B\frac{1-c^{n\theta}}{1-c^{\theta}} \qquad (n=1,\ 2,\ 3\ .....),$$

$$(26b) \qquad Y_{n-t\theta_1}=c^{t_1}Y_{n\theta}+\alpha\theta(1+\alpha c)(1-c^{t_1})-\alpha t_1 \qquad (n=0,\ 1,\ 2\ .....,\ 0\leqq t_1\leqq\theta-1),$$

where α is as shown above and

$$B=c_\theta+\alpha c[\theta(1+\alpha c)(1-c^{\theta-1})-(\theta-1)].$$

Thus, in both cases shown above on Postulate 7a, reinvestment and depreciation are not identical due to endogeneous causes.

Next we turn to suppose that

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6 Here we have cyclical fluctuations of income with period θ. This shows so-called echo-principle. See J. Einarsen, "Reinvestment Cycles," *Review of Economic Statistics*, 1938, pp. 1-10, reprinted in *Readings in Business Cycles and National Income*, ed. by A. H. Hansen and R. V. Clemence, pp. 293-309. and J. Tinbergen and J. J. Polak, *The Dynamics of Business Cycles*, pp. 157-158, 176 ff.

7b Probability that capital goods operate for τ periods is $p(\tau)$. Since, hereon,

$$Q_{(\tau)t}=p(\tau)G_t\,, \tag{27}$$

we have, taking account of (10) and (11),

$$K_t=\sum_{\tau=1}^{m}\sum_{\rho=1}^{\tau}p(\tau)G_{t-\rho}\,, \tag{28}$$

$$H_t=\sum_{\tau=1}^{m}p(\tau)G_{t-\tau}\,. \tag{29}$$

Thus, if autonomous investment appears only in the period 0, equations (4), (13), (19) and (29) give us the time path of total investment which is also the path of reinvestment for $t>0$. This path is shown as

$$G_t=\sum_{\tau=1}^{m}p(\tau)G_{t-\tau}\,, \tag{30}$$

It can be verified that investment G, therefore reinvestment R, does converge to a finite level.[7,8] Insertion of this limit value into

$$W_t=\sum_{\tau=1}^{m}\sum_{\rho=1}^{\tau}\frac{1}{\tau}p(\tau)G_{t-\rho}\,, \tag{31}$$

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[7] The characteristic equation of (27) is

$$f(\lambda)\equiv\lambda^m-p(1)\lambda^{m-1}-p(2)\lambda^{m-2}-\cdots\cdots-p(m)=0,$$

where $p(m)>0$, $p(\tau)\geqq 0$ and $\sum_{\tau=1}^{m}p(\tau)=1$. Clearly, as the Descartes' rule of signs shows the number of positive real roots of this equation is only one (single), and this positive root is unity. On the other hand, roots of this equation are also eigenvalues of non-negative matrix

$$\mathbf{M}\equiv\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & & \vdots \\ \vdots & \vdots & \vdots & & 1 \\ p(m) & p(m-1) & p(m-2) & \cdots & p(1) \end{pmatrix},$$

and *vice versa*. Moreover, as far as $p(m)>0$ and at least one of $p(\tau)$ $(\tau\neq m)$ is not zero, **M** is *acyclic* and therefore *primitive*. Thus it is affirmable that any root of $f(\lambda)=0$ other than unity is less than unity in modulus. (R. M. Solow, "On the Structure of Linear Models," *Econometrica*, 1952, p. 40, G. Frobenius "Über Matrizen aus nicht negativen Elementen," *Sitzungsberichte der Koniglichen Preussischen Akademie der Wissenschaften*, 1912, S. 456.) Also see J. R. Hicks, *loc cit.*, pp. 180-182, W. Feller, *An Introduction to Probability Theory and Its Application*, vol. 1, pp. 243 ff., 272-277.

[8] Mr. Hicks (*lic cit.* p. 42) thought, on Postulate 7b, that it would be allowed to neglect reinvestment cycles. However, because that investment appears more or less concentrating at a phase of business cycles and also the tapering of oscillations of reinvestment takes a much of time, we should not be able to neglect reinvestment cycles in a study of business fluctuatious.

which is obtained through (8) and (27), shows that *in limit* depreciation and reinvestment become equal. It is easily observed that net income eventually goes back to its original level. Thus, in this case, we get *in limit* the same result as the net-approach shows.

The situations we have got on Postulate 7a is somewhat different from what occur in the net approach. But the former does not contradict with the latter. If we assume autonomous investment in every period which is equal to the difference of reinvestment and depreciation, and neglect both reinvestment and depreciation, then we should have the same time path of various items as what has been shown above, by course of the net-approach. Only difference is in that what we dealt as endogeneous causes are now considered to be exogeneous. However, this difference would bring out further variations, if induced investment were taken account of, in the studies in economic fluctuations.

III

Dynamization of *Keynesian Economics* may be said to have started on the recognition of duality of investment. Investment, on the one hand, constitutes part of demand, and so gives an impules to output and income, and, on the other hand, increases the stock of capital, and, therefore, capacity of production. Such view is accepted generally of investment in net terms. Total investment, as mentioned above consists of new investment and reinvestment. Also it may, from a different viewpoint, be classfied into two categories; net investment and replacement. The former means investment for the purpose of increase in capacity of production and the latter for maintenance of it. In the net-approach, investment is generally considered to correspond to new investment, on the one hand, and to net investment, on the other hand. However, these two items do not necessarily coincident. When an amount of outlay becomes able to purchase more efficient equipment than ever due to, say, technological progress, reinvestment brings about augmentation in capacity, so it contains not only replacement but net investment.[9] On the contrary, if capital goods are more expensive, it may impossible to purchase, for amortization funds, equipment as of productivity as original one. Then, in such case, to replacement there corresponds both reinvestment and new investment. Therefore invest-

[9] Cf. A. Murad, "Net Investment and Industrial Progress," in *Post Keynesian Economics*, ed. by K. K. Kurihara, pp. 227-250.

ment in net terms is well defined only when a given amount of outlay is always just enough to obtain equipment of the same productivity.

We, however, no more concerned with this problem, and turn to a comment on continued equilibrium growth of economy, where not only the temporary equilibrinm but also the equality of actual output and normal output holds. Of course the constancy of the rate of growth may not always possible. In order that it is possible, for example, the following postulates may be sufficient;

8 The ratio of normal output to the stock of capital is constant.

9 If in a period equilibrium is realized, entrepreneurs intend to adjust their capital so as to maintain the rate of growth of output of the preceding period.

Herein, we must recognize these postulates in *gross* terms. For Postulate 8 is concerned with technological relation, in which total output and operating stock of capital must be connected, and owing to Postulate 1 capacity of capital stock is reckoned in gross terms. It is also clear due to our discussion in section **I** that Postulate 9 must be considered in gross terms.

If depreciation is proportional to gross stock of capital in each period, the recognition of Postulate 8 and 9 in net terms may result in the same conclusions as when they are considered in gross terms. And if gross income grows at a constant rate, net income increases at the same rate. For identity (3) tells us

$$g_t^* - w_t = n_t(g_t - w_t), \tag{32}$$

where g^* g and w are respectively the rate of growth of gross income, net income and depreciation and n is the ratio of net income to gross income. On Postulate 8 when equilibrium growth is realized, the rate of growth of gross capital stock is g^*. and if the proportionality of depreciation and capital stock is assumed then the left side of (32) is zero, Therefore g is equal to g^*. It may be observed that on Postulate 7a depreciation is proportional to gross capital stock. So on this postulate it is not illigitimate to consider equilibrium growth in net terms, though the vaguness of difinition of equilibrium due to the inequality of depreciation and reinvestment, must be reminded.[10] However, if different assumption is made on the life time of capital equipment, this does not necessarily fit.

At any rate, in so far as we must consider equilibrium, the reaction

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[10] Cf. E. D. Domar, "Dpreciation, Replacement, and Growth," *Economic Journal*, 1953, pp. 1-32, reprinted in his *Essays in the Theory of Economic Growth*, pp. 154-194.

of entrepreneurs and so on, at first, in gross terms, the analysis of economic growth must find its root defined in gross terms, even if it can be explained in net terms. Discussions in net terms is valid only if it corresponds to the analysis in gross terms.

Summarizing our argument, we may say that (1) various economic relations, with a few exceptions such as consumption function, must be, at first, reckoned in gross terms, and (2) even though there may be some cases where the net-approach gives valid conclusions, it is so only when this approach reflects legitimatly the economic relations which are well defined in gross terms. Cases shown in this paper are not more than simple examples for this assertion.